INTEGRATIVE REDUCTION, CONFIRMATION, AND THE SYNTAX-SEMANTICS MAP

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ABSTRACT. Different pairs of scientific theories stand in different relations. The present paper identifies a new type of intertheoretic relation, Integrative Reduction, that is instantiated in linguistic syntax and semantics. We show its commonalities with Nagelian reduction and establish their salient differences. To assess the epistemic value of Integrative Reduction, we analyze it in the framework of Bayesian confirmation theory. More specifically, we show that the prior and posterior probabilities and the degree of confirmation of the conjunction of syntax and semantics is higher after the Integrative Reduction than after the Nagelian reduction of syntax to semantics.

Keywords Intertheoretic relations, Bayesian confirmation, Bayesian networks in philosophy, Reduction, Syntax-semantics map.

1. INTRODUCTION

Reductive relations between theories take a central place in contemporary philosophy of science. Their standard analysis is familiar: A scientific theory $\mathcal{T}_2$ (paradigmatically, thermodynamics) is reduced to a theory $\mathcal{T}_1$ (paradigmatically, statistical mechanics) by connecting their non-logical vocabularies via bridge laws, substituting terms from $\mathcal{T}_1$ by their counterparts from $\mathcal{T}_2$, and (thus) deriving every proposition in $\mathcal{T}_2$ from corresponding propositions in $\mathcal{T}_1$, cf. (Nagel, 1961). While recent research on intertheoretic relations has advanced our knowledge of different types of reduction, its focus on the natural sciences has created a bias towards a certain class of relations. The present paper seeks to counterbalance this trend: Abstracting from Richard Montague's theory of natural language syntax and semantics, we identify a new type of intertheoretic relation that is instantiated in linguistics.

Our new type of intertheoretic relation, which we call *Integrative Reduction*, shares many salient properties of Nagelian reduction.[1] Like Nagelian reduction, Integrative Reduction aims to derive propositions of the reduced theory ($\mathcal{T}_2$) from propositions of the reducing theory ($\mathcal{T}_1$). To this end, it establishes connections between their non-logical vocabularies and substitutes terms in propositions of

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[1]For many years, Nagelian reduction has been considered a dead end. The present paper rejects this assumption. While Nagel's original model of reduction (Nagel, 1961) has different problems, the latter are overcome in Schaffner's extension (Nagel, 1977; Schaffner, 1974), cf. (Dijzadji-Bahmani et al., 2010b). For the present purposes, it will suffice to focus only on the Nagelian model. We present a Schaffner-style extension of our new model in Section 6.

the reducing theory by their reduced-theory counterparts. Integrative Reduction differs from Nagelian reduction with respect to its deductive strength and the connection between terms in the two theories' non-logical vocabularies. In contrast to Nagelian reduction, Integrative Reduction does not assume a bijective relation between terms in the vocabulary of the reduced and the reducing theory, or between their associated objects. Thus, a property of an object in the theory $\mathcal{T}_2$ may correspond to different properties of its counterpart in $\mathcal{T}_1$ (i.e. may be multiply realizable). Correspondingly, bridge laws and their associated propositions may take the form of disjunctions. While a change in the property of an object in $\mathcal{T}_1$ will, thus, not entail a change in the property of its $\mathcal{T}_2$-counterpart, the converse cannot be excluded.

But the described directedness is not the defining feature of Integrative Reduction. Rather, the latter is defined by a special kind of connectability property. As noted above, Nagelian reduction associates, with every scientific theory $\mathcal{T}$, a set of non-logical constants $\mathcal{L}$. Our new type of reduction identifies this set with the union of the set $\mathcal{L}_p$ (for 'primitive' language), whose referential domain contains primitive objects, and the set $\mathcal{L}_d$ (for 'derived' language), whose associated objects are constructions out of primitive objects. Bridge laws are taken to reflect the dependency relation between both kinds of objects. As a consequence, propositions of the reduced or the reducing theory will no longer be independent, as is commonly assumed, cf. (Dijzadji-Bahmani et al., 2010b). Rather than being able to reduce a theory's propositions one-by-one (i.e. sequentially), its propositions are now reduced simultaneously (i.e. integratively).

