(Some More) Vagueness

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In these notes, I made extensive use of Rosanna Keefe's excellent book: *Theories of Vagueness* (Cambridge University Press, 2000).

Three features of vague predicates: (a) borderline cases

- It is common to think of vague predicates as involving three features (Keefe [2000], pp. 6-8):
- (a) **Vague predicates admit *borderline cases***: cases in which it is unclear whether the predicate applies or not.
- An example: Otávio is borderline bald --- not clearly bald and not clearly not bald.
- There does not seem to be a fact of the matter as to whether Otávio is bald. No amount of information about the exact number of Otávio’s hair seems to help one decide whether he is bald. It is indeterminate whether this is the case.
- This seems to violate bivalence: ‘Otávio is bald’ is neither true nor false.
- This seems to violate excluded middle: ‘Either Otávio is bald or he is not bald’ does not seem to hold either.

Three features of vague predicates: (b) no sharp boundaries

- (b) **Vague predicates *lack* (or *seem to lack*) *sharp boundaries***: they seem to lack well-defined extensions. There does not seem to be a sharp boundary between bald people and non-bald ones.
- If one considers a line of people ordered by the number of their hair, no sharp line can be drawn on the cases in which ‘bald’ applies. The predicate seems to have fuzzy boundaries.
- This seems to violate classical logic’s requirement that all predicates have well-defined extensions, that they do not involve fuzzy boundaries.
- The lack of sharp boundaries is closely connected with the presence of borderline cases:
- (i) The lack of sharp boundaries between bald and non-bald seems to yield a region of borderline cases of being bald (the so-called *penumbra*).
- (ii) If the range of borderline cases between bald and non-bald were sharply bounded, ‘bald’ would have a sharp boundary as well.

Three features of vague predicates: (c) sorites paradoxes

- (c) **Vague predicates are susceptible to *sorites paradoxes***: One hair cannot make a difference as to whether a person is bald or not: such a minuscule deviation is too small to matter.
- (T) If a person *A* is not bald, and another person *B* has one less hair than *A*, then *B* is not bald either.
- Consider a line of people, starting with someone full of hair, and each person on the line with one less hair than the previous person.
- By repeatedly applying (T), one would conclude that each person on the line is not bald, no matter how down the line one goes.
- This entails that a person with no hair at all is not bald, which is undeniably false.

Sorites paradoxes

- The Sorites paradox can be stated in at least two forms (Keefe [2000], pp. 18-19):
- *First form*: Suppose that x_i is a sequence of objects [such as, (adult) people a hundredth of an inch shorter than the previous ones, and x_1 is 7 feet tall], and suppose that F is a predicate [such as 'is tall'], so that both (1) and (2) are true:
- (1) Fx_1
- (2) For every i, if Fx_i, then Fx_{i+1}
- However, for some n [suppose that x_n is 4 feet tall], (3) is clearly false:
- (3) Fx_n
- But the conclusion is clearly absurd!

Sorites paradoxes

- *Second form*: The first premise (1) is the same, but the second is replaced by a series of particular conditionals:
- (1) x_1 is tall.
- ($2C_1$) If x_1 is tall, then x_2 is tall too.
- ($2C_2$) If x_2 is tall, then x_3 is tall too.
- ($2C_3$) If x_3 is tall, then x_4 is tall too, and so on.
- However, once again, a clearly false conclusion follows:
- (3) A four-feet person is tall.

Responses to the sorites paradox

- Responses to the sorites paradoxes can be grouped into (at least) four approaches (Keefe [2000], p. 19-25):
- (A) One can question the validity of the argument, and insist that the truth of the premises does not guarantee the truth of the conclusion.
- (B) One can deny the second premise of the argument: either by questioning the truth of the quantified inductive premise (2) or by denying at least one of the particular conditionals ($2C_i$).
- (C) One can question the truth of premise (1).
- (D) One can embrace the validity of the argument and the truth of the premises, and insist that this establishes the incoherence of the relevant predicate.

Responses to the sorites paradox: (A) questioning validity

- (A) One can question the validity of the sorites argument. There are three ways of doing that:
- (A1) In the many-conditionals version of the sorites paradox, one can deny the validity of modus ponens (for a discussion, see Dummett [1975] and Keefe [2000], p. 20).
- (1) x_1 is tall.
- ($2C_1$) If x_1 is tall, then x_2 is tall too.
- ($2C_2$) If x_2 is tall, then x_3 is tall too.
- ($2C_3$) If x_3 is tall, then x_4 is tall too, and so on.
- Dummett [1975] does *not* recommend this option, since he takes the validity of modus ponens as constitutive of the meaning of the conditional.
- But is this the case?

