1 Background: Garber, Jeffrey and Earman

By the time Einstein had formulated the general theory of relativity ($H$), the evidence regarding the perihelion of Mercury ($E$) — which Newtonian theory was unable to adequately explain — had long been known \[4\]. Indeed, it is not implausible to suppose that Einstein was *certain* (in 1915) that $E$ was true. Nonetheless, it is widely accepted that Einstein learned *some* proposition $X$ (in 1915) which had the effect of confirming $H$ (i.e., rationally raising Einstein’s credence in $H$).

Garber \[2\] proposes that what Einstein learned was a *logical* fact (i.e., that $X = \text{“}H\text{ entails }E\text{”}$). By adding an additional atomic statement “$X$” to the $H, E$-language (and interpreting “$X$” extra-systematically as “$H$ entails $E$”), Garber showed how it was possible to write down Bayesian models of this sort, having the following desired confirmation-theoretic property $$(†) \quad \Pr(H \mid X) > \Pr(H).$$

Garber did not, however, endorse specific constraints on $\Pr(\cdot)$ that would ensure $(†)$.

Subsequently, Jeffrey \[3\] and Earman \[1\] offered such constraints. Their approach requires the addition of $X$ and a fourth atomic statement “$Y$” to the $H, E$-language, where $Y$ is extra-systematically interpreted as “$H$ entails $\sim E$” (viz., $H$ refutes $E$). In these traditional Garber-style approaches, the background extra-systematic constraints on $\Pr(\cdot)$ consist of the following pair of “*modus ponens* principles” for the extra-systematic entailment relation.$^1$

$$\begin{align*}
(1) \quad \Pr(E \mid H \& X) &= 1 \\
(2) \quad \Pr(E \mid H \& Y) &= 0
\end{align*}$$

In addition to (1) and (2), Jeffrey offers the following additional extra-systematic constraint:

$$\begin{align*}
(3) \quad \Pr(X \vee Y) &= 1
\end{align*}$$

Informally, (3) says that *either* $H$ entails $E$ *or* $H$ refutes $E$. It is straightforward to show that

\[1\] Of course, Garber-style approaches also rest on the background assumption that $\Pr(E) = 1$. We’ll return to this background $E$-certainty assumption in the next section.
(1)–(3) jointly entail the desired confirmation-theoretic conclusion (†). This was Jeffrey’s Garberian approach to the old evidence problem.

As Earman rightly points out, Jeffrey’s constraint (3) is not very plausible as a rational stricture on Einstein’s credences, prior to his learning \(X\) (in 1915). There was no good reason for Einstein to be certain (prior to learning \(X\)) that \(H\) either entails or refutes \(E\). Earman offers the following alternative additional extra-systematic constraint:

\[
(4) \quad \text{Pr}(H \mid X) > \text{Pr}(H \mid \sim X \& \sim Y)
\]

Earman shows that (1), (2) and (4) jointly entail (†); and, he argues that (4) is a plausible assumption regarding Einstein’s credences (in 1915). We agree that (4) is more plausible than Jeffrey’s (3). For one thing, (4) is not a numerical constraint, but merely an ordinal constraint on Einstein’s 1915 credences. Moreover, because Einstein was certain of \(E\), it is reasonable to suppose that he would have judged that \(X\) confers a greater probability on \(H\) than \(\sim X \& \sim Y\) does. Having said that, we think (4) is a little too close to the desired conclusion (†) itself, since both require credal comparisons involving \(\text{Pr}(H \mid X)\).

In the next section, we show how to improve upon the Garber-style approaches of Jeffrey and Earman.

