Paradoxes of Consistency & (Revising) The Logic of Belief

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[1]This talk draws heavily on joint with with Kenny Easwaran (USC).

- I will consider several (4) kinds of epistemic paradoxes, which seem to show that (classical) deductive consistency is *not* a rational (*formal, coherence*) requirement for belief.
- These paradoxes will be in (what I take to be) increasing order of difficulty. Only the final one involves self-reference. The first three involve (large-ish) *minimal* inconsistencies.
- After introducing the first three paradoxes, I will discuss two common types of responses to them (or responses to paradoxes like them) that have appeared in the literature.
- I will explain why I don't find those responses completely satisfying. And, this will lead to a new proposal for *revising* the classical "logic of belief" (*viz.*, classical coherence).
- I will present a new *framework* [8] for grounding formal coherence requirements. It is a *natural and non-ad hoc generalization* (to *belief*) of Joyce's [13, 12] framework for grounding epistemic coherence requirements for *credence*.

- Before we dive in — some notation and basic assumptions.
  - $B(p) \stackrel{\text{def}}{=} S$ believes that p. $D(p) \stackrel{\text{def}}{=} S$ disbelieves that p.

☞ We will be using *simple, idealized models* of epistemic agents (*viz.*, epistemically rational agents). As with all models, *only some aspects* of epistemic agents are modeled.

- I'll assume the following about B, D. [The first 4 are integral, but the last one is just for simplicity. More on these later.]
  - **Logical Omniscience**. If $p \dashv\vDash q$, then $B(p) \equiv B(q)$.
  - **Incompatibility**. $B(p) \Rightarrow \neg D(p)$.
  - **Opinionation**. $B(p) \vee D(p)$.
  - **Accuracy conditions**. $B(p)$ [$D(p)$] is accurate iff p is T [F].

  - **(NC)** $D(p) \equiv B(\neg p)$.
- Our S's make judgments regarding *all* p's in some *finite* Boolean algebra $\mathcal{B}$ (generated by some $\mathcal{L}_P$). I will use $\mathfrak{B}$ to denote the set of *all* of S's judgments (B's and D's) over $\mathcal{B}$.

- Classical deductive consistency is implicated in some well-known *paradoxes*. The two most infamous paradoxes (of the sort I have in mind) are the Lottery and the Preface.
  - (I) **Lottery Paradox** ([15],[6]). For each ticket i, it is highly probable that i is a loser (L_i). So, it would seem reasonable to be such that $B(L_i)$, for each i. However, this inevitably renders our set $\mathfrak{B}$ *inconsistent*, since we *know* that $(\exists i)(\neg L_i)$.
  - (II) **Preface Paradox** ([17],[4]). Let $\mathbf{B} \subset \mathfrak{B}$ be the set containing *all* of your *reasonable* (1st–order) beliefs. This $\mathbf{B}$ is an incredibly rich and complex set of judgments. You're fallible (*i.e.*, your 1st–order evidence is *sometimes misleading*). So, it seems reasonable to believe that *some* B's in $\mathbf{B}$ are false. However, adding *this* (2nd–order) belief to $\mathbf{B}$ renders $\mathfrak{B}$ *inconsistent*.
- There has been a ton of discussion of these paradoxes in the literature. Roughly, there are two sorts of responses:
  - (1) Dogmatically *maintain* consistency as a (*global*) CR for $\mathfrak{B}$.
    - Pollock ([18]): *all* inconsistent $\mathfrak{B}$'s engender "*collective defeat*," which mandates *suspension of belief* (wrt *some* p's).

- Pollock-style responses are implausible. As Christensen [4] nicely explains, it just seems wrong to claim that *suspension* is *reasonable/supported*, for *any* salient p. [More generally, (1) has implausible consequences about *evidential support*.]

(2) Radically *abandon all* (formal) coherence requirements for $\mathfrak{B}$.

