A New Gibbardian Collapse Theorem for the Indicative Conditional

Branden Fitelson

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Let $\mathscr{L}$ be a sentential (object) language containing atoms 'A', 'B', ..., and two *logical* connectives '&' and '$\rightarrow$'. In addition to these two *logical* connectives, $\mathscr{L}$ will also contain another binary connective '$\rightsquigarrow$', which is intended to be interpreted as the English indicative. In the meta-language for $\mathscr{L}$, we will have two meta-linguistic operations: '$\Vdash$' and '$\vdash$'. '$\Vdash$' is a binary relation between individual sentences in $\mathscr{L}$. It will be interpreted as "single premise entailment" (or "single premise deducibility in $\mathscr{L}$"). '$\vdash$' is a monadic predicate on sentences of $\mathscr{L}$. It will be interpreted as "logical truth of the logic of $\mathscr{L}$" (or "theorem of the logic of $\mathscr{L}$"). We will not presuppose anything about the relationship between '$\Vdash$' and '$\vdash$'. Rather, we will state explicitly all assumptions about these meta-theoretic relations that will be required for Gibbard's Theorem.

Below, I report a new version of Gibbardian Collapse. First, two preliminary remarks: (a) the "if... then" and "and" I'm using in the meta-meta-language of $\mathscr{L}$ to state the assumptions of the theorem are assumed to be classical, and (b) these assumptions are all *schematic* (*i.e.*, they are to be interpreted as allowing *any instances* that can be formed from sentences of $\mathscr{L}$).

We begin with seven (7) background assumptions, which are purely formal renditions of some of Gibbard's presuppositions in his collapse argument. Think of this as a (very weak) *background logic* for $\langle \rightsquigarrow, \& \rangle$.

1. $\vdash (p \,\&\, q) \rightsquigarrow q$
   - (1) is a (*right*) *conjunction-elimination axiom* for $\langle \rightsquigarrow, \& \rangle$. This also holds in all theories of the conditional of which I am aware.
2. If $p \Vdash q$ and $\vdash p$, then $\vdash q$.
   - (2) is a basic assumption about the relationship between $\Vdash$ and $\vdash$, which says that if p entails q and p is a theorem, then q is a theorem.
3. If $\vdash p \rightarrow q$, then $p \Vdash q$.
   - (3) is one direction of the deduction theorem for the logical conditional $\rightarrow$.
4. $p \rightsquigarrow q \Vdash p \rightarrow q$.
   - (4) asserts that the indicative conditional entails the logical conditional. This is one of Gibbard's main assumptions (that the indicative conditional is *at least as strong as* the logical conditional).
5. If $\vdash p \rightarrow (q \rightarrow r)$, then $\vdash (p \,\&\, q) \rightarrow r$.
   - (5) is a (theoremhood) form of the *import* law for the logical conditional.

My previous collapse theorems made use of *either* (a) (full) import-export for the indicative, *or* (b) merely export for the indicative. That is, they made use (at least) of the following assumption:

6. If $\vdash (p \,\&\, q) \rightsquigarrow r$, then $\vdash p \rightsquigarrow (q \rightsquigarrow r)$.

In light of an example due to Paolo Santorio, I got to thinking about whether the following alternative (mixed) principle would suffice for collapse.

7. *If* $\vdash (p \,\&\, q) \rightarrow r$, *then* $\vdash p \rightsquigarrow (q \rightsquigarrow r)$.

(7) asserts that if $(p \,\&\, q) \rightarrow r$ is a (logical) theorem, then $p \rightsquigarrow (q \rightsquigarrow r)$ is a theorem (for the indicative). As it happens, (7) *does* suffice for collapse, given (1)–(5). That brings us to our new collapse result.

Collapse. $p \rightarrow q \Vdash p \rightsquigarrow q$.

We have the following theorem (proof omitted).

Theorem. Assuming (1)–(5), (7) entails **Collapse**.