2 A New Garber-Style Approach

We like Garber’s idea of adding a pair of extra-systematic statements (and extra-systematic credal constraints) to the \(H, E\)-language. But, we think the existing implementation of this general strategy has two main shortcomings. First, we think interpreting “\(X\)” and “\(Y\)” as “\(H\) entails \(E\)” and “\(H\) refutes \(E\)” is unduly restrictive. It is more plausible to suppose that what is learned in cases of old evidence (viz., \(X\)) may not (always) be a logical fact. To be more precise, let “\(X\)” and “\(Y\)” be interpreted as follows:

- \(X \equiv H\) adequately explains (or accounts for) \(E\).
- \(Y \equiv \text{some alternative to } H \ (H') \text{ adequately explains (or accounts for) } E\).

What really matters here is not whether \(H\) entails \(E\) (or \(\sim E\)), but whether \(H\) adequately explains (or accounts for) \(E\) and/or whether some alternative scientific theory \(H'\) adequately explains (or accounts for) \(E\). It may be that \(H\) adequately explains \(E\) in a deductive-nomological sense. But, why not allow for the possibility that \(H\) (and/or \(H'\)) explains \(E\) in a non-deductive-nomological way? In this regard, we think that the original Garber-style approaches are too narrow in their explanatory scope.
A second problem with the traditional Garber-style approaches is that they have required extra-systematic credal constraints [e.g., \( \Pr(E) = 1 \), (3) and (4)] that are either implausibly strong or too close to the desired confirmation-theoretic conclusion (†) itself. This defect is also remedied by moving to our alternative, explanatory extra-systematic interpretation of \( X \) and \( Y \). Consider the following four ordinal constraints on \( \Pr(\cdot | \cdot) \).

\[
\begin{align*}
(5.1) & \quad \Pr(H | X & \sim Y) > \Pr(H | \sim X & \sim Y) \\
(5.2) & \quad \Pr(H | X & \sim Y) > \Pr(H | \sim X & Y) \\
(5.3) & \quad \Pr(H | X & Y) > \Pr(H | \sim X & Y) \\
(5.4) & \quad \Pr(H | X & Y) \geq \Pr(H | \sim X & \sim Y)
\end{align*}
\]

Let’s examine each of four constraints, in turn. Suppose that \( H \) is the only scientific theory that adequately explains \( E \). Constraints (5.1) and (5.2) assert that \( H \) is more probable, given this supposition \( (X & \sim Y) \) than it is given either the supposition that no scientific theory adequately explains \( E \) (i.e., given \( \sim X & \sim Y \)) or the supposition that some alternative (\( H' \)) to \( H \) is the only scientific theory that adequately explains \( E \) \( (\sim X & Y) \). In other words, (5.1) and (5.2) say that \( H \)'s being the only adequate explanation of \( E \) confers a greater probability on \( H \) than any possibility which implies that \( H \) does not adequately explain \( E \). These two constraints strike us as being completely uncontroversial.

Constraint (5.3) also seems extremely plausible. It asserts that \( H \) is less probable, given the supposition that some alternative (\( H' \)) to \( H \) is the only scientific theory that adequately explains \( E \) (i.e., given \( \sim X & Y \)) than it is given the supposition that both \( H \) and some alternative scientific theory (\( H' \)) adequately explain \( E \) \( (X & Y) \). The fourth and final credal comparison (5.4) says that \( H \) is at least as probable, given the supposition that both \( H \) and some alternative scientific theory (\( H' \)) adequately explain \( E \) (i.e., given \( X & Y \)) as it is given the supposition that no scientific theory adequately explains \( E \) (i.e., given \( \sim X & \sim Y \)). One might maintain that it would be reasonable to rank \( \Pr(H | X & Y) \) strictly higher in one’s comparative confidence ranking than \( \Pr(H | \sim X & \sim Y) \). After all, \( X & Y \) implies that \( H \) does adequately explain \( E \) (which is already known), whereas \( \sim X & \sim Y \) implies that \( H \) does not adequately explain \( E \). On the other hand, one might also reasonably maintain that these two suppositions \( (X & Y \) and \( \sim X & \sim Y \) \) place \( H \) and its alternatives on a par with respect to explaining \( E \), and so they shouldn’t confer different probabilities on \( H \). Both of these positions are compatible with (5.4). The only thing (5.4) rules out is the claim that \( H \) is more probable given \( E \)’s inexplicability \( (\sim X & \sim Y) \) than it is given \( E \)’s multiple explicability by both \( H \) and some alternative to \( H \) \( (X & Y) \). As such, (5.4) also seems eminently reasonable.
As it happens, the desired (Garberian) confirmation-theoretic conclusion (†) follows from (5.1)–(5.4) alone. To be more precise, we can prove the following general result.