- Christensen [4] suggests we *do everything with credences*, which (a) *do* have a formal coherence requirement (namely, *probabilism*), and (b) have no trouble dealing with the Lottery or the Preface. We should *learn to live without B/D*.
- Kolodny [14] thinks we *cannot live without B/D*, but we *should* learn to live without (formal, rational) coherence requirements for them. Instead, they have *only evidential requirements*, which are *satisfied* in the Lottery & Preface.

- I would prefer a different response than *either* (1) *or* (2). But, while (2) goes *too far*, there is a kernel of truth to it.
  - Christensen's focus on credences is useful. We can learn a lot from how *their* coherence requirements are *grounded*.
  - Kolodny's focus on evidence/justification is also crucial.

- Here is another epistemic paradox, but this one is couched (first) in terms of *comparative confidence*. I will use $\ulcorner p \sim q \urcorner$ to express the claim that $\ulcorner S$ *is equally confident in* p and $q \urcorner$.

(III.1) S observes a bank robbery. S gets a good look at the robber (r_0), who has a full head of hair. The police create n perfect duplicates of the robber (r_i). They remove i hairs from the head of r_i, and make a *line-up*: $r_0, r_1, r_2, r_3, \ldots, r_N$. They show S *only pairs*: $\langle r_0, r_1\rangle$, $\langle r_1, r_2\rangle$, $\langle r_2, r_3\rangle, \ldots, \langle r_{N-1}, r_N\rangle$. Let $p_i \stackrel{\text{def}}{=} r_i$ is the robber ($p_i \stackrel{\text{def}}{=} r_i = r_0$). On the basis of the visual evidence S obtains from seeing the pairs, S comes to be such that $p_i \sim p_{i+1}$, for each i. But, at the end of the day, the police show S *one last pair*: $\langle r_0, r_N\rangle$, where r_N has *zero* hairs on his head. Of course, S is such that: $p_0 \nsim p_N$.

- This one is not so easy for the probabilist to handle. All forms of probabilism will require that $\sim$ *is transitive* [10].
- Interestingly, there is a full belief version of (III) that the probabilist *can* handle (Christensen-style). We will also be able to handle the B-version of (III) in our framework.

- Instead of using $p \sim q$, we will use $B(p \approx q)$, where this is interpreted as: S *believes that* p *and* q *are equally likely*.

(III.2) S observes a bank robbery. S gets a good look at the robber (r_0), who has a full head of hair. The police create n perfect duplicates of the robber (r_i). They remove i hairs from the head of r_i, and make a *line-up*: $r_0, r_1, r_2, r_3, \ldots, r_N$. They show S *only pairs*: $\langle r_0, r_1\rangle$, $\langle r_1, r_2\rangle$, $\langle r_2, r_3\rangle, \ldots, \langle r_{N-1}, r_N\rangle$. Let $p_i \stackrel{\text{def}}{=} r_i$ is the robber ($p_i \stackrel{\text{def}}{=} r_i = r_0$). On the basis of the visual evidence S obtains from seeing the pairs, S comes to be such that $B(p_i \approx p_{i+1})$, for each i. But, at the end of the day, the police show S *one last pair*: $\langle r_0, r_N\rangle$, where r_N has *zero* hairs on his head. Of course, S is s.t.: $D(p_0 \approx p_N)$.

- It's interesting that this one becomes easier to handle (both for probabilists and for us [10]), if it is couched in terms of *beliefs about likelihoods*, rather than *comparative confidence*.
- In any case, (III.2) is similar to the Lottery and the Preface, because it involves a *large-ish, minimal inconsistent* $\mathfrak{B}$. [Here, I presuppose S *knows* the relation $\approx$ is *transitive*.]

- We would suggest that these paradoxes indicate that consistency is *too strong* to be a CR for full belief.
- Typically, such paradoxes involve a *conflict* between a *consistency* requirement and an *evidential* requirement, which requires *believing what is justified/supported*.