The present paper investigates the epistemic payoff of Integrative Reduction. In a recent article, Dijzadji-Bahmani et al. (2010b) have shown the positive impact of the establishment of Nagelian reductive relations on the confirmation of theories by evidence. Our paper follows their example. To enable the use of the Bayesian apparatus, we interpret propositions of the reduced and reducing theory as objects of probabilistic evaluation. We assess the value of Integrative Reduction by comparing the (prior and posterior) probabilities and the degree of confirmation of the conjunction of integratively reduced propositions with the probabilities and degree of confirmation of the conjunction of Nagelian and of unreduced pairs of propositions.

The paper is organized as follows: Sections 2 and 3 present Montague's formal framework for the analysis of natural language syntax and semantics, and review relevant concepts from Bayesian confirmation and network theory. The remaining sections focus on the simultaneous development and evaluation of our model of Integrative Reduction. Section 4 investigates our theories' probabilities and confirmation after the establishment of Nagelian reduction relations. Section 5 demonstrates the epistemic value of Integrative Reduction: Following the introduction of object-relating *types*, we investigate the probabilities and the degree of confirmation of three different cases, stipulating multiple, two, or a single basic type for the establishment of a relation between syntactic and semantic propositions. We observe that, given the derivability of all propositions, the assumption of a minimal number of primitive (types of) objects yields the highest probabilities and effects a maximal flow of confirmation between the two theories. We close by showing how the resulting model of Integrative Reduction can be incorporated into a sophisticated variant of Schaffner's revised model of Nagelian reduction.

2. Montague Grammar

We begin with a presentation of the two theories that we aim to relate. Section 2.1 states their propositions and proposition-connecting mechanism. Section 2.2 compares the Montagovian account of the syntax-semantic relation with the Nagelian model of intertheoretic reduction. To enable a Bayesian analysis of our new type of intertheoretic relation, Section 2.3 identifies syntactic and semantic propositions with the objects of probabilistic evaluations.

2.1. **Montague's 'Two Theories' Theory.** Richard Montague's 'Universal Grammar' provides a formal framework for the analysis and interpretation of natural language syntax, based on (Montague, 1970a; 1973; 1970b). Montague conceives of natural languages as interpreted formal systems: The syntax of a language (hereafter 'Categorial Grammar', CG) is specified through the enumeration of grammatical categories, CAT $= \{\mathrm{N(oun)}, \mathrm{V(erb)}, \mathrm{S(entence)}, \dots\}$, their associated structures ('expressions'), $\mathcal{E} = \{\mathcal{E}_{\mathrm{N}}, \mathcal{E}_{\mathrm{V}}, \mathcal{E}_{\mathrm{S}}, \dots\}$ (with $\mathcal{E}_{\mathrm{N}} = \{\mathsf{John}, \mathsf{Mary}, \mathsf{Fido}, \dots\}$), and the definition of rules, $\mathbb{G} = \{\mathbb{G}_{\mathrm{S}}, \dots\}$, governing the behavior of syntactic operations like concatenation and conjunction. The latter apply to tuples of expressions to yield unique complex expressions. Montague syntax thus constitutes an algebra, $\mathcal{A}_{\mathrm{CG}} = \{\{\mathcal{E}_{\mathrm{N}}, \mathcal{E}_{\mathrm{V}}, \dots\}, \mathbb{G}_{\mathrm{S}}, \dots\}$, over the set of basic expressions. A language (e.g. English) is identified with the closure of the set $\{\mathcal{E}_{\mathrm{N}}, \mathcal{E}_{\mathrm{V}}, \dots\}$ under the rules of the algebra.[2]

