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Goodman’s “Grue” Argument in Historical Perspective

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Overview of Today’s Talk

- Today, my main aim will be to sketch and trace some important consequences of the following (crude) analogy: entailment : inference : : confirmation : evidential support
- I will focus on arguments against classical deductive and inductive logic (“relevantist” and “grue” arguments).
- The talk is mainly defensive. I won’t offer positive accounts of the “paradoxical” cases I will discuss (but, see “Extras”).
- I’ll begin with Harman’s defense of classical deductive logic against certain (epistemological) “relevantist” arguments.
- Then, I’ll argue that if you like Harman’s defensive move in the deductive case, you should like a similar defense of inductive logic (from Goodman’s “grue”) even more.
- I will indicate how a “Harmanian maneuver” might be used to defend either Hempelian or Carnapian inductive logic.
- I will focus mainly on defending Carnapian IL from “grue”.

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- Here is a (naive) “reductio” of classical deductive logic:
  1. For all sets of statements X and all statements p, if X is inconsistent, then p is a logical consequence of X.
  2. If an agent S’s belief set B entails p (and S knows B ⊨ p), then it would be reasonable for S to infer/believe p.
  3. Even if S knows their belief set B is inconsistent (and, hence, that B ⊨ p, for any p), there are still some p’s such that it would not be reasonable for S to infer/believe p.
  4. \(\therefore\) Since (1)–(3) lead to absurdity, our initial assumption (1) must have been false — *reductio* of the “explosion” rule (1).

- Harman [9] would concede that (1)–(3) are inconsistent, and (as a result) that *something* is wrong with premises (1)–(3).

- But, he would reject the relevantists’ diagnosis that (1) must be rejected. I take it he’d say it’s (2) that is to blame here.

- (2) is a *bridge principle* [13] linking entailment and inference.

- (2) is correct only for consistent B’s. [Even if B is consistent, the correct response may rather be to reject some B’s in B.]

- The choice of *inconsistent* belief set B is intentional here. In such contexts, there is a *deep disconnect* between (known) entailment relations and (kosher) inferential relations.

- Will a more sophisticated DBP (2’) help here? A *dilemma*:
  1. (2’) will be *too weak* to yield a (classically) valid “reductio”. *or*
  2. (2’) will be *false*. [Our original BP (2) falls under this horn.]

- Let B be S’s belief set, and let q be the conjunction of the elements B_i of B. Here are two more candidate BP’s:
  1. If S knows that B ⊨ p, then S should not be such that *both*: S believes q, and S does not believe p.
  2. If S knows that B ⊨ p, then S should not be such that *both*: S believes each of the B_i ∈ B, and S does not believe p.
  3. (2’) is *false* (preface paradox) and *too weak* (it’s wide scope).
  4. (2’) may be true, but it is also *too weak*. [It’s wide scope, and the agent can reasonably disbelieve *both q and p*.]
Consider the following two inductive arguments:

\[(\alpha_1) \quad (E_1) \quad a \text{ is a green emerald.} \quad \therefore (H_1) \quad \text{All emeralds are green.} \]

\[(\alpha_2) \quad (E_2) \quad a \text{ is a grue emerald.} \quad \therefore (H_2) \quad \text{All emeralds are grue.} \]

A “potted history” version of Goodman’s argument [8]:

1. Arguments (\alpha_1) and (\alpha_2) have the same logical form.
2. Argument (\alpha_1) is “inductively valid” (i.e., E_1 confirms H_1).
3. (\alpha_2) is not “inductively valid” (i.e., E_2 does not confirm H_2).
4. “Inductive validity” is not merely a matter of logical form.

My talk today aims mainly to undermine Goodman’s argument (in FF&F [8]) for premises (2) and (3).

Sidebar: I also think (1) is question-begging. I won’t be able to get to this today, but see my “Extras” slides for more.