☞ Ideally, we want coherence requirements for full belief that are entailed by *both alethic and evidential* considerations.

- Our framework yields just such CR's, in "3 easy steps".
- **Step 1**: Define the *vindicated (viz., perfectly accurate) judgment set*, at w. ["Judgments of the omniscient S at w."]
  - $\mathring{\mathfrak{B}}_w$ contains $B(p)$ [$D(p)$] iff p is true (false) at w.
- **Step 2**: Define a notion of "distance between $\mathfrak{B}$ and $\mathring{\mathfrak{B}}_w$". That is, a measure of *distance from vindication* $d(\mathfrak{B}, \mathring{\mathfrak{B}}_w)$.
  - $d(\mathfrak{B}, \mathring{\mathfrak{B}}_w) \stackrel{\text{def}}{=}$ the number of inaccurate judgments in $\mathfrak{B}$ at w.
- **Step 3**: Adopt a *fundamental epistemic principle*, which *uses* $d(\mathfrak{B}, \mathring{\mathfrak{B}}_w)$ to ground a formal coherence requirement for $\mathfrak{B}$.

- Given our choices at Steps 1 and 2, there is ***a*** choice we can make at Step 3 that will yield *consistency* as a CR for $\mathfrak{B}$.

  Possible Vindication (PV). There exists *some* possible world w at which *all* of the judgments in $\mathfrak{B}$ are accurate. Or, to put this more formally in terms of d: $(\exists w)[d(\mathfrak{B},\mathring{\mathfrak{B}}_w) = 0]$.

- Possible vindication is *one way* we could go here. But, our framework is *much more general* than the classical one. It allows for *many other* choices of fundamental principle.
- Like Joyce [13, 12] — who makes the *analogous* move with *credences*, to ground *probabilism* — we *retreat* from (PV) to the *weaker*: *avoidance of (weak) dominance in* $d(\mathfrak{B},\mathring{\mathfrak{B}}_w)$.

  Weak Accuracy-Dominance Avoidance (WADA).

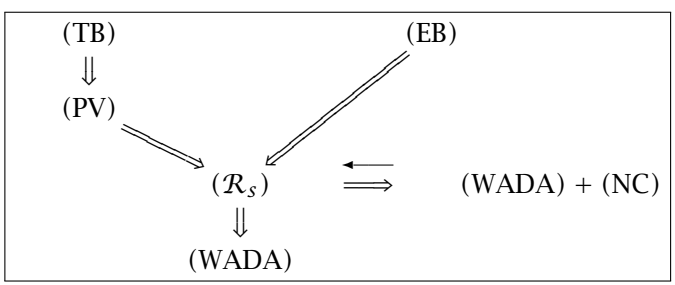
  There does *not* exist an alternative belief set $\mathfrak{B}'$ such that:
  (i) $(\forall w)[d(\mathfrak{B}',\mathring{\mathfrak{B}}_w) \leq d(\mathfrak{B},\mathring{\mathfrak{B}}_w)]$, and
  (ii) $(\exists w)[d(\mathfrak{B}',\mathring{\mathfrak{B}}_w) < d(\mathfrak{B},\mathring{\mathfrak{B}}_w)]$.

- Completing Step 3 in *this* way leads to a *new* CR for $\mathfrak{B}$.

- The new coherence requirement implied by this application of our framework has just the sort of properties we wanted.
- We wanted a coherence requirement that (like consistency) was motivated by considerations of accuracy (ideally, *entailed by* alethic requirements such as consistency/PV).
- But, we also wanted a coherence requirement that was *strictly weaker* than deductive consistency — in such a way that it is *also entailed by our evidential requirements*.
- It can be shown that we have met both of these *desiderata*, provided that we accept the following *necessary condition* for the satisfaction of our evidential requirements (EB).
  - **Necessary Condition for Satisfying (EB)**. S satisfies (EB), *i.e.*, S's judgments are *justified*, ***only if*** $(\mathcal{R}_s)$ there exists *some* probability function $\Pr(\cdot)$ which *probabilifies each of her beliefs and dis-probabilifies each of her disbeliefs*.
- There is *dis*agreement about *which* $\Pr(\cdot)$ is the requisite one for (EB) [3, 19, 11], but there is *agreement* that (EB) $\Rightarrow$ $(\mathcal{R}_s)$.

