What chance-credence norms should not be

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The question

How should our credences in propositions concerning objective chances relate to our credences in other propositions?

- Enumerate the possible chance-credence norms.
- Show that one *prima facie* plausible one in fact behaves very badly in the circumstances in which it is designed to be used.

Terminology: credences

- Let $\mathcal{F}$ be the algebra of propositions about which our agent has an opinion. (Assume $\mathcal{F}$ is finite.)
- Let $b_t : \mathcal{F} \to [0, 1]$ be her *credence function* at t.
- Let E_t be her total evidence at t.

Terminology: chances

- The *ur-chance function* at world w is the probability function ch_w such that, if H_{tw} is the history of w up to time t, then the chances in w at t are given by $ch_w(\cdot|H_{tw})$.
- Given a probability function ch, let

$$C_{ch} \equiv \textit{The ur-chances are given by } ch.$$

Thus, C_{ch} is true at w iff $ch = ch_w$.
(Assume our agent has an opinion about only finitely many possible ur-chance functions.)

The putative chance-credence norms

(PP) $b_t(A|C_{ch}) = ch(A|E_t)$. (Lewis 1980)

(NP) $b_t(A|C_{ch}) = ch(A|E_t \wedge C_{ch})$. (Hall 1994)

(IP) $b_t(A) = \sum_{ch} b_t(C_{ch}) ch(A|E_t)$. (Ismael 2008)

Toy example

Suppose we know that the world contains only four coin tosses.

Sixteen possible worlds:

$$\text{HHHH, HHHT, HHTH, } \ldots, \text{ TTTH, TTTT.}$$

Five possible ur-chance functions for the reductionist:

$$ch_0(\text{Heads}) = 0 \quad ch_1(\text{Heads}) = \tfrac{1}{4} \quad ch_2(\text{Heads}) = \tfrac{1}{2}$$

$$ch_3(\text{Heads}) = \tfrac{3}{4} \quad ch_4(\text{Heads}) = 1$$

$$\begin{aligned}
C_{ch_0} &\equiv \text{TTTT} \\
C_{ch_1} &\equiv \text{TTTH} \vee \text{TTHT} \vee \text{THTT} \vee \text{HTTT} \\
C_{ch_2} &\equiv \text{HHTT} \vee \text{HTHT} \vee \text{THTH} \vee \text{TTHH} \vee \text{THHT} \vee \text{HTTH} \\
C_{ch_3} &\equiv \text{HHHT} \vee \text{HHTH} \vee \text{HTHH} \vee \text{THHH} \\
C_{ch_4} &\equiv \text{HHHH}
\end{aligned}$$

Self-undermining ur-chance functions

Definition

An ur-chance function ch is **self-undermining** in the presence of evidence E if $ch(C_{ch}|E) < 1$.

In our example, the self-undermining ur-chance functions are: ch_1, ch_2, ch_3.

For example:

$$\begin{aligned} ch_3(C_{ch_1}) &= ch_3(\text{TTTH}) + \ldots + ch_3(\text{HTTT}) \\ &= 4 \times \left(\frac{3}{4}\right) \times \left(\frac{1}{4}\right)^3 \\ &= \frac{3}{64} > 0. \end{aligned}$$

So $ch_3(C_{ch_3}) < 1$.

Self-undermining and chance-credence norms

Theorem
If there is at least one chance function that is self-undermining in the presence of E_t, then (PP) *cannot be satisfied at t.*

Proof. If ch is self-undermining in the presence of E_t, then

$$ch(C_{ch}|E_t) < 1 = b_t(C_{ch}|C_{ch})$$

□

Theorem
Whatever the ur-chance functions are like, (NP) *can be satisfied at any time.*

Theorem
Whatever the ur-chance functions are like, (IP) *can be satisfied at any time.*

Three problems for (IP)

The reductionist must choose between (NP) and (IP).

- **The Problem of Updating**
  There is no satisfactory updating rule that is consistent with (IP).
- **The Problem of Determinism**
  - In the absence of evidence, (IP) demands certainty in determinism.
  - In the presence of little evidence, (IP) demands certainty about future chance events.
- **The Problem of Deference**
  If (IP) formalizes deference, then ur-chance functions don't defer to themselves.

Thus, the reductionist ought to choose (NP).

The Problem of Updating

Bayesian Conditionalization (BC)

It ought to be the case that:

$$b_{t'}(A) = b_t(A|E_{t'})$$

providing $b_t(E_{t'}) > 0$.

The Problem of Updating

Theorem

If

- b_t *satisfies* (NP)*;*
- $b_{t'}$ *is obtained from* b_t *in accordance with* (BC)

then

- $b_{t'}$ *satisfies* (NP)*.*

The Problem of Updating

Theorem

There are b_t *and* $b_{t'}$ *such that*

- b_t *satisfies* (IP)*;*
- $b_{t'}$ *is obtained from* b_t *in accordance with* (BC)

and yet

- $b_{t'}$ *does not satisfy* (IP)*.*

What's so good about (BC)?

