

## Goodman's "Grue" Argument in Historical Perspective

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## Overview of Today's Talk

- Today, my main aim will be to sketch and trace some important consequences of the following (crude) analogy:  
entailment : inference :: confirmation : evidential support
- I will focus on arguments *against* classical deductive and inductive logic ("relevantist" and "grue" arguments).
- The talk is mainly *defensive*. I won't offer positive accounts of the "paradoxical" cases I will discuss (but, see "Extras").
- I'll begin with Harman's defense of classical deductive logic against certain (epistemological) "relevantist" arguments.
- Then, I'll argue that *if* you like Harman's defensive move in the deductive case, you should like a similar defense of inductive logic (from Goodman's "grue") *even more*.
- I will indicate how a "Harmanian maneuver" might be used to defend either Hempelian or Carnapian inductive logic.
- I will focus mainly on defending Carnapian IL from "grue".

- Here is a (naïve) "*reductio*" of classical deductive logic:
  - (1) For all sets of statements  $X$  and all statements  $p$ , if  $X$  is inconsistent, then  $p$  is a logical consequence of  $X$ .
  - (2) If an agent  $S$ 's belief set  $B$  entails  $p$  (and  $S$  knows  $B \models p$ ), then it would be reasonable for  $S$  to infer/believe  $p$ .
  - (3) *Even if*  $S$  knows their belief set  $B$  is inconsistent (and, hence, that  $B \models p$ , for *any*  $p$ ), there are still *some*  $p$ 's such that it would *not* be reasonable for  $S$  to infer/believe  $p$ .
  - (4)  $\therefore$  Since (1)-(3) lead to absurdity, our initial assumption (1) must have been false — *reductio* of the "explosion" rule (1).
- Harman [9] would concede that (1)-(3) are inconsistent, and (as a result) that *something* is wrong with premises (1)-(3).
- But, he would reject the relevantists' diagnosis that (1) must be rejected. I take it he'd say it's (2) that is to blame here.
- ☞ (2) is a *bridge principle* [13] linking *entailment* and *inference*.
- (2) is correct *only* for *consistent*  $B$ 's. [*Even if*  $B$  is consistent, the correct response *may* rather be to *reject* some  $B_i$ 's in  $B$ .]

- The choice of *inconsistent* belief set  $B$  is intentional here.
- In such contexts, there is a *deep disconnect* between (known) *entailment* relations and (kosher) *inferential* relations.
- Will a more sophisticated DBP (2') help here? A *dilemma*:
  - (2') will be *too weak* to yield a (classically) *valid* "*reductio*".
  - or
  - (2') will be *false*. [Our original BP (2) falls under this horn.]
- Let  $B$  be  $S$ 's belief set, and let  $q$  be the conjunction of the elements  $B_i$  of  $B$ . Here are two more candidate BP's:
  - (2'<sub>1</sub>) If  $S$  knows that  $B \models p$ , then  $S$  should *not* be such that *both*:  $S$  believes  $q$ , *and*  $S$  does not believe  $p$ .
  - (2'<sub>2</sub>) If  $S$  knows that  $B \models p$ , then  $S$  should *not* be such that *both*:  $S$  believes each of the  $B_i \in B$ , *and*  $S$  does not believe  $p$ .
- (2'<sub>2</sub>) is *false* (preface paradox) *and* too weak (it's wide scope).
- (2'<sub>1</sub>) *may* be true, but it is also *too weak*. [It's wide scope, and the agent can reasonably disbelieve *both*  $q$  and  $p$ .]


## “Potted History” Version of Goodman’s Argument

- Consider the following two inductive arguments:

( $\mathcal{A}_1$ )  $(E_1)$   $a$  is a green emerald.  $\vdots (H_1)$  All emeralds are green. | ( $\mathcal{A}_2$ )  $(E_2)$   $a$  is a grue emerald.  $\vdots (H_2)$  All emeralds are grue.