- Here are the logical relationships between key norms:

The Truth Norm for Belief:	(TB)
	$\Downarrow \not\Uparrow$
Possible Vindication (*viz.*, consistency):	(PV)
	$\Downarrow \not\Uparrow$
☞ Weak Accuracy-Dominance Avoidance:	(WADA)
	$\Uparrow \not\Downarrow$
Evidential Requirement for Belief:	(EB)



- For a more detailed map (with 10 norms), see Extras (16,17).

- (WADA) leads to a coherence requirement that is *strictly weaker* than consistency — *even if* (NC) $D(P) \equiv B(\neg P)$. [We think (WADA) + (NC) is *equivalent* to $(\mathcal{R}_s)$. See Extras (15–17).]
- Moreover, in general, *minimal* inconsistent $\mathfrak{B}$'s (of cardinality *greater than 2*) will *not violate* (WADA)/$(\mathcal{R}_s)$.
- ☞ So, if we adopt (WADA)/$(\mathcal{R}_s)$, rather than (PV)/consistency, "paradoxes" (I)–(III) do *not* imply violations of our CR for $\mathfrak{B}$.
- But, if S *does* violate (WADA)/$(\mathcal{R}_s)$, this reveals *two* defects:
  - There must exist some judgment in $\mathfrak{B}$ that is *inaccurate*. [This *alethic defect* follows from: (PV) $\Rightarrow$ (WADA)/$(\mathcal{R}_s)$.]
  - *And*, there must exist some judgment in $\mathfrak{B}$ that is *not supported by "the evidence"* — *whatever* "the evidence" is! [This *evidential defect* follows from: (EB) $\Rightarrow$ (WADA)/$(\mathcal{R}_s)$.]
- Our coherence requirements are *points of agreement* between alethic *vs* evidential perspectives (*i.e.*, points of contact between truth/accuracy *vs* justification/evidence).

- So far, our "paradoxes" are far removed from semantic paradoxes. [But, (III) does flirt with the sorites paradox.]
- Michael Caie [2] writes about an example involving self-reference, which causes problems for Joyce-style (accuracy-dominance) arguments for *probabilism*.
- There are analogous examples in our full belief framework.

  (IV) Consider the following self-referential statement:

  (P) S does not believe that P. [$\neg B(\ulcorner P \urcorner)$.]
- In order to be able to say anything interesting about (P), we'll need to relax our simplifying assunption (NC).
- Moreover, if S is *opinionated*, then (next slide) the best she can do (from an accuracy-dominance-avoidance perspective) is to be such that *either* $\{B(P), B(\neg P)\}$ *or* $\{D(P), D(\neg P)\}$.
- So, we will relax both *both* **(NC)** *and* **Opinionation**. We will now allow S to *suspend judgment* regarding p [$S(p)$]. But, it's not clear whether adding suspension will help us here.