Goodman’s argument against inductive logic is analogous to the (unsound) argument above against classical deductive logic. This is what the rest of the talk will aim to establish.

Carnapian confirmation (i.e., later Carnapian theory [14]) is based on probabilistic relevance, not deductive entailment:

- E confirms H, relative to K iff Pr(H | E & K) > Pr(H | K), for some “suitable” conditional probability function Pr(· | ·).
- Note how this is an explicitly 3-place relation. Hempel’s was only 2-place. This is because Pr (unlike \(\rightarrow\)) is non-monotonic.
- Carnap thought “suitable Pr” meant “logical Pr” in a very strong/naive sense. But, Goodman’s argument (charitably reconstructed) will work against any probability function Pr.

Carnap’s theory implies only 1 of our 3 Hempelian claims:

(EQC). It does not imply either (NC) or (M) (see [4]/[14]).

- This will allow Carnap IL to avoid facing the full brunt of Goodman’s “grue” (but, it will still face a serious challenge).
- For Carnap, confirmation is a logical relation (akin to entailment). Like entailment, confirmation can be applied, but this requires epistemic bridge principles [akin to (2)].
- Carnap [1] discusses various bridge principles. The most well-known of these is the requirement of total evidence.
There is just one more ingredient in Goodman's argument:
- The agent S who is assessing the evidential support that \( E \) provides for \( H_1 \) vs \( H_2 \) in a Goodmanian “grue” context \( C_G \) has \( Oa \) as part of their total evidence in \( C_G \), e.g., [2, [16].]

Now, we can run the following Goodmanian “reductio”:
- (i) \( E \) confirms \( H \), relative to \( K \) iff \( \Pr(H \mid E \& K) > \Pr(H \mid K) \).
- (ii) \( E \) evidentially supports \( H \) for \( S \) in \( C \) iff \( E \) confirms \( H \), relative to \( K \), where \( K \) is \( S \)’s total evidence in \( C \).
- (iii) The agent \( S \) who is assessing the evidential support \( E \) provides for \( H_1 \) vs \( H_2 \) in a Goodmanian “grue” context \( C_G \) has \( Oa \) as part of their total evidence in \( C_G \) [i.e., \( K \equiv Oa \)].
- (iv) If \( K \equiv Oa \), then—c.p.—\( E \) confirms \( H_1 \) relative to \( K \) iff \( E \) confirms \( H_2 \) relative to \( K \), for any \( \Pr \) [i.e., (\‡) holds, \( \forall \Pr \)’s].
- (v) Therefore, \( E \) evidentially supports \( H_1 \) for \( S \) in \( C_G \) if and only if \( E \) evidentially supports \( H_2 \) for \( S \) in \( C_G \).
- (vi) \( E \) evidentially supports \( H_1 \) for \( S \) in \( C_G \), but \( E \) does not evidentially support \( H_2 \) for \( S \) in \( C_G \).
- \( \vdash \) (i)–(vi) lead to an absurdity. Hence, our initial assumption (i) must have been false. Carnapian inductive logic refuted?
Three Salient Quotes from Goodman [8]

**Quote #1** (page 67): “Just as deductive logic is concerned primarily with a relation between statements — namely the consequence relation — that is independent of their truth or falsity, so inductive logic . . . is concerned primarily with a comparable relation of confirmation between statements. Thus the problem is to define the relation that obtains between any statement $S_1$ and another $S_2$ if and only if $S_1$ may properly be said to confirm $S_2$ in any degree.”

**Quote #2** (73): “Confirmation of a hypothesis by an instance depends . . . upon features of the hypothesis other than its syntactical form”.

**Quote #3** (page 73): “…the fact that a given man now in this room is a third son does not increase the credibility of statements asserting that other men now in this room are third sons, and so does not confirm the hypothesis that all men now in this room are third sons.”

Carnap never re-wrote the part of LFP [1] that discusses the (RTE), in light of a probabilistic relevance (“increase in firmness” [1]) notion of confirmation. This is too bad.