**Theorem.** (5.1)–(5.4) jointly entail (†).

**Proof.** Let \( a \equiv \Pr(H \mid X \& \sim Y) \), \( b \equiv \Pr(H \mid X \& Y) \), \( c \equiv \Pr(H \mid \sim X \& \sim Y) \), \( \delta \equiv \Pr(H \mid \sim X \& Y) \), \( x \equiv \Pr(\sim Y \mid X) \), and \( y \equiv \Pr(\sim Y \mid \sim X) \). Given these assignments, (5.1)–(5.4) are as follows.

(5.1) \[ a > c \]
(5.2) \[ a > \delta \]
(5.3) \[ b > \delta \]
(5.4) \[ b \geq c \]

Suppose that \( a \in (0, 1] \), \( \delta \in [0, 1) \), and \( b, c, x, y \in (0, 1) \). Then, (5.1)–(5.4) jointly entail

\[ ax + b(1 - x) > cy + \delta(1 - y). \]

And, by the law of total probability, we have:

\[ \Pr(H \mid X) = ax + b(1 - x) \]
\[ \Pr(H \mid \sim X) = cy + \delta(1 - y) \]

Thus, (5.1)–(5.4) jointly entail \( \Pr(H \mid X) > \Pr(H \mid \sim X) \), which entails \( \Pr(H \mid X) > \Pr(H) \).

We think our Theorem undergirds a superior Garber-style approach to the problem of old-evidence. Specifically, our approach has the following three distinct advantages over traditional Garber-style approaches.

(i) Our approach does not require the assumption that \( \Pr(E) = 1 \). It may be true that our constraints (5.1)–(5.4) are most plausible given the background assumption that \( E \) is known with certainty. But, we think (5.1)–(5.4) retain enough of their plausibility, given only the weaker assumption that \( E \) is known with near certainty (i.e., \( \Pr(E) \approx 1 \)).

The only two conditional credences that may reasonably take extremal values here are \( a \) and \( \delta \). If \( H \) is the only theory that adequately explains \( E \), then it may be reasonable to assign \( H \) maximal credence. And, if some alternative to \( H \) (\( H' \)) is the only theory that adequately explains \( E \), then it may be reasonable to assign minimal credence to \( H \). This is why we allow \( a \in (0, 1] \) and \( \delta \in [0, 1) \). The other conditional credences involved in our Theorem (i.e., \( b, c, x, y \)) should, in general, take non-extreme values.

(ii) If we relax the standard Garberian assumption that \( \Pr(E) = 1 \), then we'll need to specify what happens to the probability of \( E \), when \( X \) is learned. Intuitively, the probability of \( E \) should not change during this learning process. As a result, models of such a learning event may need to be more sophisticated than our present (naïve strict conditionalization) approach would suggest.
(ii) Our approach rests only on ordinal assumptions (5.1)–(5.4), which are *neither* implausibly strong *nor* too close to the desired confirmation-theoretic conclusion (†).

(iii) Our approach is not restricted to cases in which $H$ (and/or alternatives $H'$) explain $E$ in a *deductive-nomological* way. That is, our approach covers all cases in which scientists come to learn that their theory *adequately explains* $E$, not only those cases in which scientists learn that their theory *entails* $E$ (or explains $E$ *deductive-nomologically*).

**References**