- A “potted history” version of Goodman’s argument ([8]):
  - Arguments ( $\mathcal{A}_1$ ) and ( $\mathcal{A}_2$ ) have the same logical form.
  - Argument ( $\mathcal{A}_1$ ) is “inductively valid” (i.e.,  $E_1$  confirms  $H_1$ ).
  - ( $\mathcal{A}_2$ ) is *not* “inductively valid” (i.e.,  $E_2$  does *not* confirm  $H_2$ ).
  - $\vdots$  “Inductive validity” is not *merely* a matter of logical form.
- My talk today aims mainly to undermine Goodman’s argument (in FF&F [8]) for premises (2) and (3).

- Sidebar: I also think (1) is *question-begging*. I won’t be able to get to this today, but see my “Extras” slides for more.

 Goodman’s argument against *inductive* logic is analogous to the (unsound) argument above against classical *deductive* logic. This is what the rest of the talk will aim to establish.


- I’ll begin by laying out the salient bits of the inductive logical (*viz.*, *confirmation*) theories of Hempel and Carnap.
- Hempel’s confirmation theory uses *entailment* to explicate “inductive logical support” (confirmation), which is a logical relation between statements. [ $E$  confirms  $H$  iff  $E = \text{dev}_E(H)$ ]
- Hempel’s theory has the following three key consequences:
  - (EQC) If  $E$  confirms  $H$  and  $E \neq E'$ , then  $E'$  confirms  $H$ .
  - (NC) For all constants  $x$  and all (consistent) predicates  $\phi$  and  $\psi$ : ‘ $\phi x \ \& \ \psi x$ ’ confirms ‘ $(\forall y)(\phi y \supset \psi y)$ ’.
  - (M) For all  $x$ , for all (consistent)  $\phi$  and  $\psi$ , and all statements  $H$ : If ‘ $\phi x$ ’ confirms  $H$ , then ‘ $\phi x \ \& \ \psi x$ ’ confirms  $H$ .
- These three properties are the crucial ones needed to reconstruct Goodman’s “grue” argument against Hempel.
- Before giving a precise reconstruction of Goodman’s “grue” argument, we’ll look at the essentials of Carnapian IL/CT. [Goodman targeted both Hempel and Carnap in FF&F [8].]

- Carnapian confirmation (i.e., *later* Carnapian theory [14]) is based on *probabilistic relevance*, *not* deductive entailment:
  - $E$  confirms  $H$ , relative to  $K$  iff  $\text{Pr}(H \mid E \ \& \ K) > \text{Pr}(H \mid K)$ , for some “suitable” conditional probability function  $\text{Pr}(\cdot \mid \cdot)$ .
    - Note how this is an *explicitly* 3-place relation. Hempel’s was only 2-place. This is because  $\text{Pr}$  (unlike  $=$ ) is *non-monotonic*.
    - Carnap thought “suitable  $\text{Pr}$ ” meant “*logical*  $\text{Pr}$ ” in a very strong/naive sense. But, Goodman’s argument (charitably reconstructed) will work against *any* probability function  $\text{Pr}$ .

 Carnap’s theory implies *only 1* of our 3 Hempelian claims: (EQC). It does *not* imply either (NC) *or* (M) (see [4]/[14]).

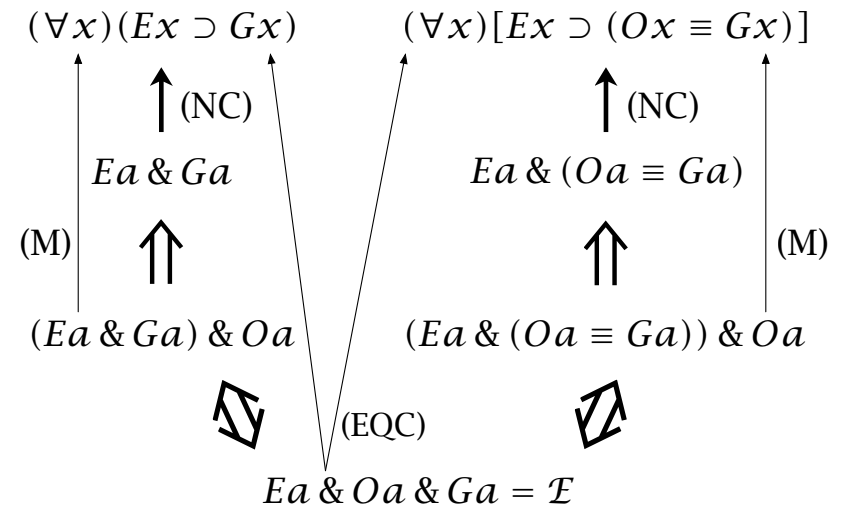
- This will allow Carnapian IL to avoid facing the full brunt of Goodman’s “grue” (but, it will still face a serious challenge).