If Carnap had discussed this (“old evidence”) issue, I suspect he would have used something like (RTE$\neg$) as his bridge principle connecting confirmation and evidential support.

Various other philosophers have proposed similar accounts of “support” as some probabilistic relation, taken relative to an “empty” (perhaps “a priori”) background $\land \lor$ probability.

- Richard Fumerton (who, unlike Williamson, is an epistemological *internalist*) proposes such a view in his [5].
- Patrick Maher [14] applies such relations extensively in his recent (neo-Carnapian) work on confirmation theory.

- So, many “Bayesians” *already* reject (RTE), for reasons that are largely independent of “gruesome” considerations.

As Tim Willimson points out [18, ch. 9], Carnap’s (RTE) must be rejected, because of the problem of old evidence [3].

If $S$’s total evidence in $C$ ($K$) entails $E$, then, according to (RTE), $E$ cannot evidentially support any $H$ for $S$ in $C$.

As a result, there are $C$’s in which we can’t use $\Pr(\cdot | K)$ — for any $\Pr$ — when assessing the *evidential import of $E$ in $C$*.

There are (basically) two kinds of strategies for revising (RTE). Carnap [1, p. 472] & Williamson [18, ch. 9] suggest:

(RTE$\neg$) $E$ evidentially supports $H$ for $S$ if $S$ possesses $E$ as evidence in $C$ and $\Pr(\cdot | E + K)$ > $\Pr(\cdot | K)$. $\{K$ is “empty”, $\Pr$: is “inductive” [14]/“evidential” [18]/“logical” [11].

Note: Hempel explicitly *required* that confirmation be taken “relative to $K_T$” in all treatments of the paradoxes [10, 11]. (RTE$\neg$) is a charitable Carnapian reconstruction of Hempel.

A more “standard” way to revise (RTE) is [(RTE$\neg$)] to use $\Pr_S(\cdot | K’)$, where $K = K’ \neq E$, and $\Pr_S$ is the credence function of a “counterpart” $S’$ of $S$ with total evidence $K’$.

So far, I have left open (precisely) what I think Bayesian confirmation theorists *should* say (*logically* & *epistemologically*) in light of Goodman’s “grue” paradox.

Clearly, BCTs will need to revise (RTE) in light of “grue”. But, the standard (RTE’) way of doing this to cope with “old evidence” isn’t powerful enough to avoid both problems.

The more draconian (RTE$\neg$) — suggested by the work of Carnap — avoids both problems, from a logical point of view (if “inductive”/“logical” probabilities exist!). But, what should would-be “Carnapians” say on the *epistemic* side?

I’m not sure what the evidential relations are in “grue” contexts (but, see “Extras”). But, *that* doesn’t undermine my line on Goodman’s “grue” *argument* against inductive logic.

- Analogy: Harman doesn’t tell us (in general) how someone *should* respond to the discovery that their beliefs are inconsistent. But, *that* doesn’t undermine Harman’s points about relevantist “reductios” of classical deductive logic.


What Could “Carnapian” Inductive Logic Be? Part I

- Many logical empiricists dreamt that inductive logic (confirmation theory) could be formulated in such a way that it supervenes on deductive logic in a very strong sense.
  - **Strong Supervenience** (SS). All confirmation relations involving sentences of a first-order language $L$ supervene on the deductive-logical (viz., syntactical) structure of $L$.

- Hempel clearly saw (SS) as a desideratum for confirmation theory. The early Carnap also seems to have (SS) in mind.
- I think it is fair to say that Carnap’s project — understood as requiring (SS) — was unsuccessful. [Note: I think this is true for reasons that are independent of Goodman’s “grue”].

- The later Carnap seems to be aware of this. Most commentators interpret this shift as the later Carnap simply giving up on inductive logic (qua logic) altogether.
- I want to resist this “standard” reading of the history.

