Emergent Chance

Christian List and Marcus Pivato*

February-April 2013

Abstract: We offer a new argument for the claim that there can be non-degenerate objective chance ("true randomness") in a deterministic world. Using a formal model of the relationship between different levels of description of a system, we show how objective chance at a higher level can coexist with its absence at a lower level. Unlike previous arguments for the level-specificity of chance, our argument shows, in a precise sense, that higher-level chance does not collapse into epistemic probability, despite higher-level properties supervening on lower-level ones. We show that the distinction between objective chance and epistemic probability can be drawn, and operationalized, at every level of description. There is, therefore, not a single distinction between objective and epistemic probability, but a family of such distinctions.

1. Introduction

There has been much debate on the question of whether there can be objective chance in a deterministic world. The "orthodox view" is that non-degenerate objective chance ("true randomness") is incompatible with determinism, and any use of probability in a deterministic world is purely epistemic, reflecting nothing but an observer's lack of complete information. This view was held by Popper (1982) and Lewis (1986) and has recently been defended by Schaffer (2007). Other authors defend "compatibilist views", according to which there *can* be non-degenerate objective chance in a deterministic world (e.g., Hoefer 2007, Ismael 2009, Sober 2010, Glynn 2010). They employ a variety of argumentative strategies, ranging from an appeal to statistical mechanics (e.g., von Plato 1982, Loewer 2001) to a semantic approach linking chance to ability (Eagle 2010).

One strategy is to argue that the objective chance of an event depends on the level of description (e.g., Loewer 2001, Glynn 2010, Strevens 2011). According to this strategy, saying that, macroscopically described, a coin toss has an objective chance of ½ of landing heads is consistent with saying that, microscopically described, the initial state of the coin determines the outcome. Furthermore, as Glynn (2010) argues, such level-specific chances can play the role we expect "objective chance" to play. However, no existing version of this strategy has been sufficiently immunized against the objection

* C. List, Departments of Philosophy and Government, London School of Economics, London WC2A 2AE, U.K.; M. Pivato, Department of Mathematics, Trent University, Peterborough, Ontario K9L 1Z8, Canada.
We are grateful to Rosa Cao for a helpful suggestion.

that so-called "higher-level chances" are best understood, not as true objective chances, but as expressing the observer's uncertain degrees of belief about the events in question, given his (or her) informational limitations.

We develop an account of objective chance as an emergent phenomenon that answers this objection. Our account is based on a formal model of the relationship between different levels of description of a system (drawing on List 2011 and Butterfield 2012) and shows how indeterminism and chance at a higher level can coexist with determinism and the absence of chance at a lower level.[1] We identify a precise sense in which higher-level chance does not collapse into epistemic probability and show that the distinction between the two can be drawn and operationalized at *every* level of description. It is therefore misleading to draw a single overall distinction between objective chance and epistemic probability. There is an entire *family* of such distinctions: one for each level.

The key insight underlying our account is that different levels of description of a system correspond to different specifications of the system's state space and its set of possible histories, at different levels of "coarse-graining", which induce different "algebras of events" on which probabilities are defined. Far from overcomplicating matters, this insight allows us to develop a parsimonious criterion of what separates objective chance from epistemic probability. What we are suggesting is no doubt implicit in earlier work on the topic (e.g., von Plato 1982), but the literature does not yet contain a satisfactory account of why the objective-epistemic distinction can be drawn at every level and how different levels are insulated from one another so as to permit objective chance as an emergent phenomenon, despite "chancy" higher-level world histories supervening on "non-chancy" lower-level ones.

2. The basic setup

We model a system whose state evolves over time (adopting the formalism from List 2011).[2] *Time* is represented by a set T of points that are linearly ordered. The *state* of the system at each time is given by an element of some *state space* S. A *history* of the system

[1] Butterfield (2012) and List (2011) discuss emergent indeterminism; we here focus on emergent chance.
[2] Other, structurally similar branching-history models include Butterfield (2012) and, in agential contexts, Belnap, Perloff, and Xu (2001).

is a temporal path through the state space, formally a function h from T into S, where, for each time t in T, $h(t)$ is the state of the system at t.

In this model, histories play the role of possible worlds. We write Ω to denote the set of all histories deemed *possible*. This could be either the set of all logically possible functions from T into S or, more plausibly, a proper subset of that universal set, so as to capture the fact that the laws of the system permit some histories while ruling out others. In the latter case, possibility (in Ω) can be understood as nomological possibility.[3]

To define determinism and indeterminism, some further terminology is needed. For any history h and any time t, we write h_t to denote the *truncated history* up to time t (defined as the restriction of the function h to all points in time up to t in the relevant linear order). A history h is *deterministic* if, for every time t, its truncation h_t has only one possible continuation in Ω, where a *possible continuation* of h_t is a history h' such that $h'_t = h_t$. A history h is *indeterministic* if, for some time t, its truncation h_t has more than one possible continuation in Ω. Thus indeterministic histories allow *branching*, while deterministic histories do not.

Probability functions, irrespective of their interpretation, are always defined on algebras of events. An event is a collection of histories, i.e., a subset of Ω. An algebra is a collection of events that is closed under union, intersection, and complementation. One example of such an algebra is the set of all possible events (i.e., the power set of Ω). However, when Ω is infinite, it is technically useful to work with smaller algebras. Typically, the structure of Ω dictates a canonical choice of algebra, which we label $\mathcal{A}(\Omega)$.[4] A probability function is a function from $\mathcal{A}(\Omega)$ into the interval from 0 to 1 with standard properties. It is *non-degenerate* if some events have probability greater than 0 and less than 1.

There can be different probability functions on the same algebra, indexed to different "locations" or "vantage points". It is widely agreed, for example, that any objective chance function, when it exists, is indexed to a particular history and time (Lewis 1986, Schaffer 2007). Chance assignments thus take the form "event E has

[3] The laws of the system may go beyond specifying modal facts (facts about what is and is not nomologically possible); the set Ω only encodes those modal facts. A family of objective chance functions, when it exists, may encode additional, probabilistic facts.

[4] For example, if Ω has a topology, $\mathcal{A}(\Omega)$ is usually the Borel sigma algebra.

objective chance p in history h at time t". Epistemic probability (or credence) functions are indexed not only to histories and times, but also to agents and/or their informational states. Epistemic probability (or credence) assignments thus take the form "agent A (with information I) in history h at time t has degree of belief p in event E".

3. Objective chance

Not every history-and-time-indexed probability function qualifies as an objective chance function. Indeed, on the orthodox view, no non-degenerate probability function does so for a deterministic history. Schaffer (2007) proposes six desiderata that a family of history-and-time-indexed probability functions must satisfy to play the "objective chance" role.[5] These express the idea that chance must relate in the right kind of way to various other pertinent concepts, such as credence, possibility, the future, intrinsicness, lawfulness, and causation. For present purposes, we accept Schaffer's claim that whichever family of probability functions "best" satisfies these desiderata represents objective chance. We call such a family, $\langle \Pr_{h,t} \rangle$ (with h ranging over Ω and t ranging over T), an *(objective) chance structure* on Ω. Translated into our framework,[6] the desiderata are as follows:

> **The chance-credence desideratum:** If an agent, in history h at time t, were to receive the information that the objective chance of some event $E \subseteq \Omega$ is p, he would set his degree of belief in E to p, no matter what other admissible

[5] Schaffer speaks of a single function with three arguments: a proposition (event), a world (history), and a time. Technically, however, only the projection of this function for a fixed history and a fixed time is a probability function. Hence it is mathematically more correct, though equivalent to what Schaffer has in mind, to speak of a *family* of history-and-time-indexed probability functions.

