Comments on Alan Hájek's "Staying Regular?"

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Alan's main points

1. The formulation of regularity
2. The arguments against regularity
3. The lessons for living without regularity

I will focus on the first and second.

Regularity

- Regularity connects probability and modality
- $C(p) = 1 \rightarrow \Box p$
- $\Diamond p \rightarrow C(p) > 0$
- Alan is correct: regularity is best understood connecting credence to epistemic modality, and it can be given up.
- But: a related principle can't be given up, and would be equally refuted by his arguments.

The Minimal Constraint

What can't be given up is a related principle about chance and how it is to be measured, the Minimal Constraint (MC):

- If the chance of p happening is 0% then p does not happen.
- If the chance of p happening is 100% then p happens.

(MC) connects chance to what happens.

(MC) is not just a truth, but a conceptual truth about chance.

Because of this the relevant conditionals are themselves necessary.

Regularity vs. (MC)

Regularity	(MC)
$C(p) = 1 \rightarrow \Box p$	$\Box(C(p) = 1 \rightarrow p)$
$\Diamond p \rightarrow C(p) > 0$	$\Box(p \rightarrow C(p) > 0)$
$C(p) = 0 \rightarrow \Box\neg p$	$\Box(C(p) = 0 \rightarrow \neg p)$

What (MC) requires

You need to combine three ideas to have a proper measure of chance, and thus validate (MC):

1. Infinitesimals: the measures of chance must form a non-Archimedean field
2. Non-locality: chance does not supervene on intrinsic properties
3. Flexibility: measures are tailored to the task

Alan rejects the first and third. Here I disagree. All three together work. The second and third correspond to answers to the two best arguments against infinitesimals.

Non-locality

A tension between isomorphism and parthood (Williamson (2007):

$$C(H1...) = \frac{1}{2}C(H2...)$$

$$C(H1...) = C(H2...)$$

Thus C(H1...)=0

Williamson takes one side in the tension, (MC) requires us to take the other side.

Chance is not intrinsic to the event.

The measure of chance must operate more globally.

Hájek's Arms Race

- Alan's arguments: too many cases of chance 0, gaps, arms race
- Alan is correct: just taking a hyperreal field of the size of the reals is not always going to work.
- Another example: transfinite sequences of coin tosses
- Arguments always just rely on cardinality

Measurement is the issue

- The quantity of chance, and its measurement
- Chance as a gradable feature of events, numbers as measures of the degree of that feature
- Compare: cardinal numbers as measures of sizes of collections.
- No lack of objectivity in the feature measured simply because of flexibility of the numbers used to measure it.

Kolmogorov redone

A probability space is a triple $< \Omega, F, P >$ where

- $F \subset \mathcal{P}(\Omega)$,
- P is a function from F to H,
- H is an *F-suitable* hyperreal field,
- and versions of the other axioms hold (more on that shortly)

What is ‘F-suitable’? So far it seems like all that is needed is for H to be large enough, say:

$$card(H) = 2^{card(F)}$$

Can we prove it? No, not yet.

Additivity

- Alan is correct: what about additivity?
- This is worse for those who don't believe in infinitesimals.
- Why countable additivity and not arbitrary additivity?
- Problems with additivity
- Outlook: non-locality suggest replacing a function based account of additivity with a global constraint based account
- Global constraints can do without least upper bound, etc.
- The task ahead

Conclusion

- Infinitesimals are required in the measure of chance
- They could be used to save regularity, but it doesn't have to be saved
- Regularity can be dropped, unorthodox Bayesianism might be a good idea, but infinitesimals should be adopted in any case.