- There are nine (9) possible pairs of attitudes (*viz.*, $\mathfrak{B}$'s) a *non*-opinionated agent can have toward $\{P, \neg P\}$. They are:
  1. $\mathfrak{B}_1 = \{B(P), B(\neg P)\}$. P is false. 1 right, 1 wrong, 0 (?).
  2. $\mathfrak{B}_2 = \{B(P), D(\neg P)\}$. P is false. 0 right, 2 wrong, 0 (?).
  3. $\mathfrak{B}_3 = \{B(P), S(\neg P)\}$. P is false. 0 right, 1 wrong, 1 (?).
  4. $\mathfrak{B}_4 = \{D(P), B(\neg P)\}$. P is true. 0 right, 2 wrong, 0 (?).
  5. $\mathfrak{B}_5 = \{D(P), D(\neg P)\}$. P is true. 1 right, 1 wrong, 0 (?).
  6. $\mathfrak{B}_6 = \{D(P), S(\neg P)\}$. P is true. 0 right, 1 wrong, 1 (?).
  7. $\mathfrak{B}_7 = \{S(P), B(\neg P)\}$. P is true. 0 right, 1 wrong, 1 (?).
  8. $\mathfrak{B}_8 = \{S(P), D(\neg P)\}$. P is true. 1 right, 0 wrong, 1 (?).
  9. $\mathfrak{B}_9 = \{S(P), S(\neg P)\}$. P is true. 0 right, 0 wrong, 2 (?).
- The "(?)" indicates my uncertainty about how to *evaluate suspensions* here. [Question: should we have $S(P) \equiv S(\neg P)$?]
- Anyhow, among the *opinionated* $\mathfrak{B}$'s [1,2,4,5], the ***only non-dominated*** $\mathfrak{B}$'s are [1,5], which ***violate*** (PV) ***and*** ($\mathcal{R}_s$)!

☞ This wreaks havoc with our framework, as it now stands.

	$\mathfrak{B}_1$	$\mathfrak{B}_2$	$\mathfrak{B}_3$	$\mathfrak{B}_4$	$\mathfrak{B}_5$	$\mathfrak{B}_6$	$\mathfrak{B}_7$	$\mathfrak{B}_8$
$\neg X \,\&\, \neg Y$	D	D	D	D	D	D	D	D
$X \,\&\, \neg Y$	D	D	D	D	D	D	D	D
$X \,\&\, Y$	D	D	D	D	D	D	D	D
$\neg X \,\&\, Y$	D	D	D	D	D	D	D	D
$\neg Y$	D	D	D	D	B	B	B	B
$X \equiv Y$	D	D	B	B	D	D	B	B
$\neg X$	D	B	D	B	D	B	D	B
X	B	D	B	D	B	D	B	D
$\neg(X \equiv Y)$	B	B	D	D	B	B	D	D
Y	B	B	B	B	D	D	D	D
$X \vee \neg Y$	B	B	B	B	B	B	B	B
$\neg X \vee \neg Y$	B	B	B	B	B	B	B	B
$\neg X \vee Y$	B	B	B	B	B	B	B	B
$X \vee Y$	B	B	B	B	B	B	B	B
$X \vee \neg X$	B	B	B	B	B	B	B	B
$X \,\&\, \neg X$	D	D	D	D	D	D	D	D

- There are judgment sets that are *non-d-dominated, but inconsistent*.
- This is true, *even if* we assume *opinionation* & (NC) $D(p) \equiv B(\neg p)$.
- The simplest possible examples of this kind involve $\mathcal{L}$'s with *at least two atomic sentences* (*e.g.*, X, Y).
- The table shows *the* 8 sets that are *non-dominated*, but *inconsistent*.
- Of course, as we look at larger and larger languages/algebras, we start seeing *more interesting* examples.
- But, even these toy examples exhibit a "preface-like" structure.
- The *logically stronger* propositions tend to be *dis*believed (and the *logically weaker* ones *believed*).

(TB) S ought believe p just in case p is true.

(PV) $(\exists w)[d(\mathfrak{B}, \mathring{\mathfrak{B}}_w) = 0]$. That is, $\mathfrak{B}$ is *deductively consistent*.

(SADA) $\nexists \mathfrak{B}'$ such that: $(\forall w)[d(\mathfrak{B}', \mathring{\mathfrak{B}}_w) < d(\mathfrak{B}, \mathring{\mathfrak{B}}_w)]$.