[6] In Schaffer's model, each possible world (history, in our terms) is equipped with its own laws; different worlds can have different laws. In our model, the laws come into play in two ways. First, they impose modal constraints on what histories are nomologically possible: they determine Ω. Second, the laws determine the chance structure. So, in one sense, the laws are the same across all possible histories. However, if h and h' are two distinct histories, the *local* properties of Ω and $\langle \Pr_{h,t} \rangle$ may be different in a neighbourhood of h than in a neighbourhood of h'. Thus, there is another sense in which the world described by h may have "different" laws than the one described by h' (in line with Schaffer's assumption of world-specific laws). Formally, these "local" laws at h and h' can still be understood as fragments of the "global" laws embodied by the whole structure of Ω and $\langle \Pr_{h,t} \rangle$.

> information he has (where this information is formally represented by a subset $I \subseteq \Omega$, containing precisely the histories consistent with the information).[7]

This is a version of Lewis's "Principal Principle", which is commonly accepted as a key constraint on the role of chance. The next two desiderata are equally natural: only events that are possible can have non-zero chance, and only future events can have non-degenerate chance.

> **The chance-possibility desideratum:** A necessary condition for an event $E \subseteq \Omega$ to have non-zero objective chance in history h at time t is that E is *possible* in h at t, meaning that E contains a continuation of h_t.

> **The chance-future desideratum:** A necessary condition for an event $E \subseteq \Omega$ to have non-degenerate objective chance in history h at time t is that E is "properly in the future" of t.

Spelling out what it means for an event E to be "properly in the future" of time t is a non-trivial matter. A simple, but ultimately unsatisfactory criterion would be that the complement of E is still possible in h at t. The chance-future desideratum would then become an immediate consequence of the chance-possibility desideratum. The two desiderata can be teased apart using a more mathematically involved construction.[8]

The following desideratum captures the idea that the objective chance of any event is determined by relevant properties of the event itself, not by extrinsic or relational properties.

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[7] The definition of "admissible information" is subtle. We adopt a permissive definition here, deeming, at any time t, any information about the past admissible. On a generous criterion, information $I \subseteq \Omega$ is admissible in history h at time t whenever I contains at least all possible continuations of h_t. A more restrictive admissibility criterion would only strengthen our conclusions. Further, since the laws of the system are encoded in Ω (and the chance structure $\langle \Pr_{h,t} \rangle$), the admissible information $I \subseteq \Omega$ can also convey information about the laws. An agent in a deterministic history can thus, in principle, fully predict the future if he learns which truncated history he is in.

[8] Let S^T be the set of *all* functions from T into S, i.e., the set of all *logically* possible histories and thus a superset of Ω. Any event E in Ω can be (non-uniquely) represented as $E = E' \cap \Omega$, for some $E' \subseteq S^T$. Heuristically, E' is a (merely) *logically* possible event, while E is the set of all histories in E' that are *nomologically* possible. For any time t, let $B(t)$ be the set of all times up to and including t, and $A(t)$ the set of all times after t. Let $S^{B(t)}$ be the set of all functions from $B(t)$ into S, and $S^{A(t)}$ the set of all functions from $A(t)$ into S. Heuristically, $S^{B(t)}$ is the set of all *logically* possible truncated histories up to and including time t, and $S^{A(t)}$ the set of logically possible future histories after time t. Then $S^T = S^{B(t)} \times S^{A(t)}$. An event E is *settled in the past* of t if $E = (P \times S^{A(t)}) \cap \Omega$ for some $P \subseteq S^{B(t)}$. An event E is *properly in the future of t* if it is not settled in the past of t. If all histories in Ω are *deterministic*, any event $E \subseteq \Omega$ is settled in the past in this technical sense.

> **The chance-intrinsicness desideratum:** For any histories h, h', any events E, $E' \subseteq \Omega$, and any times t, t', if the triple (E, h, t) is an intrinsic duplicate of the triple (E', h', t'), the objective chance of E in h at t is the same as that of E' in h' at t'.

The precise definition of an "intrinsic duplicate" is difficult and raises a number of philosophical issues beyond the scope of this paper.[9] Informally, if all intrinsic properties of (E, h, t) are exactly replicated in (E', h', t'), for instance in two separate runs of the same experiment, then the objective chance facts should be the same.

The next desideratum requires that objective chances must be determined by the laws of the system, as opposed to, for instance, the attitudes of the observer.

> **The chance-lawfulness desideratum:** There is a set of laws at the level of Ω that determines the chance structure on Ω.

For example, there are physical laws that imply that a photon has a chance of ½ of passing through each of the two symmetrical slits in the classic double-slit experiment.

To state the final desideratum, some preliminary definitions are needed. Let C and E be two events, occurring at times t_C and t_E, respectively.[10] We say that C *appears causally relevant to* E in history h at time t (before t_E) if t_C is before t_E and $\Pr_{h,t}(E|C) \neq \Pr_{h,t}(E)$. Informally, this means that, in the context of the other facts that obtain in history h at time t, the occurrence of C alters the chance of the subsequent occurrence of E. We require t_C to be before t_E to rule out backwards causation. The causal relation is dependent on a specific history and time because certain background

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[9] Schaffer restricts this requirement to triples (E',h',t') and (E,h,t) in which $h' = h$, since he allows different worlds to have different laws (in our terminology, these would be "local" laws, as discussed in an earlier footnote). Although we have no problem with making the desideratum more restrictive, our formulation can be defended against the "different laws" objection by using a sufficiently stringent criterion of when (E',h',t') and (E,h,t) are intrinsic duplicates. Here is one operationalization (though undoubtedly a departure from what Schaffer has in mind). Let π be a permutation (one-to-one, onto function) on S. Then π induces a permutation π^* on S^T. We call π a *symmetry* if $\pi^*(\Omega) = \Omega$ and, for any h in Ω, t in T, and $E \subseteq \Omega$, we have $\Pr_{h',t}(\pi^*(E)) = \Pr_{h,t}(E)$ where $h' = \pi^*(h)$. Heuristically, π transforms the state of the system in a way that preserves all causally relevant features. Examples might be shifting all particles in the universe five metres in a particular direction or assigning a unique integer label to every electron and exchanging the even- and odd-numbered electrons. All "intrinsic" features of the system are preserved under such a symmetry. On this operationalization, (E',h',t') is an "intrinsic duplicate" of (E,h,t) if there is some symmetry π such that $h'(t') = \pi(h(t))$ and $E' = \pi^*(E)$.

[10] For a technical treatment of the time of an event, see an earlier footnote.

conditions may be necessary for C to be causally relevant to E. (For example, lighting a match cannot "cause" a fire unless there is already flammable material nearby.[11]) Finally, the expression "appears causally relevant" acknowledges that "true" causality may require more than just a probabilistic relationship between C and E. For our purposes, we need not take stand on what else causality involves. The last desideratum can now be stated as follows:

> **Chance-causation desideratum:** If some event C appears causally relevant to another event E in history h at time t, then C must happen after time t and before E.