"Carnapian“ Counterexamples to (NC) and (M)

(K) Either: (H1) there are 1000 green emeralds 900 of which have been examined before $t$, no non-green emeralds, and 1 million other things, or (¬H) there are 1,000 black ravens, 1 white raven, and 1 million other things.

- Let $E \equiv Ra \& Ba$ (a randomly sampled from universe). Then:
  \[
  \Pr(E | H & K) = \frac{1000}{1000100} < \frac{1000}{1001000} = \Pr(E | \neg H & K)
  \]

- ∴ This $K/Pr$ constitutes a counterexample to (NC), assuming a “Carnapian” theory of confirmation. This model can be emulated in the later Carnapian $\lambda/y$-systems [14].

- Let $Bx \equiv x$ is a black card, $Ax \equiv x$ is the ace of spades, $Jx \equiv x$ is the jack of clubs, and $K \equiv a$ card $a$ is sampled at random from a standard deck (where $Pr$ is also standard):
  \[
  \begin{align*}
  Pr(Aa | Ba & K) &= \frac{1}{26} > \frac{1}{52} = Pr(Aa | K) \\
  Pr(Aa | Ba & Ja & K) &= 0 < \frac{1}{52} = Pr(Aa | K).
  \end{align*}
  \]

A “Carnapian” Counterexample to (‡)

(K) Either: (H1) there are 1000 green emeralds 900 of which have been examined before $t$, no non-green emeralds, and 1 million other things in the universe, or (H2) there are 100 green emeralds that have been examined before $t$, no green emeralds that have not been examined before $t$, 900 non-green emeralds that have not been examined before $t$, and 1 million other things.

- Imagine an urn containing true descriptions of each object in the universe ($Pr \equiv$ urn model). Let $E \equiv “Ea \& Oa \& Ga”$ be drawn. $E$ confirms $H_1$ but $E$ disconfirms $H_2$, relative to $K$:
  \[
  \Pr(E | H_1 & K) = \frac{900}{1001000} > \frac{100}{1001000} = \Pr(E | H_2 & K)
  \]

- This $K/Pr$ constitutes a counterexample to (‡), assuming a “Carnapian” theory of confirmation. This probability model can be emulated in the later Carnapian $\lambda/y$-systems [14].

(B) Either: (H1) there are 1000 black ravens, no nonblack ravens, and 1 million other things, or (¬H) there are 1,000 black ravens, 1 white raven, and 1 million other things.

- Let $E \equiv Ra \& Ba$ (a randomly sampled from universe). Then:
  \[
  \Pr(E | H & K) = \frac{1000}{1000100} < \frac{1000}{1001000} = \Pr(E | \neg H & K)
  \]

- ∴ This $K/Pr$ constitutes a counterexample to (NC), assuming a “Carnapian” theory of confirmation. This model can be emulated in the later Carnapian $\lambda/y$-systems [14].

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  \[
  \Pr(E | H & K) = \frac{1000}{1000100} < \frac{1000}{1001000} = \Pr(E | \neg H & K)
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- Let $E \equiv Ra \& Ba$ (a randomly sampled from universe). Then:
  \[
  \Pr(E | H & K) = \frac{1000}{1000100} < \frac{1000}{1001000} = \Pr(E | \neg H & K)
  \]
What Could “Carnapian” Inductive Logic Be? Part II

- I propose a different reading of the later Carnap, which makes him much more coherent with the early Carnap.
- I propose *weakening* the supervenience requirement in such a way that it (a) ensures this coherence, and (b) maintains the “logicality” of confirmation relations in Carnap’s sense.
- Let $\mathcal{L}$ be a formal language strong enough to express the fragment of probability theory Carnap needs for his later, more sophisticated confirmation-theoretic framework.
- **Weak Supervenience** (WS). All confirmation relations involving sentences of a first-order language $\mathcal{L}$ supervene on the deductive-logical (viz., syntactical) structure of $\mathcal{L}$.
- Happily, $\mathcal{L}$ is pretty weak (Carnap’s $c$-theories are *decidable*). So, even by early (logicist) Carnapian lights, satisfying (WS) is sufficient to ensure the “logical determinateness” of $c$.
- The specific (WS) approach I favor takes confirmation to be a 4-place relation: between $E, H, K$, and a Pr-model $\mathbb{M}$.

