Comments on Khoo & Mandelkern

Branden Fitelson

04/12/17

Thanks 10^6 to Justin & Matt (J&M) for their highly edifying paper [7]. They have really helped me to better understand the significance of my two recent (strengthened) triviality results for the indicative conditional [6, 5, 4]. I will pick up where they left off — and discuss some *probabilistic* phenomena in the vicinity.[1]

1 J&M's Probabilistic Counterexample to LIE

> Suppose that John has just rolled a fair six-sided die, but the result is hidden. Let $Q \stackrel{\text{def}}{=}$ John rolled an even number and $P = R \stackrel{\text{def}}{=}$ John rolled a prime. Then, consider this dialogue:

Smith : If John rolled an even number, he rolled a prime. [$Q > R$]

Jane : If John rolled a prime, then what Smith said is true. [$P > (Q > R)$]

If John rolled a prime and John rolled an even number, he rolled a prime. [$(P \,\&\, Q) > R$]

Then, we have the following facts.

(1) $\Pr(R \mid Q \,\&\, P) = \Pr(R \mid P) = 1$.

(2) $\Pr(R \mid Q \,\&\, \overline{P}) = \Pr(R \mid \overline{P}) = 0$.

(3) $\Pr(R \mid Q) = \frac{1}{3} < \Pr(R) = \frac{1}{2}$.

(4) $\Pr(R \mid Q \,\&\, P) = 1 \gg \Pr(R) = \frac{1}{2}$.

In words (sort of),

(1) If P, then Q is irrelevant to R. [Explains why $P > (Q > R)$ sounds meh?]

(2) If $\overline{P}$, then Q is irrelevant to R. [Explains why $\overline{P} > (Q > R)$ sounds meh?]

(3) Q is negatively relevant to R (unconditionally). [Explains why $Q > R$ sounds bad?]

(4) $Q \& P$ is (strongly) positively relevant to R (unconditionally). [Explains why $(P \& Q) > R$ sounds good?]

That is, J&M's counterexample to LIE is a case of *Simpson's Paradox*. I suspect that Simpson's Paradox cases are of interest here, more generally. If *confirmation* (*i.e., relevance*) relations are pertinent when evaluating indicative conditionals [2], then there will be an intimate connection between (putative) counterexamples to IE principles and Simpson's Paradox. To be more precise, in [3], I show that — if probabilistic relevance is important for the evaluation of indicative conditionals — then *all* Simpson's Paradox cases will be potential counterexamples to IE. In the next section, I discuss another case (with a non-extremal Simpson's Paradoxical structure), which may cause trouble for *non*-logical versions of IE.

[1] I don't have time to discuss my strengthened probabilistic triviality result [5] here (or J&M's terrific analysis of it). But, they argue convincingly that it enjoys a dialectical position similar to my Gibbardian result. That is, many of the usual replies to Lewis's original triviality results (*e.g.*, denying "closure under conditionalization") are not effective against it.

2 A (Non-Anaphoric?) Counterexample to *Non*-Logical IE?

Imagine admissions at a fictitious university, and consider these three claims about a (randomly chosen) applicant.[2] $R \overset{\text{def}}{=}$ applicant is admitted, $P \overset{\text{def}}{=}$ applicant applies to the honors program, and $Q \overset{\text{def}}{=}$ applicant is female. Finally, suppose that the admissions statistics (or propensities, if you like) are as follows.

	Q	$\overline{Q}$	Overall
P	$0.64 = \Pr(R \mid Q \,\&\, P)$	$0.108696 = \Pr(R \mid \overline{Q} \,\&\, P)$	$0.331933 = \Pr(R \mid P)$
$\overline{P}$	$0.992966 = \Pr(R \mid Q \,\&\, \overline{P})$	$0.992757 = \Pr(R \mid \overline{Q} \,\&\, \overline{P})$	$0.9928 = \Pr(R \mid \overline{P})$
Overall	$0.979138 = \Pr(R \mid Q)$	$0.980226 = \Pr(R \mid \overline{Q})$	$0.98 = \Pr(R)$

That is, we have the following facts.

(1) $\Pr(R \mid Q \,\&\, P) = 0.64 \gg \Pr(R \mid P) \approx 0.33$.

(2) $\Pr(R \mid Q \,\&\, \overline{P}) = 0.992966 \gtrapprox \Pr(R \mid \overline{P}) = 0.9928$.

(3) $\Pr(R \mid Q) = 0.979138 \lessapprox \Pr(R) = 0.98$.

(4) $\Pr(R \mid Q \,\&\, P) = 0.64 \ll \Pr(R) = 0.98$.

In words,

(1) If P, then Q is *strongly positively relevant to* R. [$P > (Q > R)$ sound good?]

- If you apply to honors, then if you're female you'll be admitted. (☺)

(2) If $\overline{P}$, then Q is *weakly positively relevant to* R. [$\overline{P} > (Q > R)$ sound OK?]

- If you apply to non-honors, then if you're female you'll be admitted.

(3) Q is *weakly negatively relevant to* R (unconditionally). [$Q > R$ sound meh?]

- If you're female, then you'll be admitted.

(4) $Q \,\&\, P$ is *strongly negatively relevant to* R (unconditionally). [$(P \,\&\, Q) > R$ sound bad?]

- If you're a female honors applicant, then you'll be admitted. (☹)

The key IE contrast here is between (1) and (4). This sounds like a potential counterexample to IE to me (and one which doesn't seem to rely on anaphora). In any case, examples such as these bring out some interesting connections between confirmation, Simpson's Paradox, indicatives, and IE [2, 3, 8].

References

[1] N. Cartwright, "Causal Laws and Effective Strategies." *Noûs* (1979): 419–437.
[2] I. Douven, "The Evidential Support Theory of Conditionals." *Synthese* (2009): 19–44.
[3] B. Fitelson. "Confirmation, Causation, and Simpson's Paradox." *Episteme*, to appear.
[4] ______. "Two new(ish) triviality results for the indicative conditional." Lecture notes, 2016.
[5] ______. "The Strongest Possible Lewisian Triviality Result." *Thought* (2015): 69–74.
[6] ______. "Gibbard's collapse theorem for the indicative conditional: an axiomatic approach." In *Automated Reasoning and Mathematics*, (2013): 181–188. Springer.
[7] J. Khoo & M. Mandelkern, "Triviality results and the relationship between logical and natural languages." Manuscript, 2017.
[8] M. Kotzen, "Conditional Oughts and Simpson's Paradox." Manuscript, 2013.

[2] This example is (loosely) based on the Berkeley Graduate School case — as analyzed by Nancy Cartwright [1].