Definition

b is *immodest* if, for all $c \neq b$,

$$\sum_{w \in W} b(w) EU(c, w) < \sum_{w \in W} b(w) EU(b, w)$$

Theorem (Greaves and Wallace)

If $b_t(\cdot|E_{t'})$ is immodest, then, for all $c \neq b_t(\cdot|E_{t'})$,

$$\sum_{w \in E_{t'}} b_t(w) EU(c, w) < \sum_{w \in E_{t'}} b_t(w) EU(b_t(\cdot|E_{t'}), w)$$

The Problem of Updating

The Brier score

$$B(b, w) := 1 - \sum_{A \in \mathcal{F}} (b(A) - v_w(A))^2$$

Theorem

Relative to B,

- b_t *is immodest over* $E_t \Leftrightarrow b_t$ *is a probability function and* $b_t(E_t) = 1$.
- (BC) *maximizes expected epistemic utility.*

The Problem of Updating

The Chance Brier score

$$C_I^E(b,w) := 1 - \sum_{A \in \mathcal{F}} (b(A) - ch_w(A|E))^2$$

Theorem

Relative to C_I^E,

- *b_t is immodest over E_t iff b_t satisfies (IP).*
- *The following updating rule minimizes expected epistemic utility:*

  $$b_{t'}(A) = \sum_{ch} b_t(C_{ch}|E_{t'})ch(A|E_{t'})$$

  Call it Ismael Conditionalization or (IC).

The Problem of Updating

The victory is shortlived...

Theorem

There are credence functions b_t *and* $b_{t'}$ *such that*

- b_t *satisfies* (IP),
- $b_{t'}$ *is obtained from* b_t *in accordance with* (IC)

and yet

- $b_{t'}$ *does not satisfy* (IP).

The Problem of Determinism

Theorem

Suppose $ch \neq ch'$ *and*

(i) ch *is not self-undermining in the presence of* E_t

(ii) $ch'(C_{ch}|E_t) > 0$

Then, if b_t *satisfies* (IP), *then* $b_t(C_{ch'}) = 0$.

The Problem of Determinism

Suppose we know that the world contains only four coin tosses.
Sixteen possible worlds:

$$\text{HHHH, HHHT, HHTH, \ldots, TTTH, TTTT.}$$

Five possible ur-chance functions for the reductionist:

$$ch_n(\text{Heads}) = \frac{n}{4} \qquad n = 0, 1, 2, 3, 4$$

- Self-undermining in the presence of $E_t = \top$: ch_1, ch_2, ch_3.
- $ch_i(C_{ch_0}), ch_i(C_{ch_4}) > 0$, for $i = 1, 2, 3$.
- Therefore, $b_t(C_{ch_i}) = 0$, for $i = 1, 2, 3$.
- Therefore, $b_t(\text{Determinism}) = b_t(C_{ch_0} \vee C_{ch_4}) = 1$.

The Problem of Determinism

Suppose we know that the world contains only four coin tosses. Sixteen possible worlds:

HHHH, HHHT, HHTH, ..., TTTH, TTTT.

Five possible ur-chance functions for the reductionist:

$$ch_n(\text{Heads}) = \frac{n}{4} \qquad n = 0, 1, 2, 3, 4$$

- Self-undermining in the presence of $E_t = \text{H}$: ch_1, ch_2, ch_3.
- $ch_i(C_{ch_4}|\text{H}) > 0$, for $i = 1, 2, 3$.
- Therefore, $b_t(C_{ch_i}) = 0$, for $i = 1, 2, 3$.
- Therefore, $b_t(C_{ch_4}) = b_t(\text{HHHH}) = 1$.

The Problem of Determinism

There is no analogous problem for (NP):

Theorem

Suppose $\lambda_{ch} \geq 0$ *for all* ch *and* $\sum_{ch} \lambda_{ch} = 1$. *Then define* b_t *as follows:*

$$b_t(A) = \sum_{ch} \lambda_{ch} ch(A|C_{ch} \wedge E_t)$$

Then b_t *satisfies* (NP).

The Problem of Deference

Do the ur-chance functions satisfy (IP)? Not all of them.

$$ch_0(A) = \sum_{i=0}^{4} ch_0(C_{ch_i})ch_i(A)$$

$$ch_4(A) = \sum_{i=0}^{4} ch_4(C_{ch_i})ch_i(A)$$

$$ch_1(\text{HHHH}) = \frac{1}{256} \neq \frac{2128}{65,536} = \sum_{i=0}^{4} ch_1(C_{ch_i})ch_i(\text{HHHH})$$

$$ch_2(\text{HHHH}) = \frac{1}{16} \neq \frac{15}{256} = \sum_{i=0}^{4} ch_2(C_{ch_i})ch_i(\text{HHHH})$$

$$ch_3(\text{HHHH}) = \frac{81}{256} \neq \frac{24,528}{65,536} = \sum_{i=0}^{4} ch_3(C_{ch_i})ch_i(\text{HHHH})$$

The Problem of Deference

- A chance-credence norm is supposed to express the intuition that agents ought to defer to the chances when they set their credences.
- If deference to the chances involves satisfying (IP) and if the chances violate (IP), then the chances do not defer to themselves.

Meta-Normative Principle

An agent ought not to defer to an epistemic expert that does not defer to itself.

The Problem of Deference

No analogous problem for (NP) (under certain assumptions):

Theorem

Suppose the possible ur-chance functions are $ch_0, \ldots, ch_n$*. Suppose that for all worlds* w*,* w' *such that* $ch_w = ch_{w'}$*, we have* $ch(w) = ch(w')$*, for all* ch*. Then each possible ur-chance function satisfies* (NP)*.*

Replies

Objection $ch(C_{ch'})$ is not defined.

Reply Yes, it is. Consider the example from above:

- $C_{ch_1} \equiv \text{TTTH} \vee \text{TTHT} \vee \text{THTT} \vee \text{HTTT}$.
- Each ch_i is defined at TTTH, TTHT, THTT, and HTTT.

And in general:

- Chance hypotheses (of the form C_{ch}) are disjunctions of world histories.
- Chances must be defined on world histories in order to define the notion of 'fit' required by the Best-System Analysis of chance.

Conclusion

Which chance-credence norm should we adopt?

- (PP): inconsistent in the presence of self-undermining chances.
- (IP): implausible consequences in the presence of self-undermining chances.
- (NP): no analogous problems.

References

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