What Could “Carnapian” Inductive Logic Be? Part IV

- The inductive logician must explain how it is that inductive logic can satisfy the following Carnapian *desiderata*.
  - The confirmation function $c_{\mathbb{M}}(H, E | K)$ quantifies a *logical* (in a Carnapian sense) relation between $E, H$, and $K$.
  - (D$_1$) “Logical determinateness” of $c$ is ensured by the move from (SS) to (WS) [from an $\mathcal{L}$-determinate to an $\mathcal{L}$-determinate $c$].
  - (D$_2$) Another aspect of “logicality” insisted upon by Carnap is that $c_{\mathbb{M}}(H, E | K)$ should *generalize* the entailment relation.
    - At least: $c_{\mathbb{M}}(H, E | K)$ should take a max (min) value when $E \& K \models H$ ($E \& K \models \neg H$) — for all (regular) Pr-models $\mathbb{M}$.
  - (D$_3$) There must be *some* interesting “bridge principles” linking $c$ and *some* relations of evidential support, in *some* contexts.
    - My basic “bridging” idea (rough): subject-context pairs $(S, C)$ will determine “epistemically appropriate” Pr-models $\mathbb{M}$.
    - (D$_2$) implies that *if* there are any such bridge principles linking *entailment* and (say) *conclusive evidence*, these will be *inherited by* $c$. So, we also inherit Harman’s problem!

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What Could “Carnapian” Inductive Logic Be? Part III

- Consequences of moving to such a 4-place $c$-relation:
  - We need not try to “construct” “logical” probability functions from the syntax of $\mathcal{L}$. This is a dead-end anyhow.
  - Indeed, on this view, inductive logic has nothing to say about the *interpretation/origin* of Pr. That is not a logical question, but a question about the application of logic.
    - Analogy: Deductive logicians don’t owe us a “logical interpretation/construction” of the *valuation function*.
  - Moreover, this leads to a vast increase in the *generality* of inductive logic. Carnap was stuck with an impoverished set of “logical” probability functions (in his $\lambda/y$-continuum).
  - On my approach, *any* probability function can be part of a confirmation relation (via $\mathbb{M}$). Which functions are “appropriate” or “interesting” will depend on applications.
  - So, some confirmation relations will not be “interesting”, *etc.* But, this (already) true of *entailments*, as Harman showed.
- Questions: Now, what is the job of the inductive logician, and how (if at all) do they interact with *epistemologists*?

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“Potted History” Version of Goodman’s Argument (#2)

- Some say that “sensitivity to choice of language” is a central/essential theme/aspect of Goodman’s argument.
- But, this *can’t* be the case, for many reasons. Here’s one:
  1. Goodman’s main target was *Hempel*.
  2. Hempel’s $c$-relation is defined in terms of $\models$.
  3. $\models$ *is* (essentially) sensitive to choice of language.
  4. Or, if $\models$ is sensitive to choice of language (and said sensitivity *is essential* to Goodman’s argument), then Goodman’s riddle is *neither new nor peculiar to induction*.
- Carnap’s *later* theories of $c$ *are* sensitive to choice of language. But, (a) Goodman was not aware of those later theories, and (b) “grue” doesn’t reveal *that* problem anyway.
- In order to pinpoint the (pernicious) language-variante of Carnap’s later $c$-theories, more sophisticated constructions are required (e.g., David-Miller-esque [15, Ch. 11] constructions).