$\neg(\exists_\beta \Box$MI) $\nexists\, \beta \subseteq \mathfrak{B}$ s.t.: $(\forall w)\left[> \frac{1}{2} \textit{ of the members of } \beta \textit{ are inaccurate at } w\right]$.

($\mathcal{R}_s$) $\exists$ a probability function $\Pr(\cdot)$ such that, $\forall p \in \mathcal{B}$:

$$B(p) \textit{ iff } \Pr(p) > \tfrac{1}{2}, \text{ and } D(p) \textit{ iff } \Pr(p) < \tfrac{1}{2}.$$

(EB) S ought believe p just in case p is supported by S's evidence.
Note: this assumes *only* $(\exists \Pr)(\exists E)[\Pr(p \mid E) > \frac{1}{2} \textit{ iff } B(p)]$.

(WADA) $\nexists \mathfrak{B}'$ s.t.: $(\forall w)[d(\mathfrak{B}', \mathring{\mathfrak{B}}_w) \leq d(\mathfrak{B}, \mathring{\mathfrak{B}}_w)] \,\&\, (\exists w)[d(\mathfrak{B}', \mathring{\mathfrak{B}}_w) < d(\mathfrak{B}, \mathring{\mathfrak{B}}_w)]$.

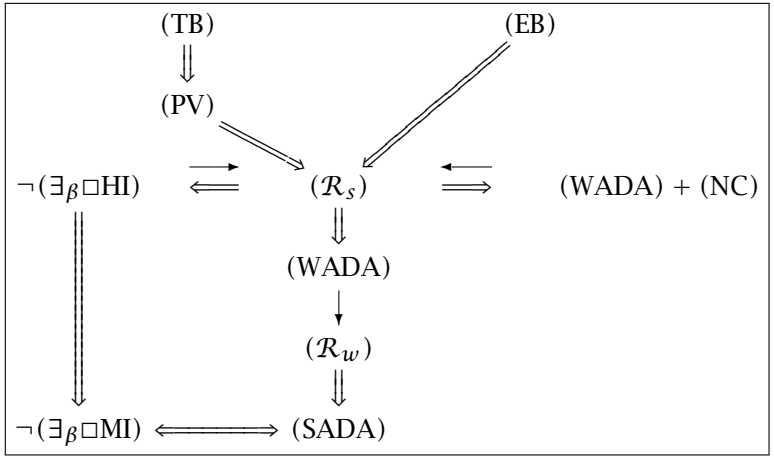
($\mathcal{R}_w$) $\exists$ a probability function $\Pr(\cdot)$ such that, $\forall p \in \mathcal{B}$:

$$B(p) \textit{ if } \Pr(p) > \tfrac{1}{2}, \text{ and } D(p) \textit{ if } \Pr(p) < \tfrac{1}{2}.$$

$\neg(\exists_\beta \Box$HI) $\nexists\, \beta \subseteq \mathfrak{B}$ s.t.: $(\forall w)\left[\geq \frac{1}{2} \textit{ of the members of } \beta \textit{ are inaccurate at } w\right]$.

(NC) S disbelieves p iff S believes $\neg p$ [*i.e.*, $D(p) \equiv B(\neg p)$].

- Here is what the logical relations look like, among all of the 10 norms for (opinionated) $\mathfrak{B}$. [Double (single) arrows represent *known* (*conjectured*) entailments. And, if there is no path, then we believe (or conjecture) that there is no entailment.]



- We (along with Rachael Briggs and Fabrizio Cariani) [1] are investigating various applications of this new approach.
- One interesting application is to *judgment aggregation*. *E.g.*,
  - Majority rule aggregations of the judgments of a bunch of agents — each of whom satisfy (PV) — *need not* satisfy (PV).
- **Q**: does majority rule preserve *our* notion of coherence, *viz.*, is (WADA) preserved by MR? **A**: yes (on simple, atomic + truth-functional agendas), but *not on all possible agendas*.
  - There are (not merely atomic + truth-functional) agendas A and sets of judges J ($|A| \geq 5$, $|J| \geq 5$) that (severally) satisfy (WADA), while their majority profile *violates* (WADA).
- *But*, if a set of judges is (severally) *consistent* [*i.e.*, satisfy (PV)], then their majority profile *must* satisfy (WADA).

