Raffman handout on sorites paradox
PIKSI workshop
NEU, Boston, July12, 2018

● Vagueness = possession of blurred boundaries of application.

● Tolerance = property of applying across incremental changes on a decisive dimension.

● Tolerance leads to the *sorites paradox* (a.k.a. "paradox of the heap").

Sorites paradox for 'old'

Consider a series of ages (of human beings) proceeding from 90 years to 10 years, ordered so that each age is one day younger than its predecessor. Presumably a difference of one day can't make the difference between being old and being not old. But if that's right, then it looks like we can generate the following simple and devastating argument, using *Modus Ponens*:

90 years is old.
If 90 is old, then 90 minus 1 day is old. (Tolerance)
If 90 minus 1 day is old, then 90 minus 2 days is old.
If 90 minus 2 days is old, then 90 minus 3 days is old.
Etc.
Therefore, 10 years is old.

But 10 years is not old, so we have a contradiction: 10 years both is, and is not, old. Where have we gone wrong? The argument is a paradox because seemingly impeccable reasoning from seemingly impeccable premises leads to a falsehood. This result suggests that the word 'old' is incoherent; its use leads to contradictions. But can such an ordinary word, which we use successfully all the time, really be incoherent? How exactly should we respond to the paradox?

There have been many attempts to solve the sorites paradox, i.e., many attempts to show that the argument is in fact fallacious: something is wrong either with the reasoning or with the premises. Often these efforts require use of a special, non-classical logic and/or semantics. We will discuss some of these at the workshop.

Raffman, handout on borderlines for PIKSI sessions on vagueness
NEU, July 12

- Standard Analyses of borderline cases:
  (i) Neither definitely Φ nor definitely not Φ
  (ii) '*x* is Φ' is neither true nor false (or: neither def true nor def false)

- On semantic theories of vagueness, borderlines are taken to be *indeterminate* with respect to being Φ; there is "no fact of the matter" as to whether *x* is Φ.
  Then '*x* is Φ' takes an intermediate truth-value, or *x* falls into a truth-value *gap*.

- Other analyses that have been proposed:
  (i) Borderlines are items about which we are uncertain as to whether they are Φ.
  (ii) Borderlines are items that can permissibly be classified either as Φ or as not Φ.
  (iii) Borderline cases are "hard cases" arising from open texture.

- From the literature:

> [V]agueness is [a matter of] lacking "sharp boundaries", of dividing logical space as a blurred shadow divides the background on which it is reflected….[This] figure equally exemplifies the idea of the borderline case, a region falling neither in light nor [in] shadow (Wright 1976, 226).

> [T]he vagueness of a vague predicate is ineradicable. Thus 'hill' is a vague predicate, in that there is no definite line between hills and mountains. But we could not eliminate this vagueness by introducing a new predicate, say 'eminence', to apply to those things which are neither definitely hills nor definitely mountains, since there would still remain things which were neither definitely hills nor definitely eminences, and so *ad infinitum* (Dummett 1978, 182).

> [The] concept of a borderline case is the concept of a case that is neither definitely in nor definitely out (Tye 1994b, 18).

> (Sainsbury 1997, 264)….[Some theorists] represent the sense of a predicate like 'green' or 'child' by its effecting a division of categorially appropriate objects into three sets. This is supposed to do justice to the actuality or possibility of borderline cases: surfaces intermediate between blue and green, people intermediate between childhood and adulthood (*ibid.*).

> [A] borderline case is an object that is neither definitely *F* nor definitely not *F*…Tarmin may have enough canine characteristics to ensure that she is definitely either a dog or a wolf, without being either definitely a dog or definitely a wolf (McGee and McLaughlin 1994, 210).

• Incompatibilist analysis

(i) For any proximate incompatible predicates 'Φ' and 'Φ*', *x* is a Φ[Φ*] borderline case if and only if *x* belongs to a Φ/Φ* ordering but is neither Φ nor Φ*.
(ii) For any predicate 'Φ', *x* is a borderline case for 'Φ' if and only if there is some proximate incompatible predicate 'Φ*' such that *x* is a Φ[Φ*] borderline case.
'*x* is Φ' is false and '*x* is not Φ' is true.

• Some additional shortcomings of the standard analysis.
(i) Two kinds of borderline cases?
(ii) Aspects of borderlines that cannot be captured without incompatibles.
(iii) Failure to capture symmetry between the two categories.

• Problems with the very idea of borderline cases for 'not-Φ'
Can find these only by locating BΦ.
B-cases of not-heap, not-bald. Being not-red doesn't look like anything.

Arguments commonly offered in support of the standard analysis:

(1) "[A] borderline case of the predicate *F* is equally a borderline case of not-*F*: it is unclear whether or not the candidate is *F*. This symmetry prevents us from simply counting a borderline *F* as not-*F*" (Keefe and Smith 1997, 7). Call this the 'argument from symmetry'.

(2) If borderline cases were not-Φ, then their status with respect to 'Φ' (and 'not-Φ') would not be indeterminate (indefinite, unclear, uncertain). There would be a "fact of the matter": borderline cases are not-Φ. Such a result runs counter to the very nature of borderline cases. Call this the 'argument from indeterminacy'.

(3) Even if we could say that borderline cases for 'Φ' are not-Φ, this would only postpone the inevitable. For then new, higher-order borderline cases would arise between the Φ items and the not-Φ (*viz*., borderline) items in a Φ-ordering that defines borderline cases for 'Φ'. And these new borderline cases would have to be neither-definitely-Φ-nor-definitely-not-Φ. Thus we would arrive at the standard analysis at one remove, as it were. Russell writes:

> Someone might seek to obtain precision in the use of words by saying that no word is to be applied in the penumbra, but unfortunately the penumbra itself is not accurately definable, and all the vaguenesses which apply to the primary use of words apply also when we try to fix a limit to their indubitable applicability" (1923, 87).

Call this the 'argument from higher-order borderline cases'.

(4) In judging a borderline case, we are apprised of all the relevant facts; in other words, nothing is hidden from us. Yet we can't tell that '*x* is Φ' is true and can't tell that '*x* is Φ' is false. Therefore the sentence must be neither true nor false, since if it were true we could tell that it was true and if it were false we could tell that it was false. Call this the 'argument from accessibility', the idea being that since no relevant facts are hidden, the truth-value of 'x is Φ', if it had one, would always be accessible to competent users of the predicate.