Formulated this way, the desideratum is essentially the well-known *Markov condition* on a stochastic process: it says that any causally relevant properties of events occurring up to time t are already encoded in the truncated history h_t. Thus, if C is causally relevant to E, then C must occur *after* time t (and of course before E itself).[12]

If we accept the six desiderata, we obtain the following conclusion, as noted by Schaffer (2007).

> **Observation 1:** There can be no non-degenerate objective chance in a deterministic history.

To see this, let h be a deterministic history, and consider, for example, the chance-credence desideratum. If an agent were to receive the information that some event E has non-degenerate objective chance p in history h at time t, he would have to set his degree of belief in E equal to p, no matter what other admissible information he has. However, the full information about the truncated history up to time t is certainly admissible. Formally, this is the subset I of Ω consisting of all possible continuations of h_t. But h is deterministic, so I is the singleton set containing only h itself. Thus, conditional on I, the agent will assign credence 0 or 1 to E, depending on whether E contains h or not. This contradicts the chance-credence desideratum, which mandated a credence p strictly between 0 and 1.

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[11] Indeed, in some contexts, it might be more correct to think of the truncated history h_t as being what "causes" E, with C only playing an "auxiliary" role. For example, if E is a forest fire, and h_t is a history in which a camper left a live ember under the ashes of his camp fire, C might be the random gust of wind that ignites the conflagration. Our definition is noncommittal on this point.

[12] On probabilistic causation, see Pearl (2000).

Similarly, consider the chance-possibility desideratum. Since h is deterministic, the truncation h_t, at any time t, has only one continuation, namely h itself. Thus, any event E is possible in h at time t if and only if E contains h itself, in which case the complement of E is impossible. By implication, $\Pr_{h,t}(E)=1$ if E contains h, and $\Pr_{h,t}(E)=0$ otherwise. Thus, in a deterministic history, the chance-possibility desideratum rules out non-degenerate objective chance.

Finally, consider the chance-future desideratum. In a deterministic history, all events are "settled in the past" (as technically explicated in an earlier footnote), and thus no event counts as being "properly" in the future. This would also follow if we defined an event's being "properly in the future" as requiring that its complement be possible, since in a deterministic history no possible event has a possible complement. Either way, the necessary condition for non-degenerate objective chance, according to the chance-future desideratum, can never be met under determinism. (This is not to deny that, in a more richly described ontology of events, some events could count as being "in the future" in some other sense, even under determinism.)

The question of whether, in a deterministic history, the other three desiderata – chance-intrinsicness, chance-lawfulness, and chance-causation – can be met by a non-degenerate chance structure is less straightforward. But in any case, it is clear that our package of six desiderata cannot be satisfied in its entirety. By contrast, in indeterministic histories, the desiderata pose no such restriction.

> **Observation 2:** There can be non-degenerate objective chance in an indeterministic history.

To see this, it suffices to construct an example. Consider a toy universe containing only one particle, whose state is fully described by its location. Suppose that space and time are both discrete, and space is one-dimensional. Thus, spatial positions can be represented by integers, and moments in time by positive integers. In other words, we can write $S=\{...,-3,-2,-1,0,1,2,3,...\}$ and $T=\{1,2,3,....\}$. Let Ω be the set of all histories where the particle begins at spatial position 0 and moves exactly one spatial position (either left or right) in each time period. Histories in Ω are non-deterministic, because any truncated history of length t can be extended to a truncated history of length t+1 in two ways. For example, the truncated history (0,1,2) can be extended to both (0,1,2,3) and (0,1,2,1). We can construct a non-degenerate objective chance structure for this toy universe as follows.

For any time t and any spatial position s, let $E_{[s \text{ at } t]}$ be the set of all histories h in Ω such that $h(t)=s$. Now let h be a specific history in Ω, and suppose $h(t\text{-}1)=s$. Then we set $\Pr_{h,t}(E_{[s+1 \text{ at } t]}) = ½$ and $\Pr_{h,t}(E_{[s-1 \text{ at } t]}) = ½$. In other words, the particle has an equal chance of moving right or left in each period. We then define the chance function $\Pr_{h,t}$ by multiplying these "one-step" chances in the obvious way. To be precise, for any positive number n, there are 2^n possible extensions of any truncated history h_t of length t to a truncated history of length t+n, and the chance function $\Pr_{h,t}$ will assign probability $1/2^n$ to each of these possible extensions. There is no barrier for this family of chance functions to satisfy all of the six desiderata listed above. This should, of course, be uncontroversial.

4. Epistemic probability

We have seen that non-degenerate objective chance can exist only in indeterministic histories in Ω. This does not mean, however, that non-degenerate probability assignments are never appropriate in deterministic histories: they may reflect our uncertain degrees of belief, given incomplete information. In fact, whether or not a history is deterministic, an agent's *epistemic* probability (or credence) function will typically be non-degenerate, unless the agent has complete information *and* there are no chance events in the world.

For example, after having watched the first half of an old recorded football match, we may assign probability $^2/_3$ to our favoured team's winning – a non-degenerate probability assignment – despite understanding that the outcome of the match is long settled: we just do not know what it is. This probability assignment simply expresses our uncertain degrees of belief; we are even dealing with a past event. On the other hand, we may also hold non-degenerate epistemic probabilities in cases involving real chance. Consider our disposition to bet on the outcome of tomorrow's football match. Ordinarily, we think that, over and above the players' skills, there is some real randomness involved here. Generally, therefore, epistemic probabilities (or credences) reflect a mix of incomplete information and objective-chance hypotheses.

Clearly, epistemic probabilities need not, and typically will not, satisfy the six desiderata on objective chance. Instead, they must satisfy the following desideratum, which ensures compatibility between an agent's epistemic probabilities (or credences) and his information:

> **Epistemic probability-possibility desideratum:** A necessary condition for an event $E \subseteq \Omega$ to be assigned non-zero epistemic probability by agent A with information I (at time t in history h) is that E is *epistemically possible* from A's perspective, meaning that $E \cap I$ is non-empty.

One may or may not wish to impose other desiderata on epistemic probabilities, such as Bayesian ones, but we need not take a stand on this here. The key point is that even in the limiting case of a completely deterministic history, non-degenerate epistemic probability is still possible – and typically entirely rational, given an agent's incomplete information. As long as the information set I is non-singleton, there is no conflict between non-degeneracy and the epistemic probability-possibility desideratum. The earlier example of the pre-recorded football match illustrates this: none of the conceivable outcomes of the match is ruled out by our information after watching the first half.

The example also motivates a simple criterion for distinguishing "pure" epistemic probability from probability assignments that are driven, at least in part, by objective chance. If we had complete information about the history of football, we would already know the outcome of the pre-recorded match before we even watched it: there would be no room for non-degenerate epistemic probabilities. However, *even* with complete information about football history, we would *still* not know the outcome of tomorrow's match. We would continue to entertain non-degenerate epistemic probabilities here, which arise from objective-chance considerations.[13] (For those who prefer a microphysical example: we certainly do not know which of the two slits in the classic double-slit experiment a photon will pass through. But even with complete information about the past, we would still assign non-degenerate probabilities to the two possibilities, since we dealing with objective chance.)

[13] There is no guarantee, of course, that such full-information epistemic probabilities will match the "true" objective chances. The fact *that* we assign non-degenerate epistemic probabilities to the different outcomes here is driven by objective chance; the question of *which* probability values we assign still reflects our information.