- For Carnap, confirmation is a *logical* relation (akin to entailment). Like entailment, confirmation can be *applied*, but this requires *epistemic bridge principles* [akin to (2)].
- Carnap [1] discusses various bridge principles. The most well-known of these is the *requirement of total evidence*.

- The Requirement of Total Evidence.** In the application of IL to a given knowledge situation, the total evidence available must be taken as a basis for determining the degree of confirmation.
- This *sounds* like a plausible principle. But, once it is made more precise, it will actually turn out to be subtly defective.
- More precisely, we have the following *bridge principle* connecting *confirmation* and *evidential support*:
  - (RTE)  $E$  evidentially supports  $H$  for  $S$  in  $C$  iff  $E$  confirms  $H$ , relative to  $K$ , where  $K$  is  $S$ ’s *total evidence* in  $C$ .
- The (RTE) has often been (implicitly) presupposed by Bayesian epistemologists (both subjective and objective).
- But, we’ll see that (RTE) is an *independently* implausible BP.
-  Moreover, Goodman’s “grue” argument will rely *more heavily* on (RTE) than the relevantists’ argument relies on (2). In this sense, Goodman’s argument will be *even worse*.
- Before reconstructing the argument, a brief “grue” primer.

- Let  $Gx \stackrel{\text{def}}{=} x$  is green,  $Ox \stackrel{\text{def}}{=} x$  is examined prior to  $t$ , and  $Ex \stackrel{\text{def}}{=} x$  is an emerald. Goodman introduces a predicate “grue”
 
$$Gx \stackrel{\text{def}}{=} x \text{ is grue} \stackrel{\text{def}}{=} Ox \equiv Gx.$$
- Consider the following two universal generalizations
  - $(H_1)$  All emeralds are green.  $[(\forall x)(Ex \supset Gx)]$
  - $(H_2)$  All emeralds are grue.  $[(\forall x)[Ex \supset (Ox \equiv Gx)]]$
- And, consider the following instantial evidential statement
  - $(E) Ea \& Oa \& Ga$
- Hempel’s confirmation theory [(EQC) & (NC) & (M)] entails:
  - $(\dagger) E$  confirms  $H_1$ , and  $E$  confirms  $H_2$ . [proof]
  - $\therefore (\ddagger) E$  confirms  $H_1$  if and only if  $E$  confirms  $H_2$ .
- What about (later) Carnapian theory? Does it entail  $(\ddagger)$ ?
  - NO! There are Carnapian *countermodels* to  $(\ddagger)$ . [See Extras.]
- So, Hempel was an easier target for Goodman than (later) Carnap (to be fair, Goodman talks only about *early* Carnap).
- Now, we’re ready to reconstruct Goodman’s argument.

