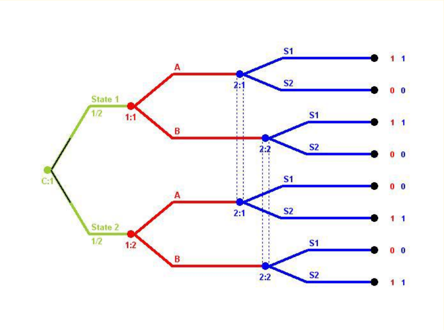
2×2 Signaling Game with Perfect Coincidence of Interests

$\pi_1(t)$ is Sender's (Player 1's) payoff in a given play of the game at period t.

$\pi_2(t)$ is Receiver's (Player 2's) payoff in a given play of the game at period t.



In the special case where the states are equiprobable, this signaling game has 11 Nash equilibria, only two of which are efficient (signaling systems).

A Conditional form of Reinforcement Learning

The state space is $\Omega = \{\omega_1, \omega_2\}$.

At her given information set $\omega \in \Omega$, Sender (Player 1) plays A with probability $\frac{\theta_A^t(\omega)}{\theta_A^t(\omega)+\theta_A^t(\omega)}$ and B with probability $\frac{\theta_B^t(\omega)}{\theta_A^t(\omega)+\theta_A^t(\omega)}$.

At his given information set $M \in \{A, B\}$, Receiver (Player 2) plays S_1 with probability $\frac{\theta_{S_1}^t(M)}{\theta_{S_1}^t(M)+\theta_{S_2}^t(\omega)}$ and S_2 with probability $\frac{\theta_{S_2}^t(M)}{\theta_{S_1}^t(M)+\theta_{S_2}^t(M)}$.

Updating Rules

For the Sender (Player 1):

$$\theta_A^{t+1}(\omega) = \theta_A^t(\omega) + 1_{[\pi_1(t)=1]}$$
$$\theta_B^{t+1}(\omega) = \theta_B^t(\omega) + 1_{[\pi_1(t)=1]}$$

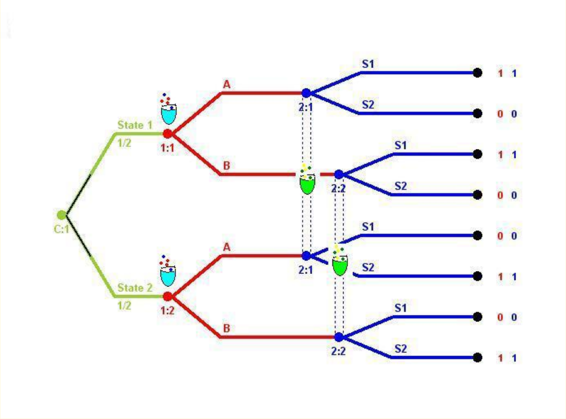
For the Receiver (Player 2):

$$\theta_{S_1}^{t+1}(M) = \theta_{S_1}^t(M) + 1_{[\pi_2(t)=1]}$$
$$\theta_{S_2}^{t+1}(M) = \theta_{S_2}^t(M) + 1_{[\pi_2(t)=1]}$$

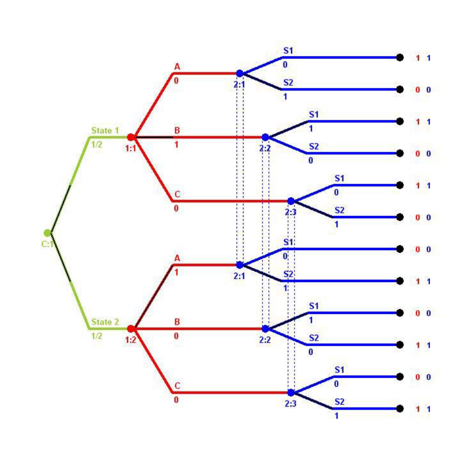
where 1_E is the indicator function.

For this particular game (with perfect coincidence of interests) these updating rules are a conditional form of Roth Erev reinforcement (1996).

Urn Interpretation: Each Player has a Polya-type urn at each information set.

