Probabilistic Coherence from a Logical Point of View

• Overview of the Talk
  - Foundation: Probabilistic Confirmation (c) from a Logical POV
    * c(h,e) as a “relevant” quantitative generalization of □(e ⊃ h)
    * c(h,e), so understood, is not Pr(e ⊃ h) or Pr(h|e), etc.
    * c(h,e) is something akin (ordinally) to the likelihood ratio
  - Defining Coherence (C) in terms of “Mutual c-Confirmation”
    * C(p,q) as a “mutual confirmation” generalization of C(p & q)
    * C(p,q), so understood, is not Pr(p & q) or Pr(q | p), etc.
    * Suggestion: C(p,q) as a function of c(p,q) and c(q,p), etc.
  - Confirmation as primitive, and coherence defined in terms of it
  - New definition of my C measure (inspired by Moretti/Douven)
  - Some Subtleties/Objections (I’ll focus on “logical” ones)

From Confirmation to Coherence I

<table>
<thead>
<tr>
<th>Confirmation (c)</th>
<th>Coherence (C)</th>
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<tbody>
<tr>
<td>Metatheoretic Concept: □(e ⊃ h)</td>
<td>Metatheoretic Concept: ω(p &amp; q)</td>
</tr>
<tr>
<td>∴ e ⊃ ~h ⇒ maximal disconfirmation</td>
<td>∴ p ⊃ q ⇒ maximal coherence</td>
</tr>
<tr>
<td>∴ e ⊃ [h</td>
<td>e ⊥] ⇒ maximal confirmation</td>
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<tr>
<td>+ Dependence is confirmation</td>
<td>+ Dependence is coherence</td>
</tr>
<tr>
<td>– Dependence is disconfirmation</td>
<td>– Dependence is incoherence</td>
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<tr>
<td>Independence is neutrality</td>
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<tr>
<td>Pr(e ⊃ h) won’t work</td>
<td>Pr(p &amp; q) won’t work</td>
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<tr>
<td>Pr(h</td>
<td>e) won’t work, etc.</td>
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<tr>
<td>Most relevance measures won’t work</td>
<td>Most relevance measures won’t work</td>
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• In the confirmation case, only a small class of candidate c-measures will work.
• And, if C is defined in terms of “mutual c”, there are also few candidates.

From Confirmation to Coherence II

• Strategy: We will construct our C measure using one of the proper c measures.
• We use a slight [new!] modification of Kemeny and Oppenheim’s c-measure F

\[ F_M(h,e) = \begin{cases} \frac{\Pr_M(e | h) - \Pr_M(e | \sim h)}{\Pr_M(e | h) + \Pr_M(e | \sim h)} & \text{if } e \not\equiv h \text{ and } e \not\equiv \sim h. \\ 1 & \text{if } e \equiv h, \text{ and } e \not\equiv \perp. \\ -1 & \text{if } e \equiv \perp. \end{cases} \]

• Let \( \mathcal{F} \) be the set containing the F values of all pairs of conjunctions of (thanks, Igor!) nonempty, disjoint subsets of the set of statements. And, C is an average of \( \mathcal{F} \). Note: F (hence C) is relativized to a (regular) Pr-model M!
• \( \mathcal{F} \) is non-trivial to visualize! I haven’t analyzed the combinatorics of \( \mathcal{F} \) yet, but I have an algorithm for generating it. See my MATHEMATICA notebook.
• I first proposed simply taking the straight average of \( \mathcal{F} \), but other averages could be given (undoubtedly, some examples will suggest unequal weights).

Some Subtleties/Objections

• Individuation: The “information sets” (collections that C measures) could be multisets/sequences of propositions, or sets of statements (tokens), etc., but not sets of propositions, unless we go anti-Stalnaker (which is controversial).
• Siebel: “if we are confronted with a pair of statements which cannot both be false together, Fitelson’s function assigns it a coherence value of at most 0.”
• True. But, this will be true for any Pr-relevance-based account (not just mine). If p and q can’t both be false, then they cannot be positively correlated! Here, correlation goes beyond a naïve generalization of the metatheoretic ω(p & q).
• Moretti (and others): On your view, logically equivalent sets of statements can have different degrees of coherence. Yep. But, this also strikes me as correct. [To my mind, \{p,q,r\} is more coherent than \{p,q\}, provided that r \equiv p.]
• Moretti: But, on your C, adding \( \top \) to a coherent set can make it incoherent! This was true on my old C. But, not on my new C. See my MATHEMATICA notebook. C(S) can be < C(S \cup \{\top\}), but this is an artifact of averaging.