☞ **Recipe**. Wherever $\mathfrak{B}$-*consistency* runs into paradox, substitute *coherence* (in *our* sense), and see what happens.

- Another possible application of our new notion of coherence is to (at least partially) fill-out David Ripley and Greg Restall's proposal about *entailment*. They suggest:

  $\Gamma \vdash \Delta \stackrel{\text{def}}{=}$ if an agent S asserts each member of Γ and denies each member of Δ, then S is (thereby) "out of bounds".
- I do not want to get into a discussion about norms for assertion/denial. But, I can offer the following alternative that seems similar *in spirit* to the Ripley/Restall idea:

  $\Gamma \Vdash \Delta \stackrel{\text{def}}{=}$ if S believes each member of Γ and disbelieves each member of Δ, then S is (thereby) *incoherent* (in our sense).
- I'm not sure how this definition would combine with Ripley's ideas about "classical logic" and intro-elim *vs* structural rules involving $\Vdash$, *etc.* We should talk about it.
- I doubt this is going to get Ripley *everything he wants*. But, if classical consistency is *not* a requirement of epistemic rationality, then this may be *the closest he can get*.

- One disanalogy with Joyce's argument for b-probabilism, is that his argument is *very flexible* about *weights* it can assign to different p's in $\mathcal{B}$. This *seems* like an advantage.
- After all, aren't *some* propositions "*more* important" ("more important" *to have accurate beliefs about*) than others?
- Sure. But, *precisely what* are we missing? 3 possibilities:
  1. Something of *pragmatic* value. *E.g.*, beliefs about my wife's health *vs.* beliefs about the # of pebbles on pebble beach.
  2. Something of *instrumental* epistemic value. *E.g.*, being right about p leads to a greater # of accurate beliefs/disbeliefs.
  3. Something ($\mathcal{X}$) of *intrinsic* epistemic value — where $\mathcal{X}$ can't supervene on considerations of *accuracy* (+ *evidence*).
- Only (3) could be a problem for us. But, what could $\mathcal{X}$ be?
- Perhaps $\mathcal{X}$ is "informativeness"? I think this is *orthogonal* to *pure accuracy*, and requires separate treatment. [Here, we'd be getting into "synchronic *closure*" requirements, *etc.*]

- Kenny is writing a paper [7] that explains how to relax the assumption of opinionation in our framework.
- Our present approach is equivalent to assigning (in)accurate judgments an "accuracy score" of $(-\mathfrak{w}) + \mathfrak{r}$ (where $\mathfrak{w} > \mathfrak{r} > 0$), and calculating the "overall accuracy score" for $\mathfrak{B}$ (at w) as the *sum* of "accuracy scores" over all $p \in \mathcal{B}$ (at w).
- Kenny's idea: allow S to *suspend on* p [$S(p)$], and then "score" suspensions with a *neutral* "accuracy score" of *zero*.
- On this neutral accuracy scheme for suspensions, we get a nice generalization of our representation Theorem (II).

(III) **Theorem**. An agent S will avoid (strict) dominance in "total accuracy score" ***if*** their $\mathfrak{B}$ can be represented as follows:

There exists a probability function $\Pr(\cdot)$ such that, $\forall p \in \mathcal{B}$:

$$B(p) \textit{ iff } \Pr(p) > \tfrac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}},$$

$$D(p) \textit{ iff } \Pr(p) < 1 - \tfrac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}},$$

$$S(p) \textit{ iff } \Pr(p) \in \left[1 - \tfrac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}}, \tfrac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}}\right].$$

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