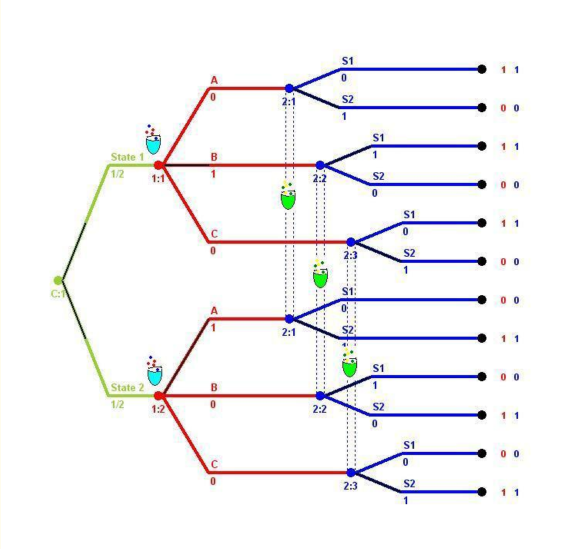


Augmented Signaling Game (after a successful interaction accompanying a draw of the "mutator")?



In the special case where the states are equiprobable, this signaling game has 68 Nash equilibria, only 6 of which are efficient.

Note that Sender (Player 1) has his original pair of urns, but Receiver (Player 2) has a new urn for her new information set



Questions for further consideration

(1) Can we get convergence to signaling systems at a rate that seems closer to the rate at which real humans learn to signal? (Perhaps?)

(2) Is there more than one way to interpret the mutator effect?

A drawback of the specific model?

At least in the 2-player case, we expect that eventually Sender and Receiver will lock into a pattern that mimics a signaling system (possibly after the introduction of sufficiently many signals). But this limiting result might take a very long time to realize.

Can we get there faster?

A variation on the Polya Urn Model?

New Updating Rules

For the Sender (Player 1):

$$\theta_A^{t+1}(\omega) = \theta_A^t(\omega) + r_{1k} 1_{[\pi_1(t)=1]}$$
$$\theta_B^{t+1}(\omega) = \theta_B^t(\omega) + r_{1k} 1_{[\pi_1(t)=1]}$$

For the Receiver (Player 2):

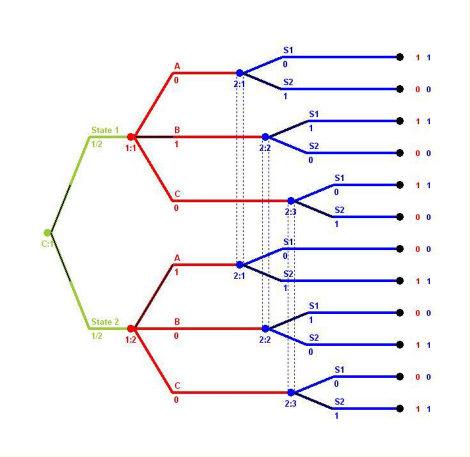
$$\theta_{S_1}^{t+1}(M) = \theta_{S_1}^t(M) + r_{2k} 1_{[\pi_2(t)=1]}$$
$$\theta_{S_2}^{t+1}(\omega) = \theta_{S_2}^t(\omega) + r_{2k} 1_{[\pi_2(t)=1]}$$

where 1_E is the indicator function and r_{ik} is a weight associated with the kth time this parameter has been reinforced. (Urn interpretation: r_{ik} is the number of balls added of a particular color the kth time this ball is drawn and the players have a “success”.)

Result for Polya Urns (Klenke 2007): Let w_n be the total number of balls of a given color after n draws of a ball of this color. If $\sum_{n=0}^{\infty} \frac{1}{w_n} < \infty$, then almost surely eventually only balls of this color will be drawn.

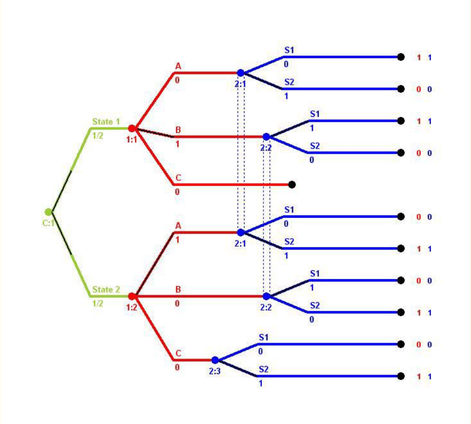
Appropriate versions of this updating rule might converge more rapidly to a pure strategy Nash equilibrium, and possibly a signaling equilibrium. But new signals might emerge even less frequently with this dynamic than they do with Alexander-Skyrms-Zabell.

Looking at the mutator effect again.



This game is a true signaling game, but it requires new acts at each of Sender's information sets, not just the set where the “mutator” appeared.

Another interpretation



This is possibly more faithful to the urn interpretation, but the generated game is technically not a signaling game.