In general, a sufficient condition for a non-degenerate probability assignment made in history h at time t to some event E to be *purely epistemic* is that it becomes degenerate, once we conditionalize on complete information about the truncated history up to time t. Formally, this yields the following test:

> **Test for pure epistemic probability:** Let E be an event, and let Pr denote a probability function held by an agent A in history h at time t. A *sufficient condition* for Pr to be *purely epistemic* with respect to E is that $\Pr(E|h_t) = 0$ or 1. Here, $\Pr(E|h_t)$ is shorthand for $\Pr(E|I)$, where I is the information that the truncated history is h_t; formally, $I = \{h' \in \Omega : h' \text{ is a continuation of } h_t\}$.

5. Emergent indeterminism

So far, we have described the system of interest at only one level, which we will now call the *lower level*. The state space S could be, for example, the set of all possible microphysical states, and Ω the set of all possible microphysical histories. Often, however, we wish to employ *higher-level descriptions*, for example by describing the state of water as liquid or frozen, rather than as a complex configuration of individual molecules, or by describing a tossed coin as landing heads or tails, rather than as following a particular finely specified physical trajectory.

We assume that (i) higher-level states and histories *supervene* on lower-level states and histories (meaning that there cannot be any variation at the higher level without variation at the lower level), and (ii) higher-level states are typically *multiply realizable* by lower-level states. There are many different configurations of water molecules that each instantiate the same state of liquid water. Similarly, there are many different physical trajectories of a coin that all correspond to landing heads.

The relationship between the different levels can be formally captured by the idea of *coarse-graining*: each higher-level state corresponds to an equivalence class of lower-level states, consisting of all its possible lower-level realizations. Call a partition of the state space S into such equivalence classes a *coarse-graining* of S, and let $\mathbb{S}$ denote the set of all equivalence classes (note the outlined letter for higher-level objects).[14] Each $\mathbb{s}$ in $\mathbb{S}$

[14] Again, we use the formalism from List (2011).

represents one higher-level state. Let σ denote the function that maps each lower-level state s in S to the corresponding higher-level state $\mathbb{s}$ in $\mathbb{S}$ (i.e., the equivalence class to which s belongs). The function σ can also be interpreted as a supervenience relation that maps subvenient lower-level states to their supervenient higher-level counterparts.

We can apply σ not only to states, but also to histories. For each lower-level history h in Ω, the corresponding higher-level history $\mathbb{h}$ is the function from T into $\mathbb{S}$ such that, for each t in T, $\mathbb{h}(t) = \sigma(h(t))$. The set of all possible higher-level histories is the projection of Ω under σ, formally $\mathbb{\Omega} = \sigma(\Omega)$.

With these definitions in place, all the concepts, definitions, and observations from the previous sections carry over, *without any formal changes*, to the system described at the higher level, where $\mathbb{S}$ is now the state space and $\mathbb{\Omega}$ the corresponding set of possible histories. All the relevant symbols in those sections must simply be replaced by their outlined counterparts.

For example, we can define determinism and indeterminism in higher-level histories in exact analogy to determinism and indeterminism in lower-level histories: a higher-level history $\mathbb{h}$ (in $\mathbb{\Omega}$) is *indeterministic* if, for some time t, its truncation $\mathbb{h}_t$ has more than one possible continuation in $\mathbb{\Omega}$, and *deterministic* otherwise. A *possible continuation* of $\mathbb{h}_t$ is defined, as before, as a history $\mathbb{h}'$ in $\mathbb{\Omega}$ such that $\mathbb{h}'_t = \mathbb{h}_t$. Similarly, a higher-level event $\mathbb{E} \subseteq \mathbb{\Omega}$ is *possible* in history $\mathbb{h}$ at time t if $\mathbb{E}$ contains a continuation of $\mathbb{h}_t$.

A key point to note is that determinism at the lower level (in Ω) is fully compatible with indeterminism at the higher level (in $\mathbb{\Omega}$).

> **Observation 3:** For suitable σ (and sufficiently large Ω), there can be indeterministic histories in $\mathbb{\Omega}$ even when all histories in Ω are deterministic (List 2011; for a structurally similar result, see Butterfield 2012).

Figure 1 provides an example of emergent indeterminism. Part (a) shows a simple system at the lower level of description (Ω). Time is plotted on the horizontal axis (T={1,2,3,...}), and the state of the system on the vertical one. Here the state space S is the set of all real numbers. The figure displays five deterministic histories, from time

$t = 1$ to time $t = 25$. Part (b) shows the same system at a higher level of description ($\mathbb{\Omega}$), obtained by coarse-graining the state space S. Specifically, $\mathbb{S}$ is the set of all integers. The coarse-graining function σ maps each real number s in S to the closest integer $\mathbb{s}$ in $\mathbb{S}$ (with the usual rounding convention). In this coarse-grained description, there are now five indeterministic histories, supervenient on the lower-level deterministic ones. In particular, they all coincide up to time $t = 9$ before diverging from one another.

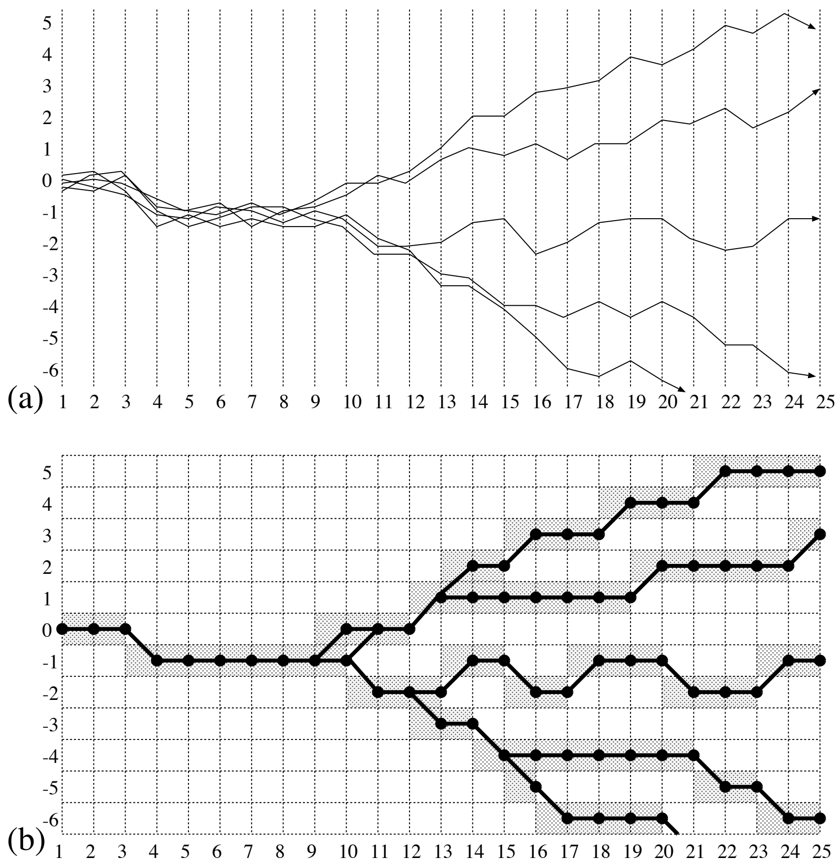

Figure 1: Emergent indeterminism

6. Objective chance at a higher level

Observation 3 shows that while a system may be deterministic at a lower level of description, indeterminism can emerge at a higher level: while the set Ω may contain only deterministic histories, a suitable coarse-graining may yield a set $\mathbb{\Omega}$ of indeterministic histories. But then Observation 2, applied to the level of $\mathbb{\Omega}$ rather than Ω, shows that $\mathbb{\Omega}$ may admit a non-degenerate objective chance structure. Thus, non-degenerate objective chance is possible at a higher level of description, *even if the system is totally deterministic at a lower level of description.*

> **Corollary of Observations 2 and 3:** There can be non-degenerate objective chance in a higher-level history (in $\mathbb{\Omega}$), even when all lower-level histories (in Ω) are deterministic. (A necessary condition for this is that the higher-level history is indeterministic, which is compatible with lower-level determinism.)