## A Proof of $(\dagger)$ From Hempel’s (NC), (M), and (EQC)



- There is just one more ingredient in Goodman’s argument:
  - The agent  $S$  who is assessing the evidential support that  $E$  provides for  $H_1$  vs  $H_2$  in a Goodmanian “grue” context  $C_G$  has  $Oa$  as part of their total evidence in  $C_G$ . (e.g., [2], [16].)
- Now, we can run the following Goodmanian “*reductio*”:
  - (i)  $E$  confirms  $H$ , relative to  $K$  iff  $\text{Pr}(H | E \& K) > \text{Pr}(H | K)$ .
  - (ii)  $E$  evidentially supports  $H$  for  $S$  in  $C$  iff  $E$  confirms  $H$ , relative to  $K$ , where  $K$  is  $S$ ’s total evidence in  $C$ .
  - (iii) The agent  $S$  who is assessing the evidential support  $E$  provides for  $H_1$  vs  $H_2$  in a Goodmanian “grue” context  $C_G$  has  $Oa$  as part of their total evidence in  $C_G$  [i.e.,  $K \models Oa$ ].
  - (iv) If  $K \models Oa$ , then—*c.p.*— $E$  confirms  $H_1$  relative to  $K$  iff  $E$  confirms  $H_2$  relative to  $K$ , for **any**  $\text{Pr}$  [i.e.,  $(\ddagger)$  holds,  $\forall$   $\text{Pr}$ ’s].
  - (v) Therefore,  $E$  evidentially supports  $H_1$  for  $S$  in  $C_G$  if and only if  $E$  evidentially supports  $H_2$  for  $S$  in  $C_G$ .
  - (vi)  $E$  evidentially supports  $H_1$  for  $S$  in  $C_G$ , but  $E$  does *not* evidentially support  $H_2$  for  $S$  in  $C_G$ .
- $\therefore$  (i)–(vi) lead to an absurdity. Hence, our initial assumption (i) must have been false. Carnapian inductive logic refuted?

- Premise (vi) is based on Goodman’s *epistemic intuition* that, in “grue” contexts,  $E$  evidentially supports  $H_1$  but *not*  $H_2$ .
- Premise (v) follows logically from premises (i)–(iv).
- Premise (iv) is a theorem of probability calculus (**any**  $\text{Pr}$ !).
  - The *c.p.* clause is  $\text{Pr}(Ea | H_1 \& K) = \text{Pr}(Ea | H_2 \& K)$ . [See [16].]
- Premise (iii) is an assumption about the agent’s background knowledge  $K$  that’s implicit in Goodman’s set-up ([2], [16]).
- Premise (ii) is (RTE). It’s the *bridge principle*, akin to (2) in the relevantists’ *reductio*. This is the premise I will focus on.
- Here are my two main points about Goodman’s argument:
  - (ii) must be rejected by Bayesians, for *independent* reasons.
  - Unlike Hempel’s theory, Carnap’s  $c$ -theory *doesn’t entail*  $(\ddagger)$ .
- This suggests Goodman’s argument is *even less a reductio* of (i) than the relevantists’ argument is a *reductio* of (1).
- Moreover, a careful reading of *Fact, Fiction, and Forecast* reveals that this *was* Goodman’s argumentative strategy.

## Three Salient Quotes from Goodman [8]

☞ The “new riddle” is *about inductive logic (not epistemology)*.

**Quote #1** (page 67): “Just as deductive logic is concerned primarily with a relation between statements — namely the consequence relation — that is independent of their truth or falsity, so inductive logic ... is concerned primarily with a comparable relation of confirmation between statements. Thus the problem is to define the relation that obtains between any statement  $S_1$  and another  $S_2$  if and only if  $S_1$  may properly be said to confirm  $S_2$  in any degree.”

**Quote #2** (73): “Confirmation of a hypothesis by an instance depends ... upon features of the hypothesis other than its syntactical form”.

☞ But, Goodman’s *methodology* appeals to *epistemic* intuitions.

**Quote #3** (page 73): “... the fact that a given man now in this room is a third son *does not increase the credibility of* statements asserting that other men now in this room are third sons, *and so does not confirm* the hypothesis that all men now in this room are third sons.”