Model-Theoretic Semantics (MS) matches the syntactic algebra on the level of natural language meaning. The interpretation function I establishes a relation between syntactic expressions and their semantic referents. For every $\mathcal{E}$-constant c (e.g. John), we assume a denotation, $[\![c]\!]$ (e.g. 🯅), such that $[\![c]\!] = I(c)$. We call the set $\{\mathcal{D}_{\mathrm{N}}, \mathcal{D}_{\mathrm{V}}, \mathcal{D}_{\mathrm{S}}, \dots\}$ (containing the domains of individual objects, properties, truth-values, etc.) of $\mathcal{E}$-denotations '$\mathcal{D}$' and stipulate that it be non-empty. Every syntactic rule, $\mathbb{G}_{k_i}$ (with $k \in$ CAT, $i \in \mathbb{N}$)[3] is associated with a semantic rule, $\mathbb{S}_{k_i}$, that governs the behavior of the syntactic operations' semantic counterparts (e.g. functional application, set intersection). The semantics of a language thus constitutes an algebra $\mathcal{A}_{\mathrm{MS}} = \{\{\mathcal{D}_{\mathrm{N}}, \mathcal{D}_{\mathrm{V}}, \dots\}, \mathbb{S}_{\mathrm{S}}, \dots\}$ over the set of basic denotations. Its interpretation is identified with the closure of this set under the rules in $\mathbb{S}$. Expressions and their denotations, as well as syntactic and semantic rules, are related via a map from the syntactic algebra to the (polynomial closure of) the semantic algebra.

Significantly, the above-described map is not strictly one-to-one. This is due to the semantic ambiguity of nouns and verbs, and attendant impossibility of mapping every element of the syntactic algebra onto a unique element of the semantic algebra. Rather than being associated with a single class of semantic referents, nouns (e.g. John) may be interpreted either as individual objects (i.e. 🯅 $= [\![\mathsf{John}]\!]'$) or as generalized quantifiers (i.e. the set, $[\![\mathsf{John}]\!]''$, of all of John's properties).[4] This is made necessary by the possibility of conjoining proper

[2] For an introduction to Montague's theory of syntax and semantics, the reader is referred to (Partee, 1997; Gamut, 1991).

[3] Our use of category indices (i.e. i) is motivated by the fact that some categories (especially the category 'S') are associated with different rules (cf. Montague's rules S4, S9, S11, S14, S17 (Montague, 1973)). Their association with semantic rules T4, T9, T11, T14, T17 preserves the above-noted correspondence. Since the remainder of this paper will only be concerned with the sentence-formation rules S4 and T4, we hereafter suppress their number.

names with quantifier phrases (cf. the expression John and every woman), and restriction of coordination to same-domain objects, cf. (Partee, 1987). To preserve function-argument structure, we similarly interpret intransitive verbs (e.g. run) either as properties of individual objects (e.g. $[\![\mathsf{run}]\!]'$) or as properties of generalized quantifiers (e.g. $[\![\mathsf{run}]\!]''$). Figure 1 (next page) illustrates the relation between elements of the syntactic and semantic algebras.

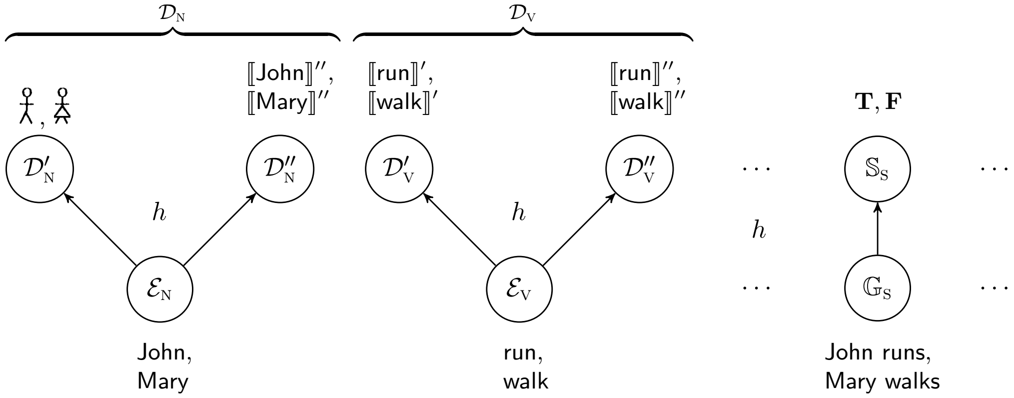

FIGURE 1. Montague's syntax-semantics map.