Responses to the sorites paradox: (A) questioning validity

- There are violations of modus ponens (see Van McGee [1985]). The 2016 US presidential election provides the context for a counterexample (see Bueno [2018] which adapts and update Van McGee's original argument):
- (P_1) If a Republican wins the election, then it if is not Donald Trump who wins, it will be Ted Cruz.
- (P_2) A Republican will win the election.
- (C) If it is not Donald Trump who wins, it will be Ted Cruz.
- The premises are true, but the conclusion is false: if it is not Donald Trump who wins the election, it will be Hillary Clinton who does. (In fact, Clinton won the popular vote by over 3 million votes!)
- If modus ponens is not generally valid, there is no reason to think that its validity is constitutive of the meaning of the conditional.
- The conditional (C) above is perfectly intelligible (and clearly false!) despite the invalidity of modus ponens.

Responses to the sorites paradox: (A) questioning validity

- (A2) In the quantified version of the sorites paradox, one can deny the validity of universal instantiation, in addition to the validity of modus ponens (for a discussion, see Dummett [1975] and Keefe [2000], p. 20).
- (1) Fx_1
- (2) For every i, if Fx_i, then Fx_{i+1}
- (2') If Fx_1, then Fx_2 (by universal instantiation from (2))
- Dummett [1975] similarly does *not* recommend this option, since he takes the validity of universal instantiation as constitutive of the meaning of the conditional.
- But is this the case?

Responses to the sorites paradox: (A) questioning validity

- There are violations of universal instantiation.
- According to some interpretations of quantum mechanics (favored by Schrödinger and Weyl), quantum particles (such as electrons) are such that identity cannot be applied to them (see French and Krause [2006]).
- If two electrons, Ike and Mike, are in the same quantum state and they are swapped around, the quantum states they are in does not change.
- This means that there is no alibi for an electron *a*: nothing in the quantum mechanical description settles the issue of whether Ike or Mike were involved in the swap (Weyl [1931]).
- Hence, the inference below, from (P) to (C) via universal instantiation, is invalid:
- (P) For all x, $x = x$.
- (C) $a = a$
- After all, on this interpretation of quantum mechanics, identity cannot be applied to an electron *a*.

Responses to the sorites paradox: (A) questioning validity

- (A3) In the many-conditionals version of the sorites paradox, one can deny the transitivity of the conditional (for a discussion, see Dummett [1975] and Keefe [2000], p. 20).
- (1) x_1 is tall.
- ($2C_1$) If x_1 is tall, then x_2 is tall too.
- ($2C_2$) If x_2 is tall, then x_3 is tall too.
- ($2C_3$) If x_3 is tall, then x_4 is tall too, and so on.
- Dummett [1975] similarly does *not* recommend this option, since this amounts to the denial of the transitivity of validity.
- But is this the case?
- It seems that the sorites paradox is a clear counterexample to the transitivity of the conditional! (Of course, one would then need to account for where the conditionals fail and why each instance of (2C*i*) seems so plausible.)

Responses to the sorites paradox: (B) questioning the second premise

- (B) One can deny the second premise of the sorites argument (Keefe [2000], pp. 19 and 21): either by questioning the truth of the quantified inductive premise (2) or by denying at least one of the particular conditionals ($2C_i$).
- (B1) In a classical context, to deny the second premise of the sorites argument amounts to provide a situation in which 'Fx_i' is true but 'Fx_{i+1}' is not (so that the conditional fails).
- This approach is implemented by epistemic theories of vagueness, which need to explain why vague predicates do not seem to lead to sharp boundaries. Ignorance is typically invoked in this context (see Williamson [1994], and, for a critical discussion, Keefe [2000], Chapter 3).

Responses to the sorites paradox: (B) questioning the second premise

- (B2) In a non-classical context, several options are available (Keefe [2000], pp. 17-18).
- (i) *Dialetheism*: According to dialetheism, some contradictions are true (Priest [2006]), but due to the use of a paraconsistent logic (da Costa, Krause, and Bueno [2007]), not everything follows from such contradictions. (Thus, contradiction and triviality are clearly distinguished.)
- A borderline case of *F* is also a borderline case of non-*F*: it is unclear whether the object in question is *F* or not.
- For the dialetheist, in a borderline case, a predication is both true *and* false. It is a truth-value *glut* (Hyde [1997], and Keefe [2000], Chapter 7, section 7).

Responses to the sorites paradox: (B) questioning the second premise

- (ii) *Supervaluationism*: in the case of borderline predications, neither ‘*F*’ is true nor ‘not-*F*’ is true. Borderline cases involve *truth-value gaps*.
- Keefe ([2000], p. 17): “a proposition involving the vague predicate ‘tall’, for example, is true (false) if it comes out true (false) on all the ways in which we can make ‘tall’ precise (ways, that is, which preserve the truth-values of uncontentiously true or false cases of ‘*a* is tall’).
- “A borderline case, ‘Tek is tall’, will be neither true nor false, for it is true on some ways of making ‘tall’ precise and false on others.
- “But a classical tautology like ‘either Tek is tall or he is not tall’ will still come out true because wherever a sharp boundary for ‘tall’ is drawn, that compound sentence will come out true.
- “In this way, the supervaluationist adopts a non-classical semantics while aiming to minimise divergence from classical logic” (Keefe [2000], p. 17).