- As Tim Williamson points out [18, ch. 9], Carnap’s (RTE) must be rejected, because of the problem of old evidence [3].
- If  $S$ ’s total evidence in  $C$  ( $K$ ) entails  $E$ , then, according to (RTE),  $E$  cannot evidentially support *any*  $H$  for  $S$  in  $C$ .
- As a result, there are  $C$ ’s in which we can’t use  $\text{Pr}(\cdot | K)$  — for *any*  $\text{Pr}$  — when assessing the *evidential import of*  $E$  in  $C$ .
- There are (basically) two kinds of strategies for revising (RTE). Carnap [1, p. 472] & Williamson [18, ch. 9] suggest:  
(RTE<sub>+</sub>)  $E$  evidentially supports  $H$  for  $S$  in  $C$  iff  $S$  possesses  $E$  as evidence in  $C$  and  $\text{Pr}_+(H | E \& K_+) > \text{Pr}_+(H | K_+)$ . [ $K_+$  is “empty”,  $\text{Pr}_+$  is “inductive” [14]/“evidential” [18]/“logical” [1].]
- Note: Hempel explicitly *required* that confirmation be taken “*relative to*  $K_+$ ” in all treatments of the paradoxes [10, 11]. (RTE<sub>+</sub>) is a charitable Carnapian reconstruction of Hempel.
- A more “standard” way to revise (RTE) is [(RTE’)] to use  $\text{Pr}_{S'}(\cdot | K')$ , where  $K \models K' \neq E$ , and  $\text{Pr}_{S'}$  is the credence function of a “counterpart”  $S'$  of  $S$  with total evidence  $K'$ .

- Carnap never re-wrote the part of LFP [1] that discusses the (RTE), in light of a probabilistic *relevance* (“increase in firmness” [1]) notion of confirmation. This is too bad.
- If Carnap had discussed this (“old evidence”) issue, I suspect he would have used something like (RTE<sub>+</sub>) as his bridge principle connecting confirmation and evidential support.
- Various other philosophers have proposed similar accounts of “support” as some probabilistic relation, taken relative to an “empty” (perhaps “*a priori*”) background  $\&/\vee$  probability.
  - Richard Fumerton (who, unlike Williamson, is an epistemological *internalist*) proposes such a view in his [5].
  - Patrick Maher [14] applies such relations extensively in his recent (neo-Carnapian) work on confirmation theory.
  - Brian Weatherson [17] uses a similar, “Keynesian” [12] inductive-probability approach to evidential support.
- So, many “Bayesians” *already* reject (RTE), for reasons that are largely *independent* of “gruesome” considerations.

- So far, I have left open (precisely) what I think Bayesian confirmation theorists *should* say (*logically & epistemologically*) in light of Goodman’s “grue” paradox.
- Clearly, BCTs will need to revise (RTE) in light of “grue”. But, the standard (RTE’) way of doing this to cope with “old evidence” isn’t powerful enough to avoid *both* problems.
- The more draconian (RTE<sub>+</sub>) — suggested by the work of Carnap — avoids both problems, from a *logical* point of view (*if* “inductive”/“logical” probabilities *exist!*). But, what should would-be “Carnapians” say on the *epistemic* side?
- I’m not sure what the evidential relations *are* in “grue” contexts (but, see “Extras”). But, *that* doesn’t undermine my line on Goodman’s “grue” *argument* against *inductive logic*.
  - Analogy: Harman doesn’t tell us (in general) how someone *should* respond to the discovery that their beliefs are inconsistent. But, *that* doesn’t undermine Harman’s points about relevantist “*reductios*” of classical deductive logic.

Overview Harman Hempel Carnap Goodman (RTE) References Extras

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## "Carnapian" Counterexamples to (NC) and (M)

(K) Either: (H) there are 100 black ravens, no nonblack ravens, and 1 million other things, or (~H) there are 1,000 black ravens, 1 white raven, and 1 million other things.

- Let  $E \stackrel{\text{def}}{=} Ra \ \& \ Ba$  ( $a$  randomly sampled from universe). Then:
 
$$\Pr(E \mid H \ \& \ K) = \frac{100}{1000100} \ll \frac{1000}{1001001} = \Pr(E \mid \sim H \ \& \ K)$$
- $\therefore$  This  $K/\Pr$  constitute a counterexample to (NC), assuming a "Carnapian" theory of confirmation. This model can be emulated in the later Carnapian  $\lambda/\gamma$ -systems [14].

