Probabilistic Coherence from a Logical Point of View

Branden Fitelson
University of California–Berkeley

- Overview of the Talk
  - Foundation: Probabilistic Confirmation ($\mathfrak{c}$) from a Logical POV
    - $\mathfrak{c}(h, e)$ as a "relevant" quantitative generalization of $\Box(e \supset h)$
    - $\mathfrak{c}(h, e)$, so understood, is not $\Pr(e \supset h)$ or $\Pr(h \mid e)$, *etc.*
    - $\mathfrak{c}(h, e)$ is something akin (ordinally) to the likelihood ratio
  - Defining Coherence ($\mathscr{C}$) in terms of "Mutual $\mathfrak{c}$-Confirmation"
    - $\mathscr{C}(p, q)$ as a "mutual confirmation" generalization of $\Diamond(p \,\&\, q)$
    - $\mathscr{C}(p, q)$, so understood, is not $\Pr(p \,\&\, q)$ or $\Pr(q \mid p)$, *etc.*
    - Suggestion: $\mathscr{C}(p, q)$ as a function of $\mathfrak{c}(p, q)$ and $\mathfrak{c}(q, p)$, *etc.*
  - Confirmation as primitive, and coherence defined in terms of it
  - New definition of my $\mathscr{C}$ measure (inspired by Moretti/Douven)
  - Some Subtleties/Objections (I'll focus on "logical" ones)

From Confirmation to Coherence I

Confirmation ($\mathfrak{c}$)	Coherence ($\mathscr{C}$)
Metatheoretic Concept: $\Box(e \supset h)$	Metatheoretic Concept: $\Diamond(p \,\&\, q)$
$\therefore e \vDash \sim h \Rightarrow$ maximal disconfirmation	$\therefore p \Dashv\vDash \sim q \Rightarrow$ maximal incoherence
$\therefore e \vDash h\ [e \nvDash \bot] \Rightarrow$ maximal confirmation	$\therefore p \Dashv\vDash q \nvDash \bot \Rightarrow$ maximal coherence
$+$ Dependence is confirmation	$+$ Dependence is coherence
$-$ Dependence is disconfirmation	$-$ Dependence is incoherence
Independence is neutrality	Independence is neutrality
$\Pr(e \supset h)$ won't work	$\Pr(p \,\&\, q)$ won't work
$\Pr(h \mid e)$ won't work, *etc.*	$\Pr(q \mid p)$ won't work, *etc.*
Most relevance measures won't work	Most relevance measures won't work

- In the confirmation case, only a small class of candidate $\mathfrak{c}$-measures will work.
- And, if $\mathscr{C}$ is defined in terms of "mutual $\mathfrak{c}$", there are also few candidates.

From Confirmation to Coherence II

- Strategy: We will construct our $\mathscr{C}$ measure using one of the proper $\mathfrak{c}$ measures.
- We use a slight [new!] modification of Kemeny and Oppenheim's $\mathfrak{c}$-measure F

$$F_{\mathcal{M}}(h, e) =_{df} \begin{cases} \dfrac{\Pr_{\mathcal{M}}(e \mid h) - \Pr_{\mathcal{M}}(e \mid \sim h)}{\Pr_{\mathcal{M}}(e \mid h) + \Pr_{\mathcal{M}}(e \mid \sim h)} & \text{if } e \nvDash h \text{ and } e \nvDash \sim h. \\ 1 & \text{if } e \vDash h, \text{ and } e \nvDash \bot. \\ -1 & \text{if } e \vDash \sim h. \end{cases}$$

- Let $\mathscr{F}$ be the set containing the F values of all pairs of conjunctions of (thanks, Igor!) nonempty, disjoint subsets of the set of statements. And, $\mathscr{C}$ is an average of $\mathscr{F}$. Note: F (hence $\mathscr{C}$) is *relativized* to a (regular) Pr-model $\mathcal{M}$!
- $\mathscr{F}$ is non-trivial to visualize! I haven't analyzed the combinatorics of $\mathscr{F}$ yet, but I have an algorithm for generating it. See my *MATHEMATICA*® notebook.
- I first proposed simply taking the straight average of $\mathscr{F}$, but other averages could be given (undoubtedly, some examples will suggest unequal weights).

Some Subtleties/Objections

- Individuation: The "information sets" (collections that $\mathscr{C}$ measures) could be *multisets/sequences of propositions*, or *sets of statements* (tokens), *etc.*, but not *sets of propositions*, unless we go anti-Stalnaker (which is controversial).
- Siebel: "if we are confronted with a pair of statements which cannot both be false together, Fitelson's function assigns it a coherence value of at most 0."
- True. But, this will be true for *any* Pr-relevance-based account (not just mine). If p and q can't both be *false*, then they cannot be *positively* correlated! Here, correlation goes beyond a naïve generalization of the metatheoretic $\Diamond(p \,\&\, q)$.
- Moretti (and others): On your view, logically equivalent sets of statements can have different degrees of coherence. Yep. But, this also strikes me as correct. [To my mind, $\{p, q, r\}$ *is* more coherent than $\{p, q\}$, provided that $r \Dashv\vDash p$.]
- Moretti: But, on your $\mathscr{C}$, adding $\top$ to a coherent set can make it *in*coherent! This *was* true on my old $\mathscr{C}$. But, *not* on my new $\mathscr{C}$. See my *MATHEMATICA*® notebook. $\mathscr{C}(\mathbf{S})$ can be $< \mathscr{C}(\mathbf{S} \cup \{\top\})$, but this is an artifact of *averaging*.