At first sight, this conclusion may seem puzzling. Have we not established that when the histories in Ω are deterministic, only degenerate objective chance structures can meet the six desiderata? However, the key insight is this: when evaluating chance and (in)determinism at a higher level of description, *only higher-level language is available.* The relevant family of history-and-time-indexed probability functions, $\langle \Pr_{\mathbb{h},t} \rangle$, now consists of functions defined on the algebra $\mathcal{A}(\mathbb{\Omega})$ rather than $\mathcal{A}(\Omega)$, and the index $\mathbb{h}$ now ranges over $\mathbb{\Omega}$ rather than Ω (while t continues to range over T).[15] Our entire analysis from the previous sections, including the desiderata, must therefore be re-applied at the level of $\mathbb{\Omega}$ rather than Ω.

Past arguments for the incompatibility of higher-level objective chance and lower-level determinism tended to make a fundamental conceptual error: they supposed that, when evaluating the chance of some higher-level event $\mathbb{E} \subseteq \mathbb{\Omega}$, we could employ a probability function indexed to a lower-level history h or conditionalize on a lower-level event $E \subseteq \Omega$, as in expressions of the form "$\Pr_{h,t}(\mathbb{E}) = 0$" or "$\Pr_{\mathbb{h},t}(\mathbb{E}|E) = 0$". But it should now be clear that this is misguided. Such expressions involve a category mistake: they mix two different levels of description.[16]

The obstacle here is conceptual, not epistemic. There are, of course, a number of epistemic questions about whether, and why, we should employ higher-level descriptions (lower-level information may or may not be accessible to us, higher-level descriptions may or may not be "reducible" to lower-level ones, and so on). We turn to these issues in

[15] Technically, $\mathcal{A}(\mathbb{\Omega})$ is a sub-algebra of $\mathcal{A}(\Omega)$.

[16] Even Glynn's (2010) defence of "indeterministic chance", whose claim about the level-specificity of chance we agree with, preserves the quantification over lower-level histories (worlds) and introduces different levels only via level-specific laws, not via explicit coarse-graining of states and histories (worlds).

Section 8. However, the conceptual point is that *when we are operating at the higher level of description, lower-level language is unavailable*. So, our claim, at this stage of the argument, is conditional: *if* we are operating at the higher level, we must stick to it. As further evidence of the pitfalls of mixing levels, note that expressions like "$\Pr_{\mathbb{h},t}(E) = 0$" or "$\Pr_{\mathbb{h},t}(\mathbb{E}|E) = 0$" are not even mathematically well-defined when the indexed history $\mathbb{h}$ and the event $\mathbb{E}$ are described at the higher level but E is described at the lower one.

In Sections 2 and 3, we laid out a theory of objective chance in the setting of indeterministic histories in Ω. But this theory applies equally well to $\mathbb{\Omega}$: simply replace every symbol with its outlined counterpart. When the higher-level analysis (in $\mathbb{\Omega}$) is correctly insulated from lower-level descriptions (in Ω), higher-level indeterminism can coexist with lower-level determinism, as we have just seen, and so the possibility of higher-level non-degenerate objective chance follows immediately from the "outlined letter" version of the framework in Sections 2 and 3.

We will now give a simple example of emergent chance (familiar from the dynamical-systems literature). Consider a system whose state space S is the interval of all real numbers between 0 and 1. Time is given by the set of positive integers, $T=\{1,2,3,...\}$. The system changes its state from one time period to next via a *transition rule,* which is formally a function f from S into itself. If s is the state at time t, then $f(s)$ is the state at time $t+1$. Thus, starting at any state s in S, we obtain the following history:

$$h(1) = s,\ h(2) = f(s),\ h(3) = f(\,f(s)\,),\ h(4) = f(\,f(\,f(s)\,)\,), \text{ and so on.}$$

The set Ω is the set of all histories that can be obtained in this way. The system is clearly deterministic.

More specifically, suppose that f is defined as follows (as illustrated in Figure 2):

$$f(s) = \begin{cases} 2s \text{ if } 0 \leq s \leq ½; \\ 2\text{-}2s \text{ if } ½ < s \leq 1. \end{cases}$$

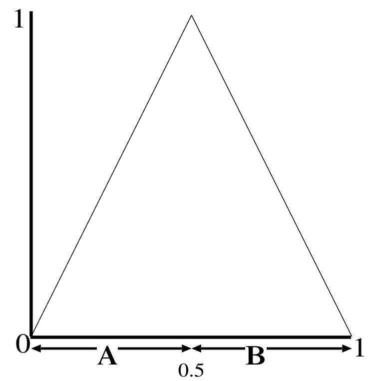

Figure 2: The transition rule *f*

Now we introduce a coarse-graining of this system. Let *A* and *B* be symbols representing higher-level states, and let $\mathbb{S}=\{A,B\}$. Define the function σ from *S* to $\mathbb{S}$ by setting

$$\sigma(s) = \begin{cases} A \text{ if } 0 \leq s \leq ½; \\ B \text{ if } ½ < s \leq 1. \end{cases}$$

By implication, σ maps each lower-level history *h* to a higher-level history $\mathbb{h}$ that takes the form of a sequence of *A*s and *B*s. For example, if we begin in the lower-level state *s*=1/7, we obtain the lower-level history *h*=(1/7, 2/7, 4/7, 6/7, 2/7, 4/7, ...). After coarse-graining via σ, this becomes the higher-level history (*A*,*A*,*B*,*B*,*A*,*B*,...). Let $\mathbb{\Omega}$ be the set of all higher-level histories that can be obtained in this way. Then $\mathbb{\Omega}$ contains *every* possible function from *T* into $\{A,B\}$. Thus, there is not only indeterminism in $\mathbb{\Omega}$, but "maximal" indeterminism: *every* truncated history up to time *t* can be extended to two truncated histories up to time *t*+1, *four* truncated histories up to time *t*+2, and so on.

