# Conditionals

Ray Briggs
Stanford University

## Example Conditionals

1. If the Axiom of Choice is true, then every set can be well ordered.
2. You will probably get lung cancer if you smoke.
3. If the syrup forms a soft ball when you drop it into cold water, then it is between 112 and 115 degrees Celsuis.
4. If kangaroos had no tails, they would topple over.
5. When I rule this land, you will be sorry.

## Why Study Conditionals?

- **decision-making**: to figure out what to do, you should consider, for each action you might choose, what would happen if you chose it.

controversial definition of expected utility:

$$EU(A) = \Sigma_i\, P(A \rightarrow O_i)U(O_i)$$

## Why Study Conditionals?

- **science and philosophy of science**: when deciding which theory to believe, you should consider, for each theory, what evidence you should expect if it is true.
- **make-believe and imagination**: what if you could talk to dolphins…

# Why Study Conditionals?

- **philosophy**: conditionals are used to formulate philosophically important concepts.
- B *counterfactually depends* on A iff:
  - If A had occurred, B would have occurred.
  - If A had not occurred, B would not have occurred.

# Why Study Conditionals?

- **philosophy**: conditionals are used to formulate philosophically important concepts.
- A is a *cause* of B iff both occurred and…
  - B counterfactually depends on A?
  - B counterfactually depends on something that counterfactually depends on A?
  - (More complicated conditions can be formulated in terms of causal graphs.)

# Why Study Conditionals?

- **philosophy**: conditionals are used to formulate philosophically important concepts.
- Dispositions
  - If X were in condition C, it would show manifestation M.
- Example: fragility
  - If this vase were dropped, it would break.

# Why Study Conditionals?

- **philosophy**: conditionals are used to formulate philosophically important concepts.
- Being a law:
  - Say that S is a *stable set* iff, for every every *s* in S proposition *P* consistent with every *s* in *S*:

    If *P* had been the case, *s* would still be the case.
  - S is a law iff it is a member of some non-maximal stable set.

# Why Study Conditionals?

- **philosophy**: conditionals are used to formulate philosophically important concepts.
- S *knows* that P iff:
  - P is true.
  - S believes that P.
  - If P had been true, S would have believed it.
  - If P had not been true, S wouldn’t have believed it.

# Why Study Conditionals?

- **philosophy**: conditionals are used to formulate philosophically important concepts.
- S did A *freely* iff:
  - S did A on purpose, and
  - if S had wanted not to do A, then S would not have done A.

# Indicative vs. Counterfactual

INDICATIVE: If Oswald didn’t kill Kennedy, then someone else did.

COUNTERFACTUAL: If Oswald hadn’t killed Kennedy, then someone else would have.

# Indicative vs. Counterfactual

INDICATIVE: If Stephanie Kwolek didn’t invent kevlar, then we can’t use it in superconductors.

COUNTERFACTUAL: If Stephanie Kwolek hadn’t invented kevlar, then we wouldn’t be able to use it in superconductors.

## Indicative vs. Counterfactual

INDICATIVE: If Tupac is still alive, then every radio station in the country is playing his music.

COUNTERFACTUAL: If Tupac was still alive, then every radio station in the country would be playing his music.

## Features to Investigate

- **Truth conditions**: what does it take for a conditional to be true?
- **Logic**: which ways of reasoning with conditionals are valid?
- **Pragmatics**: what happens when you assert a conditional

## Common Inferences

- **Modus Ponens:** $A \rightarrow B, A \vdash B$
- **Pseudo Modus Ponens:** $A \rightarrow B \vdash \neg A \vee B$
- **Modus Tollens:** $\vdash A \rightarrow B, \neg B \vdash \neg A$
- **Conditional Proof:** If $A \vdash B$ , then $\vdash A \rightarrow B$
- **Deduction Theorem:** If $A1, A2, \ldots, An, B \vdash C$ , then $A1, A2, \ldots, An \vdash B \rightarrow C$
- **Import:** $A \rightarrow (B \rightarrow C) \vdash (A \wedge B) \rightarrow C$
- **Export:** $(A \wedge B) \rightarrow C \vdash A \rightarrow (B \rightarrow C)$

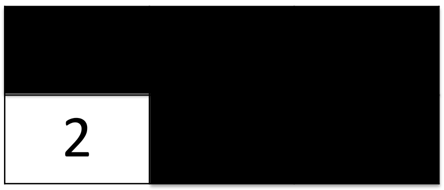
## Material Conditional

| **A** | **B** | **A → B** |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | T | T |

## Bad Results About Probability

Suppose you roll a fair die. Consider the probability of:

- *If you roll an even number, you roll a two*.
- The material conditional "you roll an even number → you roll a two"?




## Exercises

- Show that (13-18) on the handout are valid if the conditional is the Material Conditional, but not if it's the Strict Conditional.
- Find counterexamples to these conditional analyses.
  - A is <u>true</u> iff, if there were an omniscient God, God would believe A.
  - X is <u>red</u> iff, if a normal perceiver looked at X under good conditions, it would look red.
  - I <u>believe</u> that A iff, were you to ask me whether P, I'd say yes.
  - A is <u>true according to a fiction</u> iff, if all of the sentences in the fiction were true, A would be true too.
- "If you eat that mushroom, then you will die." Is this conditional indicative, counterfactual, or neither?

## The Strict Conditional