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- Let  $Bx \stackrel{\text{def}}{=} x$  is a black card,  $Ax \stackrel{\text{def}}{=} x$  is the ace of spades,  $Jx \stackrel{\text{def}}{=} x$  is the jack of clubs, and  $K \stackrel{\text{def}}{=} a$  card  $a$  is sampled at random from a standard deck (where  $\Pr$  is also standard):
  - $\Pr(Aa \mid Ba \ \& \ K) = \frac{1}{26} > \frac{1}{52} = \Pr(Aa \mid K)$ .
  - $\Pr(Aa \mid Ba \ \& \ Ja \ \& \ K) = 0 < \frac{1}{52} = \Pr(Aa \mid K)$ .

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## A "Carnapian" Counterexample to ( $\ddagger$ )

(K) Either: ( $H_1$ ) there are 1000 green emeralds 900 of which have been examined before  $t$ , no non-green emeralds, and 1 million other things in the universe, or ( $H_2$ ) there are 100 green emeralds that have been examined before  $t$ , no green emeralds that have not been examined before  $t$ , 900 non-green emeralds that have not been examined before  $t$ , and 1 million other things.

- Imagine an urn containing true descriptions of each object in the universe ( $\Pr \stackrel{\text{def}}{=} \text{urn model}$ ). Let  $\mathcal{E} \stackrel{\text{def}}{=} "Ea \ \& \ Oa \ \& \ Ga"$  be drawn.  $\mathcal{E}$  confirms  $H_1$  but  $\mathcal{E}$  disconfirms  $H_2$ , relative to  $K$ :
 
$$\Pr(\mathcal{E} \mid H_1 \ \& \ K) = \frac{900}{1001000} > \frac{100}{1001000} = \Pr(\mathcal{E} \mid H_2 \ \& \ K)$$
- This  $K/\Pr$  constitute a counterexample to ( $\ddagger$ ), assuming a "Carnapian" theory of confirmation. This probability model can be emulated in the later Carnapian  $\lambda/\gamma$ -systems [14].

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## What Could "Carnapian" Inductive Logic Be? Part I

- Many logical empiricists dreamt that inductive logic (confirmation theory) could be formulated in such a way that it *supervenes* on deductive logic in a *very strong* sense.
  - Strong Supervenience (SS)**. All confirmation relations involving sentences of a first-order language  $\mathcal{L}$  supervene on the *deductive-logical* (*viz.*, syntactical) structure *of*  $\mathcal{L}$ .
- Hempel clearly saw (SS) as a *desideratum* for confirmation theory. The early Carnap also seems to have (SS) in mind.
- I think it is fair to say that Carnap's project — understood as requiring (SS) — was unsuccessful. [Note: I think this is true for reasons that are *independent* of Goodman's "grue".]
- The later Carnap seems to be aware of this. Most commentators interpret this shift as the later Carnap simply *giving up* on inductive logic (*qua logic*) altogether.
- I want to resist this "standard" reading of the history.

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## What Could “Carnapian” Inductive Logic Be? Part II

- I propose a different reading of the later Carnap, which makes him much more coherent with the early Carnap.
- I propose *weakening* the supervenience requirement in such a way that it (a) ensures this coherence, and (b) maintains the “logicality” of confirmation relations in Carnap’s sense.
- Let  $\mathcal{L}$  be a formal language strong enough to express the fragment of probability theory Carnap needs for his later, more sophisticated confirmation-theoretic framework.
  - **Weak Supervenience (WS)**. All confirmation relations involving sentences of a first-order language  $\mathcal{L}$  supervene on the *deductive-logical* (*viz.*, syntactical) structure of  $\mathcal{L}$ .
- Happily,  $\mathcal{L}$  is pretty weak (Carnap’s  $c$ -theories are *decidable*). So, even by early (*logicist*) Carnapian lights, satisfying (WS) is sufficient to ensure the “logical determinateness” of  $c$ .
- The specific (WS) approach I favor takes confirmation to be a 4-place relation: between  $E, H, K$ , and a Pr-model  $\mathcal{M}$ .