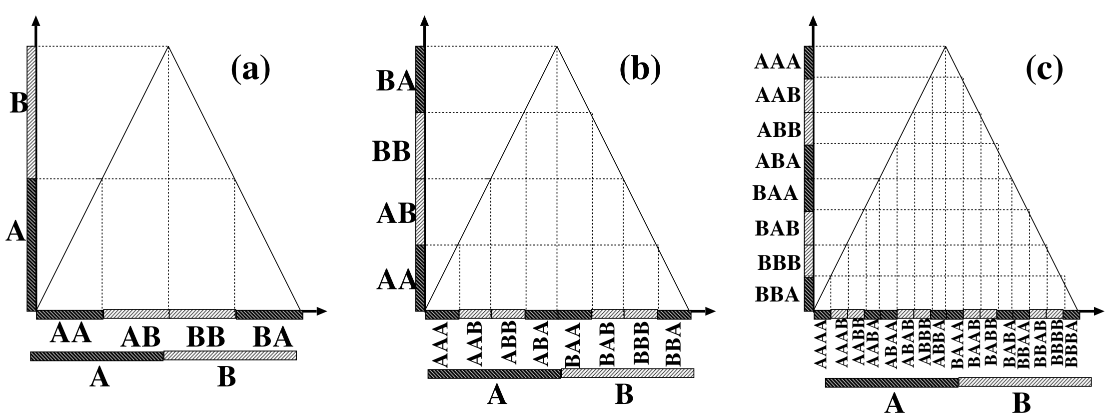

Figure 3: Higher-level histories generated by the function *f* and the partition {A,B}

To see how a non-degenerate chance structure on $\mathbb{\Omega}$ arises in a very natural way, consider Figure 3. Suppose a higher-level history $\mathbb{h}$ begins with A at time $t = 1$. Then $\mathbb{h}$ must be the coarsened counterpart of a lower-level history h beginning at some state s between 0 and ½. There are two possibilities: either $0 \leq s \leq \frac{1}{4}$, or $\frac{1}{4} < s \leq \frac{1}{2}$. In the first case, $f(s)$ (and thus $h(2)$) must be between 0 and ½, and so $\mathbb{h}(2) = A$. In the second case, $f(s)$ (and thus $h(2)$) must be between ½ and 1, and so $\mathbb{h}(2) = B$. Similarly, if $\mathbb{h}$ begins with B at time $t = 1$, its lower-level realizer must begin with some state between ½ and 1. Here, either $\frac{1}{2} < s < \frac{3}{4}$, or $\frac{3}{4} \leq s \leq 1$. In the first case, $\mathbb{h}(2) = B$; in the second, $\mathbb{h}(2) = A$.

So, depending on where exactly in the interval S the lower-level state falls at time $t = 1$, we obtain higher-level histories beginning with (A,A), (A,B), (B,B), or (B,A). These correspond exactly to four sub-intervals of S, each of length ¼, as shown in Figure 3(a).

What happens at time $t = 3$? We must now consider *eight* sub-intervals of S, each of length ⅛, as illustrated in Figure 3(b). Which of these we start in determines the higher-level history up to time $t = 3$. For example, if $\frac{1}{4} < s \leq \frac{3}{8}$, then $\frac{1}{2} < f(s) \leq \frac{3}{4}$, and so $\frac{1}{2} < f(f(s)) \leq 1$. Thus $\sigma(s)=A$, while $\sigma(f(s))=B$ and $\sigma(f(f(s)))=B$. It follows that $\mathbb{h}(s)$ begins with (A,B,B).

To determine the higher-level history up to time $t = 4$, we must consider sixteen sub-intervals of S, each of length $^1/_{16}$. These correspond to the sixteen possible truncated histories of length 4, as illustrated in Figure 3(c).

By iterating this argument, we see that, for any time t, the interval S can be subdivided into 2^t subintervals, each of length $1/2^t$, which correspond to the 2^t possible truncated histories of length t in $\mathbb{\Omega}$. This symmetry suggests a chance structure for the higher-level system, where each of these 2^t truncated histories has an equal chance of occurring. A higher-level history can then be seen as a sequence of random choices between A and B, both having probability ½, and with different choices independent of one another, as in a sequence of fair coin tosses. In other words, the higher-level chance structure is that of a classic *Bernoulli process*. There is clearly no barrier for this chance structure to satisfy the six desiderata on objective chance.[17]

[17] The higher-level system $\mathbb{\Omega}$ admits many other non-degenerate chance structures. The one described here has some claim to being the most "natural" one, since it is invariant under an exchange of the symbols A

One might object that the emergence of non-degenerate objective chance in this example is an artifact of the excessively coarse partition of S into only two sub-intervals, from 0 to ½ and from ½ to 1, which we labelled A and B. But non-degenerate objective chance also emerges from finer partitions. Suppose, for example, we partition S into four sub-intervals of length 1/4 each, labelled $\{AA,AB,BB,BA\}$, as in Figure 2(a). Then an argument similar to the one just given shows that, for any higher-level history $\mathbb{h}$ (now a function from T into $\mathbb{S} = \{AA,AB,BB,BA\}$), if $\mathbb{h}(t) = AA$ (for example), then we must have $\Pr_{\mathbb{h},t}[\mathbb{h}(t+1)=AA] = ½$ and $\Pr_{\mathbb{h},t}[\mathbb{h}(t+1)=AB] = ½$. Similar points apply if $\mathbb{h}(t)$ is AB, BB, or BA. If, instead, we partition S into eight sub-intervals of length 1/8 each, labelled $\{AAA,AAB,ABB,ABA,BAA,BAB,BBB,BBA\}$, as in Figure 2(b), then non-degenerate chance emerges again: for any higher-level history $\mathbb{h}$ (now over an even finer $\mathbb{S}$), if $\mathbb{h}(t) = BBB$ (for example), we have $\Pr_{\mathbb{h},t}[\mathbb{h}(t+1)=BBA] = ½$ and $\Pr_{\mathbb{h},t}[\mathbb{h}(t+1)=BBB] = ½$.

Indeed, a non-degenerate chance structure emerges for *any* finite partition of the interval S. The reason is that lower-level histories of the system are extremely sensitive to small perturbations. To see this, suppose that s and s' are two points in the interval S, which generate lower-level histories h and h', corresponding to higher-level histories $\mathbb{h}$ and $\mathbb{h}'$ via some coarse-graining function σ. Suppose s and s' are very close together. The distance between $f(s)$ and $f(s')$ will then typically be *twice* the distance between s and s'.[18] And the distance between $f(f(s))$ and $f(f(s'))$ will typically be *twice* that between $f(s)$ and $f(s')$ (hence *four* times the distance between s and s'),[19] and so on. In this way, the lower-level histories h and h' will rapidly diverge from each other. This, in turn, will lead the corresponding higher-level histories $\mathbb{h}$ and $\mathbb{h}'$ to come apart eventually. Even if two higher-level histories $\mathbb{h}$ and $\mathbb{h}'$ agree for their first two million entries, there is no reason for $\mathbb{h}(2,000,001)$ to be the same as $\mathbb{h}'(2,000,001)$.

and B. But it is not necessary for our purposes to show that this chance structure is in any way unique or canonical. What matters is that it can meet the six desiderata (at the level of $\mathbb{\Omega}$).

[18] This is true as long as s and s' are both in sub-interval A, or both in sub-interval B.

[19] This is true as long as $f(s)$ and $f(s')$ are both in sub-interval A, or both in sub-interval B.

7. The objective-epistemic distinction at every level

In Section 4, we drew the distinction between objective chance and epistemic probability at the lower level of description (i.e., in Ω). However, the *same* distinction can be drawn at the higher level (in $\mathbb{\Omega}$) and, indeed, at *every* level of description. Furthermore, our earlier operational test applies in each case. While objective chance is represented by whichever family of history-and-time-indexed probability functions "best" satisfies the six desiderata *at the relevant level*, we also have a test for purely epistemic probability. A non-degenerate probability assignment at a given level meets the sufficient condition for being purely epistemic if it becomes degenerate once we conditionalize on complete information about the truncated history *at that level*.