Responses to the sorites paradox: (B) questioning the second premise

- According to the supervaluationist, the second premise of the sorites argument, the quantified sentence ‘for every i, if Fx_i, then Fx_{i+1}’ is false.
- After all, for each way of making F precise, that is, for each F^* that preserves the uncontentious cases in which the predicate holds (or does not hold), there is a last x_i that is F^* and a first x_{i+1} that is *not*-F^*.
- However, there are no sharp boundaries, given that ‘Fx_i and not-Fx_{i+1}’ is *not* true on *all* ways of making F precise, and thus is neither true nor false (Keefe [2000], p. 21).

Responses to the sorites paradox: (B) questioning the second premise

- (iii) *Degree theories*: Borderline cases involve a third truth value ("indeterminate", "indefinite"): it is indeterminate or indefinite whether *a* is tall or not (Keefe [2000], p. 17).
- More generally, degree theories involve "a whole spectrum of truth-values from 0 to 1, with complete falsity as degree 0 and complete truth as degree 1.
- "Borderline cases each take some value between 0 and 1, with 'x is red' gradually increasing in truth-value as we move along the colour spectrum from orange to red. This calls for an infinite-valued logic or a so-called 'fuzzy logic'" (Keefe [2000], p. 17).
- In some of these degree theories, the second premise of the sorites argument and its negation both receive an indefinite value (Keefe [2000], p. 21). As a result, the premise is not true.

Responses to the sorites paradox: (B) questioning the second premise

- Other versions of degree theories also undermine premise 2 of the sorites paradox (Keefe [2000], pp. 21-22).
- They insist that the inductive premise, strictly speaking, is not true, although it is *nearly* true.
- The predications Fx_i involve gradually decreasing degrees of truth that range from full truth (degree 1) to full falsity (degree 0).
- If one considers consecutive predications, there is never a significant drop in the degree of truth from one Fx_i to another. (This accounts for the tolerance of the relevant vague predicate.)
- As a result, the conditional premise 'if Fx_i, then Fx_{i+1}' is not true in general, but it can be nearly true.

Responses to the sorites paradox: (C) questioning the first premise

- (C) One can question the truth of premise (1) of the sorites (Keefe [2000], p. 22-24).
- The sorites comes as an upward paradox or as downward one:
- (i) *Upward sorites* (S^+):
- (P_1) One grain of sand is not a heap.
- (P_2) If something is not a heap, adding one grain of sand to it will not turn it into a heap.
- Therefore, there are no heaps (no matter how large is the number of grains of sand that are piled up).
- Similar arguments, *mutatis mutandis*, can be used to conclude that there are no bald people, no tall people, etc.
- Thus, vague predicates lack serious application: either they apply to nothing ('is a heap') or they apply to everything ('is not a heap') (Keefe [2000], p. 22).

Responses to the sorites paradox: (C) questioning the first premise

- (ii) *Downward sorites* (S^-) (Keefe [2000], p. 23):
- (P_1) Ten thousand grains make a heap.
- (P_2) If something is a heap, removing a single grain from it still leaves a heap.
- Conclusion: a single grain of sand is a heap.
- The conclusion of the downward sorites (S^-) is clearly incompatible with the conclusion of the upward sorites (S^+), according to which there are no heaps.
- One could then deny the premise of the downward sorites in light of the upward sorites: if there are no heaps (conclusion of (S^+)), then it is false that ten thousand grains make a heap (first premise of (S^-)) (Keefe [2000], p. 23).
- *Alternatively*, one could use the conclusion of the downward sorites (S^-) to deny the premise of the upward sorites: if a single grain is a heap (conclusion of (S^-)), then it is not the case that one grain of sand is not a heap (premise of (S^+)).
- How could one choose which premise to deny?

Responses to the sorites paradox: (D) embracing the incoherence

- (D) One can embrace the validity of the Sorites argument and the truth of the premises, and insist that this establishes the incoherence of the relevant predicate (Dummett [1975], and Keefe [2000], pp. 24-25).
- According to Dummett, the sorites paradox makes explicit the incoherence of the rules governing the application of vague predicates: by following these rules, speakers may end up with contradictions.
- This means taking *both* the upward and the downward sorites at face value.

Responses to the sorites paradox: (D) embracing the incoherence

- Keefe's ([2000], p. 24) objection: "The acceptance of such pervasive inconsistency is highly undesirable and such pessimism is premature; and it is even by Dummett's own lights a pessimistic response to the paradox, adopted as a last resort rather than as a positive treatment of the paradox that stands as competitor to any other promising alternatives.
- "Communication using vague language is overwhelmingly successful and we are never in practice driven to incoherence […]. And even when shown the sorites paradox, we are rarely inclined to revise our initial judgement of the last member of the series.
- "It looks unlikely that the success and coherence in our practice is owed to our grasp of inconsistent rules" (Keefe [2000], p. 24).
- An account is indeed needed of why, despite the incoherence of vague predicates, one does not derive a contradiction from reasoning with such predicates.
- But Keefe's last point begs the question: a paraconsistent view can be used here.