

# A New Gibbardian Collapse Theorem for the Indicative Conditional

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Let  $\mathcal{L}$  be a sentential (object) language containing atoms ' $A$ ', ' $B$ ', ..., and two *logical* connectives '&' and ' $\rightarrow$ '. In addition to these two *logical* connectives,  $\mathcal{L}$  will also contain another binary connective ' $\rightsquigarrow$ ', which is intended to be interpreted as the English indicative. In the meta-language for  $\mathcal{L}$ , we will have two meta-linguistic operations: ' $\Vdash$ ' and ' $\vdash$ '. ' $\Vdash$ ' is a binary relation between individual sentences in  $\mathcal{L}$ . It will be interpreted as "single premise entailment" (or "single premise deducibility in  $\mathcal{L}$ "). ' $\vdash$ ' is a monadic predicate on sentences of  $\mathcal{L}$ . It will be interpreted as "logical truth of the logic of  $\mathcal{L}$ " (or "theorem of the logic of  $\mathcal{L}$ "). We will not presuppose anything about the relationship between ' $\Vdash$ ' and ' $\vdash$ '. Rather, we will state explicitly all assumptions about these meta-theoretic relations that will be required for Gibbard's Theorem.

Below, I report a new version of Gibbardian Collapse. First, two preliminary remarks: (a) the "if... then" and "and" I'm using in the meta-meta-language of  $\mathcal{L}$  to state the assumptions of the theorem are assumed to be classical, and (b) these assumptions are all *schematic* (i.e., they are to be interpreted as allowing *any instances* that can be formed from sentences of  $\mathcal{L}$ ).

We begin with seven (7) background assumptions, which are purely formal renditions of some of Gibbard's presuppositions in his collapse argument. Think of this as a (very weak) *background logic* for  $\langle \rightsquigarrow, \& \rangle$ .

1.  $\vdash (p \& q) \rightsquigarrow q$ 
  - (1) is a (*right*) *conjunction-elimination axiom* for  $\langle \rightsquigarrow, \& \rangle$ . This also holds in all theories of the conditional of which I am aware.
2. If  $p \Vdash q$  and  $\vdash p$ , then  $\vdash q$ .
  - (2) is a basic assumption about the relationship between  $\Vdash$  and  $\vdash$ , which says that if  $p$  entails  $q$  and  $p$  is a theorem, then  $q$  is a theorem.
3. If  $\vdash p \rightarrow q$ , then  $p \Vdash q$ .
  - (3) is one direction of the deduction theorem for the logical conditional  $\rightarrow$ .
4.  $p \rightsquigarrow q \Vdash p \rightarrow q$ .
  - (4) asserts that the indicative conditional entails the logical conditional. This is one of Gibbard's main assumptions (that the indicative conditional is *at least as strong as* the logical conditional).
5. If  $\vdash p \rightarrow (q \rightarrow r)$ , then  $\vdash (p \& q) \rightarrow r$ .
  - (5) is a (theoremhood) form of the *import* law for the logical conditional.

My previous collapse theorems made use of *either* (a) (full) import-export for the indicative, *or* (b) merely export for the indicative. That is, they made use (at least) of the following assumption:

6. If  $\vdash (p \& q) \rightsquigarrow r$ , then  $\vdash p \rightsquigarrow (q \rightsquigarrow r)$ .

In light of an example due to Paolo Santorio, I got to thinking about whether the following alternative (mixed) principle would suffice for collapse.

7. If  $p \& q \Vdash r$ , then  $\vdash p \rightsquigarrow (q \rightsquigarrow r)$ .

(7) asserts that if  $p \& q$  entails  $r$ , then  $p \rightsquigarrow (q \rightsquigarrow r)$  is a theorem (for the indicative). As it happens, (7) *does* suffice for collapse, given (1)–(5). That brings us to our new collapse result.

**Collapse.**  $p \rightarrow q \Vdash p \rightsquigarrow q$ .

We have the following theorem (proof omitted).

**Theorem.** Assuming (1)–(5), (7) entails **Collapse**.