> **Test for pure epistemic probability, where $\mathbb{\Omega}$ is the level-specific set of histories:** Let $\mathbb{E} \subseteq \mathbb{\Omega}$ be an event, and let Pr denote a probability function held by an agent A in history $\mathbb{h} \in \mathbb{\Omega}$ at time t, with Pr defined on $\mathcal{A}(\mathbb{\Omega})$. A *sufficient condition* for Pr to be *purely epistemic* with respect to $\mathbb{E}$ is that $\Pr(\mathbb{E}|\mathbb{h}_t) = 0$ or 1. As before, $\Pr(\mathbb{E}|\mathbb{h}_t)$ stands for $\Pr(\mathbb{E}|\mathbb{I})$, where $\mathbb{I}$ is the information that the truncated history is $\mathbb{h}_t$; formally $\mathbb{I} = \{\mathbb{h}' \in \mathbb{\Omega} : \mathbb{h}'$ is a continuation of $\mathbb{h}_t\}$.

Consider the example of two dice being thrown onto a gaming table. Perhaps the system of tumbling dice admits a microphysical description Ω that is completely deterministic. However, as explained in Section 6, this system may also admit a higher-level description $\mathbb{\Omega}$ in which the objective chance that the gambler is about to throw snake-eyes (a pair of ones) is $^1/_{36}$. Now suppose the gambler has already thrown the dice, but the result is hidden from your view by a barrier. The gambler can see the dice, but you cannot. There is no longer any non-degenerate objective chance here; either the dice came up snake-eyes, or they did not. The objective chance of this event is now either zero or one. But from *your* perspective, with limited information, the epistemic probability of the event (your credence) remains $^1/_{36}$. Once the barrier is removed, however, you will assign probability 0 or 1. In this story, there is both objective chance (about how the dice will land in the future) and epistemic probability (about how the dice have already landed).

For another example, consider the sort of uncertainty confronted by meteorologists. Perhaps the Earth's atmosphere admits a microphysical description Ω that is completely deterministic. However, this system may also admit a higher-level description $\mathbb{\Omega}$, in which the objective chances of future weather events are non-degenerate. Meteorologists gather data from a large array of weather sensors (thermometers, hygrometers, barometers, etc.) and analyze it with computers to predict tomorrow's weather. These predictions are uncertain, and this uncertainty arises in part from the existence of non-degenerate objective chance about tomorrow's weather *at the level of* $\mathbb{\Omega}$. Indeed, meteorologists model the weather as a stochastic system. However, meteorologists also confront another kind of uncertainty. Their network of sensors is sparse. The current meteorological conditions at some location X between two sensors are unknown. But the meteorologists can assign a probability distribution to the current conditions at X. This is a purely *epistemic* probability; if there *had* been a sensor at X, the epistemic probability for the conditions at X would be degenerate, because the meteorologists would know the actual conditions at X.

Finally, consider an example from the social world. The police in a big city wish to forecast crime rates in various neighbourhoods, in order to organize effective patrols. Whether or not there is some physical or neuropsychological level (Ω) at which each individual crime is pre-determined, at the ordinary human or social level ($\mathbb{\Omega}$) the police will have to treat patterns of crime as involving non-degenerate objective chance. The chance of various crimes happening will differ from neighbourhood to neighbourhood: there is a higher chance of petty theft and pickpocketing at the railway station than on a quiet residential street. The probabilities in question would not become degenerate even if the police had complete information about the human and social history up to now. Contrast this with a murder investigation in which the police assign probability $^1/_3$ to the hypothesis that Jones did it. This probability is purely epistemic. Conditional on complete historical information (at the level of $\mathbb{\Omega}$), it would collapse into 0 or 1, since the relevant history would settle the matter.

8. Why coarse-graining is inevitable

Higher-level objective chances arise from coarse-graining, but they are not just epistemic probabilities in disguise. It is true that coarse-graining may be a response to incomplete information about a system's lower-level state. But this is not the only reason for coarse-graining. Even if we had complete or almost-complete information about the lower-level state, coarse-graining would still be necessary for agents such as ourselves to avoid the unmanageable complexity of the lower level, for several reasons.

First, the lower-level dynamics of many systems is *chaotic*. This means that even very small errors in our measurement of the current state of the system can translate into very large errors in our predictions of the system's future behaviour. (A simple example is the system shown in Figures 2 and 3 in Section 6.) Since some tiny amount of measurement error is inevitable, this means that prediction is in practice impossible, at the lower level. So an agent who insists on maintaining a detailed lower-level representation of the world will have no predictive ability. However, under a suitable coarse-graining, the chaotically diverging trajectories at the lower level of description can perhaps be amalgamated into a single, predictable trajectory at some higher level – or at least, into a higher-level stochastic process with a manageable amount of randomness. Weather forecasting is an example. So, by using a higher-level description, the agent makes his predictive task much easier.

Also, even if the agent could make *perfect* measurements, or even if the lower-level dynamics were *not* chaotic, an accurate predictive model of the lower level may still be infeasible due to computational complexity. For example, the dynamics of trillions of molecules of water and other organic compounds ricocheting around a tea cup is chaotic and astronomically complex to model at a lower level. But it is very easy at a higher level to model what happens when you add boiling water to tea leaves.

Furthermore, the agent is not ultimately interested in the lower-level state of the system. He is interested in higher-level questions. Trivial examples are: is the tea brewed? Is it cool enough to drink? Is it strong? In principle, this higher-level information could be recovered from a complete lower-level description of the system, but this would again be extremely computationally complex.

Indeed, in some cases, this computation may not even be possible. The "coarse-graining" map σ may be a well-defined mathematical object, but there is no reason to assume that it admits a finite description in any formal language available to us. Via σ, each higher-level history $\mathbb{h}$ corresponds to an equivalence class *H* of lower-level histories. Unfortunately, however, the simplest description of *H* may be just an enumeration of its elements. If *H* contains infinitely many elements, then it may not even be describable by *any* finite sentence. This is not an outlandish possibility; there is a sense in which "almost all" subsets of Ω admit no finite description.[20]

Even if *H is* finitely describable, it is still possible that the shortest description of *H* is astronomically large: it may contain as many symbols as there are atoms in the Milky Way galaxy. And even if the description of *H* is finite and of manageable length, it is still possible that it is *formally undecidable* whether any particular lower-level history belongs to *H* or not. Thus, even with a complete lower-level description of the system, questions about its higher-level history may be formally undecidable. If we are ultimately interested in such higher-level questions, this formal undecidability would make the lower-level description effectively useless.

For these reasons, higher-level descriptions should not be regarded as simply resulting from "epistemic failure". They should be regarded as offering effectively "autonomous" conceptual schemes, which may be just as appropriate for describing the world as their lower-level counterparts – and sometimes, indeed, more appropriate. These considerations echo familiar arguments against the reducibility of higher-level properties to lower-level ones, due to multiple realizability (e.g., Fodor 1974, Putnam 1975), and for non-reductive physicalism more generally (e.g., Jackson and Pettit 1990, List and Menzies 2009).

