Comments on “From Social Choice to Theory Choice”

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• 1. Arrow’s theorem tells us nothing about theory choice on the basis of accuracy, simplicity, scope and so on.
• 2. There are many good ways of ranking theories by their overall merit.
• In particular, Morreau argues that the unrestricted domain assumption of Arrow’s theorem fails.
• Here I’ll focus on this last claim.
• One of the explicit assumptions of Arrow’s theorem is that we require a social choice function to apply to any collection of preferences.
• That is, there should be no restrictions on the preference profiles of the members of the population.
• In the application of Arrow’s theorem to theory choice, this amounts to the assumption that there are no restrictions on the way the various theoretical virtues order the theories in question.
• Morreau objects.
Consider Simplicity

- Morreau argues that in some cases it is straightforward that one theory is simpler than another.
- When this is the case, simplicity orderings that do not respect this ordering are not admissible.
- If this is right, Unrestricted Domain is not satisfied and Arrow’s theorem has no purchase here.
- The most compelling case for Morreau’s simplicity claim is the class of straight lines versus the class of quadratic functions.
Consider the boundary value problem of finding a function satisfying $d^2y/dx^2 = 0$ on the interval $(a, b)$, with boundary data $f(a) = c$ and $f(b) = d$.

The solution to this problem is the straight line $y = (d - c)x/(b - a) + (cb - ad)/(b - a)$.

If the differential equation is not given, we have an interpolation problem.

We are tempted to solve this problem with the same linear function.

Why?
The Simplicity Of Lines

• If the space is Euclidean, lines are the shortest distances between two points.
• Lines require only two parameters.
• Lines are smooth (in the sense of being $C^\infty$).
• Lines are solutions to differential equations of the form $\frac{dy}{dy} = k$ or $\frac{d^2y}{dx^2} = 0$.
• Lines have zero curvature everywhere.
• Morreau takes lines to be simpler than quadratics on the basis of the number of free parameters.
• Simplicity in some cases can be operationalised in terms of the number of free (independently adjustable) parameters.
• This might work for polynomials in the context of curve fitting problems but clearly has limited scope.
• For example, linear functions in Euclidian space \( f(x) = ax + c \) have two free parameters; so do exponential functions \( g(z) = ae^z + c \) in the complex plane.
• How do we rank these for simplicity? Are they really the same?
Can The Different Senses of Simplicity Come Apart?

- Yes!
- E.g. change the metric on the space and different curves deliver the shortest distances.
- E.g. in two dimensions, smoothness and area minimising come apart.
- So once we set the problem up in a particular way, lines are simpler than quadratics.
- Isn’t that all Morreau needs?
- Maybe .... but why think that the setup (the assumptions about the underlying space, the metric and the like) can be settled in advance and independently of the simplicity issues in question?
Ecological Theory 1: Lotka-Volterra

This model describes the population of the predator and the prey via two coupled first-order differential equations:

\[
\frac{dV}{dt} = rV - \alpha VP \\
\frac{dP}{dt} = \beta VP - qP
\]

Where \( V \) is the population of the prey; \( P \) is the population of the predator; 
\( r \) is the intrinsic rate of increase in prey population; \( q \) is the per capita death rate of the predator population; 
\( \alpha \) is a measure of capture efficiency; and 
\( \beta \) is a measure of conversion efficiency.
Ecological Theory 2: Inertial Population Growth

\[ \frac{d}{dt} \left( \frac{1}{N} \frac{dN}{dt} \right) = \psi \left( N, \frac{1}{N} \frac{dN}{dt} \right) \]

Where \( N \) is the abundance of the population under consideration and \( \psi \) is some function of \( N \), and the per capita rate of change of \( N \).
Here the debate proceeds in terms of both the details of the theory and the relevant background assumptions. For example, there is debate about how to trade off the degree of the differential equations with the number of parameters. The dimensionality of the space is genuinely in dispute here. The interpretation of the model (and the parameters) is also important (not just how many parameters there are). I think this case is not anomalous.
Questions

- Am I just quibbling about Morreau’s choice of examples?
- I don’t think so.
- Are there any circumstances (toy or otherwise) where one theory is unequivocably simpler than another?
- In my darker moments, I think not ... but I’d like to be convinced.
- Can we get by with toy examples?
- No.
- Do we need real case studies from the history of science?
- Yes.
- If so how many?
- It depends on what is supposed to be established.