## What Could “Carnapian” Inductive Logic Be? Part III

- Consequences of moving to such a 4-place  $c$ -relation:
  - We need not try to “construct” “logical” probability functions from the syntax of  $\mathcal{L}$ . This is a dead-end anyhow.
  - Indeed, on this view, inductive logic has nothing to say about the *interpretation/origin* of Pr. That is *not a logical* question, but a question about the *application* of logic.
    - Analogy: Deductive logicians don’t owe us a “logical interpretation/construction” of the *valuation function*.
  - Moreover, this leads to a vast increase in the *generality* of inductive logic. Carnap was stuck with an impoverished set of “logical” probability functions (in his  $\lambda/\gamma$ -continuum).
    - On my approach, *any* probability function can be part of a confirmation relation (*via*  $\mathcal{M}$ ). Which functions are “appropriate” or “interesting” will depend on *applications*.
    - So, some confirmation relations will not be “interesting”, *etc.* But, this is (already) true of *entailments*, as Harman showed.
  - Questions: Now, what *is* the job of the inductive logician, and how (if at all) do they interact with *epistemologists*?

## What Could “Carnapian” Inductive Logic Be? Part IV

- The inductive logician must explain how it is that inductive logic can satisfy the following Carnapian *desiderata*.
  - The confirmation function  $c_{\mathcal{M}}(H, E | K)$  quantifies a *logical* (in a Carnapian sense) relation between  $E, H$ , and  $K$ .
    - ( $\mathcal{D}_1$ ) “Logical determinateness” of  $c$  is ensured by the move from (SS) to (WS) [from an  $\mathcal{L}$ -determinate to an  $\mathcal{L}$ -determinate  $c$ ].
    - ( $\mathcal{D}_2$ ) Another aspect of “logicality” insisted upon by Carnap is that  $c_{\mathcal{M}}(H, E | K)$  should *generalize* the entailment relation.
      - At least:  $c_{\mathcal{M}}(H, E | K)$  should take a max (min) value when  $E \& K \models H$  ( $E \& K \models \sim H$ ) — *for all (regular) Pr-models*  $\mathcal{M}$ .
  - ( $\mathcal{D}_3$ ) There must be *some* interesting “bridge principles” linking  $c$  and *some* relations of evidential support, in *some* contexts.
    - My basic “bridging” idea (rough): subject-context pairs  $\langle S, C \rangle$  will determine “epistemically appropriate” Pr-models  $\mathcal{M}$ .
    - ( $\mathcal{D}_2$ ) implies that *if* there are any such bridge principles linking *entailment* and (say) *conclusive evidence*, these will be *inherited* by  $c$ . So, we also inherit Harman’s problem!

## “Potted History” Version of Goodman’s Argument (#2)

- Some say that “sensitivity to choice of language” is a central/essential theme/aspect of Goodman’s argument.
- But, this *can’t* be the case, for many reasons. Here’s one:
  1. Goodman’s main target was *Hempel*.
  2. Hempel’s  $c$ -relation is defined in terms of  $\models$ .
  3.  $\models$  is *not* (essentially) sensitive to choice of language.
  4. Or, if  $\models$  is sensitive to choice of language (and said sensitivity is *essential* to Goodman’s argument), then Goodman’s riddle is *neither new nor peculiar to induction*.
- Carnap’s *later* theories of  $c$  are sensitive to choice of language. But, (a) Goodman was not aware of those later theories, and (b) “grue” doesn’t reveal *that* problem anyway.
- In order to pinpoint the (pernicious) language-variance of Carnap’s later  $c$ -theories, more sophisticated constructions are required (*e.g.*, David-Miller-esque [15, Ch. 11] constructions).