

Testing Arguments for Validity and Soundness

Philosophy 1115 (Logic)
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1 Visualizing the Procedure for Validity/Soundness Testing

Figure 1 provides a series of questions (and their possible answers), which will help us to determine whether an argument is valid (or sound). In the next section, I will apply this method to several arguments from our introductory lectures.

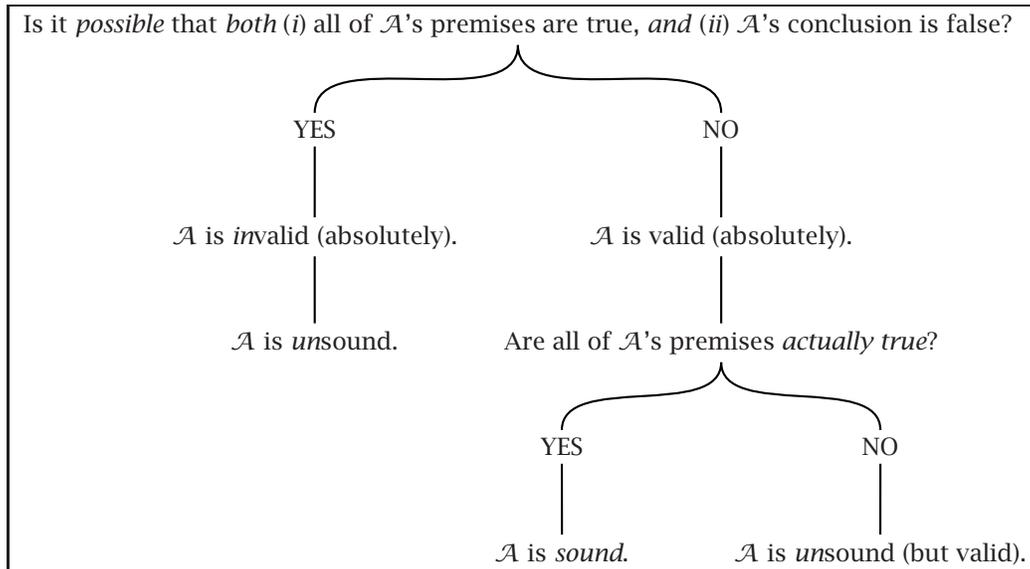


Figure 1: Testing an argument \mathcal{A} for (absolute) validity and soundness.

2 Applying the Test to Some Examples

2.1 Example #1 — An “Easy” Valid Argument

Recall our first example from last time:

Dr. Ruth is a man.

\mathcal{A}_1 : If Dr. Ruth is a man, then Dr. Ruth is 10 feet tall.
 \therefore Dr. Ruth is 10 feet tall.

The method depicted visually in Figure 1 leads to the following sequence of questions (and answers) about argument \mathcal{A}_1 .

Q₁: Is it possible that both (i) all of the premises of \mathcal{A}_1 are true, and (ii) the conclusion of \mathcal{A}_1 is false?

A₁: NO. Imagine a world in which it is true that Dr. Ruth is a man and it is true that if Dr. Ruth is a man, then Dr. Ruth is 10 feet tall. Any possible world of this kind will also be a possible world in which Dr. Ruth is 10 feet tall. So, there is no possible world in which (i.e., it is impossible that) both (i) and (ii) obtain. Therefore, \mathcal{A}_1 is valid.

Q₂: Are all of \mathcal{A}_1 's premises actually true?

A₂: NO. In fact, neither of \mathcal{A}_1 's premises is true in the actual world. Therefore, \mathcal{A}_1 is *unsound* (but *valid*, nonetheless!).

2.2 Example #2 — A “Tricky” Valid Argument

\mathcal{A}_2 : Branden weighs 200 lbs and Branden does not weigh 200 lbs.
 \therefore The moon is made of green cheese.

This time, we have the following sequence of questions (and answers) about argument \mathcal{A}_2 .

Q₁: Is it possible that both (i) all of the premises of \mathcal{A}_2 are true, and (ii) the conclusion of \mathcal{A}_2 is false?

A₁: NO. Try to imagine a possible world in which the premise of \mathcal{A}_2 is true and the conclusion of \mathcal{A}_2 is false. This would have to be a world in which *all* of the following three propositions are true:

- (1) Branden weighs 200 lbs.
- (2) Branden does not weigh 200 lbs.
- (3) The moon is not made of green cheese.

Of course, there is no problem imagining a world in which (3) is true (our very own actual world will do just fine!). But, there can be *no* possible world in which *both* (1) *and* (2) are true simultaneously, since (2) is just the *denial* of (1). So, there is no possible world in which (*i.e.*, it is *impossible* that) both (i) and (ii) obtain. Therefore, \mathcal{A}_2 is valid.

Q₂: Are all of \mathcal{A}_2 's premises *actually true*?

A₂: NO. In fact, \mathcal{A}_2 's premise is false in *all* possible worlds (not just ours!). Therefore, \mathcal{A}_2 *unsound* (but *valid*, nonetheless!).

2.3 Example #3 — Another “Tricky” Valid Argument

\mathcal{A}_3 : Glass is a liquid.
∴ If Branden is 10 feet tall, then Branden is 10 feet tall.

Q₁: Is it *possible* that *both* (i) all of the premises of \mathcal{A}_3 are true, *and* (ii) the conclusion of \mathcal{A}_3 is false?

A₁: NO. Try to imagine a possible world in which the premise of \mathcal{A}_3 is true and the conclusion of \mathcal{A}_3 is false. This would have to be a world in which *both* of the following two propositions are true:

- (1) Glass is a liquid.
- (2) *It is not the case that* if Branden is 10 feet tall, then Branden is 10 feet tall.

Of course, there is no problem imagining a world in which (1) is true (our very own actual world will do just fine!). But, there is *no* possible world in which (2) is true. Statements of the form “If *p*, then *p*” are called *tautologies* (this term will be defined and discussed in chapter 3) — they are *necessarily true* (*i.e.*, it is *impossible* for them to be false). So, there is no possible world in which (*i.e.*, it is *impossible* that) both (i) and (ii) obtain. Therefore, \mathcal{A}_3 is valid.

Q₂: Are all of \mathcal{A}_3 's premises *actually true*?

A₂: YES. In the actual world, glass *is* a liquid. Therefore, \mathcal{A}_3 is *sound*!

2.4 Example #4 — An Invalid Argument

Most professional basketball players are over 6 feet tall.
 \mathcal{A}_4 : Joe is a professional basketball player.
∴ Joe is over 6 feet tall.

Q: Is it *possible* that *both* (i) all of the premises of \mathcal{A}_4 are true, *and* (ii) the conclusion of \mathcal{A}_4 is false?

A: YES. It is easy to imagine a world in which *most* professional basketball players are over 6 feet tall, but *some* (*e.g.*, Joe) are *not*.¹ So, it is possible that both (i) and (ii) obtain. Therefore, \mathcal{A}_4 is *invalid* (*i.e.*, *NOT* valid) and *unsound*.

¹If this “most” were changed to “all,” then argument \mathcal{A}_4 *would* be valid. *Why?*