9. An objection

We have seen that higher-level objective chance is distinct from epistemic probability, even when there is lower-level determinism, and that the two kinds of probability can be distinguished *at every level of description*. A critic might object that "true" objective

[20] Technically, the class of all subsets of Ω that admit a finite description is *countable.* But the class of *all* subsets of Ω is *uncountable*. So the former class is a very small subclass of the latter.

chance can only ever exist at the lowest or most fundamental level, and that if there is determinism at that level, then any kind of higher-level chance can only be an epistemic phenomenon. This is what Schaffer (2007) seems to suggest. What can be said in response to this objection?

First, by lumping all higher-level probability together into the same "epistemic" category, the critic loses the ability to distinguish between the *chance* of the dice coming up snake-eyes in the next round (a future event that is "random" from a higher-level perspective) and our *credence* in the hypothesis that, in the last round, the outcome was snake-eyes (a past event that is already settled, but which we have not observed). The second of these probabilities would become degenerate if we received complete information about the truncated higher-level history up to now; the first probability would not. The critic is unable to make this distinction.

Secondly, it is an open question whether there is a lowest or most fundamental level. In another article, Schaffer criticizes the common assumption "that there is a bottom level which is *fundamental*" (Schaffer 2003, p. 498). He notes that this assumption often underpins "an ontological attitude according to which the entities [for our purposes: properties] of the fundamental level are *primarily* real, while any remaining contingent entities [properties] are at best derivative, if real at all" (p. 498). Schaffer argues that there is "no evidence" in support of this assumption and that, if we drop it (while upholding "a hierarchical picture of nature as stratified into *levels*", which he considers "plausible as reflected in the structure and discoveries of the sciences"), we arrive at "a far more palatable metaphysic in which … all entities [for us: properties] are equally real" (p. 498). This line of thinking is entirely consistent with our claim that higher-level chance can be just as "objective" as lower-level chance when it exists.

As we have emphasized, the distinction between objective and epistemic probability is a level-specific one. Furthermore, even if there were a lowest level, this would not necessarily be the most appropriate one for conceptualizing the chance events we are interested in. As the sciences have taught us, the most appropriate level of description may depend on the phenomena in question, and typical special-science phenomena, such as those in our examples of emergent chance, warrant a higher level of description than fundamental physical ones.

10. The mutual embeddability of deterministic and indeterministic systems

Two final results should give further pause to those who think that, once we have identified a level at which the world is deterministic, we cannot treat any higher-level indeterminism or chance as "objective".

> **Observation 4:** *Any* deterministic system can be expressed as emerging from a more fine-grained indeterministic system.

> **Observation 5:** *Any* indeterministic system can be expressed as emerging from a more fine-grained deterministic system.

Observation 4 shows that, even if we identified an apparently fundamental level of description at which a given system is deterministic, we would not be able to rule out the possibility of indeterminism at an even lower level. And similarly, Observation 5 shows that indeterminism at an apparently fundamental level would be fully consistent with determinism at an even more fundamental one.

To prove Observation 4, let T be any set of times, $\mathbb{S}$ any state space, and $\mathbb{\Omega}$ any set of deterministic histories (i.e., functions from T into $\mathbb{S}$). Now let $S = \mathbb{S} \times \{0,1\}$. In other words, S is the set of all ordered pairs of the form $(\mathbb{s}, b)$, where $\mathbb{s}$ is an element of $\mathbb{S}$, and b is either 0 or 1. Any S-valued history (i.e., any function h from T into S) is thus a combination of two functions: a function $\mathbb{h}$ from T into $\mathbb{S}$, and a function β from T into $\{0,1\}$. Let Ω be the set of all histories $(\mathbb{h}, \beta)$, where $\mathbb{h}$ is any element of $\mathbb{\Omega}$, and β is any possible function from T into $\{0,1\}$. It is clear that Ω is a set of *indeterministic* histories; any length-t truncated history $(\mathbb{h}_t, \beta_t)$ in Ω has two possible extensions to a truncated history of length t+1: one where $\beta(t+1)=0$, and one where $\beta(t+1)=1$. Now we define the function σ from S to $\mathbb{S}$ by setting $\sigma(\mathbb{s}, b) = \mathbb{s}$ for any $\mathbb{s}$ in $\mathbb{S}$ and b in $\{0,1\}$. Then σ is a coarse-graining map that converts the (indeterministic) histories of Ω into the (deterministic) histories of $\mathbb{\Omega}$.

To prove Observation 5, let T be any set of times, $\mathbb{S}$ any state space, and $\mathbb{\Omega}$ any set of indeterministic histories (i.e., functions from T into $\mathbb{S}$). Now define $S = \mathbb{\Omega} \times T$. In other words, S is the set of all ordered pairs of the form $(\mathbb{h}, t)$, where $\mathbb{h}$ is any element of

$\mathbb{\Omega}$, and t is some point in time. For any history $\mathbb{h}$ in $\mathbb{\Omega}$, we define a function h from T into S by setting $h(t) = (\mathbb{h}, t)$, for all t in T. Clearly, this is a completely deterministic history. (Heuristically, the lower-level world consists of a "book" and a "clock". The "book" is a complete record of the entire history of the higher-level world, both past and future. It is represented by $\mathbb{h}$, and it never changes. The "clock" is represented by the t-coordinate, which simply records the current time, and thus evolves in an entirely predictable way.) Let Ω be the set of all lower-level histories obtained in this way; then Ω is a deterministic system. Finally, we define a function σ from S to $\mathbb{S}$ by setting $\sigma(\mathbb{h}, t) = \mathbb{h}(t)$, for any $\mathbb{h}$ in $\mathbb{\Omega}$ and t in T. Then σ is a coarse-graining map that converts the (deterministic) histories of Ω into the (indeterministic) histories of $\mathbb{\Omega}$.

Of course, these are purely mathematical constructions, which only provide a proof of possibility. We do not claim that the lower level of any physical system would have the structure described in the previous two paragraphs. In reality, the lower level would presumably be some system of interacting particles and fields, of the kind described in modern microphysical theories. But these examples illustrate that there is no necessary entailment from indeterminism at a higher level to indeterminism at a lower level, or vice versa.

Furthermore, these constructions can be iterated indefinitely; it is perfectly possible to have a deterministic higher-level system that is a coarse-graining of an indeterministic lower-level system, which is in turn a coarse-graining of an even *lower*-level deterministic system, and so on. There could be an *infinite* hierarchy of such systems, with no "rock bottom" level; it could be "turtles all the way down".[21]

> **Observation 6:** An infinite hierarchy of levels, forever alternating between deterministic and indeterministic sets of level-specific histories, is a coherent (and unfalsifiable) possibility.

[21] Mathematically, this can be obtained through an *inverse limit* construction. As noted, Schaffer (2003) argues that such a metaphysics of "infinite descent" is perfectly coherent, and that it is in some ways *more* plausible than positing a "fundamental level". However, he does not examine the question of determinism in this setting of "infinite descent".

In this scenario, it would make no sense to ask whether the system was "really" deterministic or indeterministic. There would be no level-independent answer to this question.

11. Conclusion

Objective chance, along with indeterminism, should be understood as a level-specific phenomenon, which stands in no conflict with determinism at other levels, both lower and higher. (Objective chance is only incompatible with determinism *at the same level*.) There is not a single distinction between objective chance and epistemic probability, but a separate distinction for each level.

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