Since the identification of the domains $\mathcal{D}_{\rm N}$ and $\mathcal{D}_{\rm V}$ with the set $\{\mathcal{D}'_{\rm N}, \mathcal{D}''_{\rm N}\}$, respectively $\{\mathcal{D}'_{\rm V}, \mathcal{D}''_{\rm V}\}$ preserves the structure of the syntactic algebra, we hereafter describe Montague's syntax-semantic map as the homomorphism h. To facilitate the representation of the Montagovian framework, we assume the existence of only three classes of expressions or objects. We demonstrate in Section 5.2 that our new model of reduction is easily extensible to the remaining categories (e.g. adjectives, adverbs, determiners and their semantic correspondents).

In line with the constructive requirements of syntax and semantics, our presentation of the Montagovian framework has only stipulated rules for the formation of complex expressions and entities (e.g. sentences and truth-values). For future use, we also assume rules for the identity of basic objects. Thus, the rules $\mathbb{G}_{\rm N}, \mathbb{G}_{\rm V}$ are simple rewrite rules, taking words in the lexicon to expressions in the syntax. The rules $\mathbb{S}_{\rm N}, \mathbb{S}_{\rm V}$ constitute their semantic counterparts. Note that, by the ambiguity noun- and verb-interpretations, the rules $\mathbb{S}_{\rm N}, \mathbb{S}_{\rm V}$ (but not $\mathbb{G}_{\rm N}, \mathbb{G}_{\rm V}$) will include two different cases, covering $\mathcal{D}'_{\rm N}$ and $\mathcal{D}''_{\rm N}$, and $\mathcal{D}'_{\rm V}$ and $\mathcal{D}''_{\rm V}$, respectively.

The formation of minimally complex sentences (e.g. John runs) is governed by the rule $\mathbb{G}_{\rm S}$, below:

Let $[AB]$ represent the concatenation of the expressions A and B. The rule $\mathbb{G}_{\rm S}$ is then defined as follows, cf. (Montague, 1973, p. 251):

$\mathbb{G}_{\rm S}$. If $R \in \mathcal{E}_{\rm V}$ and $j \in \mathcal{E}_{\rm N}$, then $[jR'] \in \mathcal{E}_{\rm S}$,

where R' is the result of replacing the main verb in R (e.g. run) by its third person singular present form (runs). The concatenation of a noun (e.g. John) with an inflected verb (runs) thus yields a sentence (John runs).

[4]Intuitively, $[\![\mathsf{John}]\!]''$ abbreviates Montague's $[\![\lambda P.P(\mathsf{John})]\!]$, where P ranges over first-order properties, with John an individual constant. Properties of generalized quantifiers (e.g. the property $[\![\mathsf{run}]\!]''$) constitute similar abbreviations.

Semantic rules follow their syntactic counterparts: Given the replacement of syntactic categories, $\mathcal{E}_k$, by referential domains, $\mathcal{D}_k$, and interpretation of concatenation and agreement as functional application, nothing changes. Clause $\mathbb{S}_{\mathrm{S}}$ (below) defines the semantic correspondent of rule $\mathbb{G}_{\mathrm{S}}$, cf. (Montague, 1973, p. 254).

$\mathbb{S}_{\mathrm{S}}$. If $[\![R]\!] \in \mathcal{D}_{\mathrm{V}}$ and $[\![j]\!] \in \mathcal{D}_{\mathrm{N}}$, then $[\![R]\!]([\![j]\!]) \in \mathcal{D}_{\mathrm{S}}$.

Note that, by the set-like character of $\mathcal{D}_{\mathrm{N}} = \{\mathcal{D}'_{\mathrm{N}}, \mathcal{D}''_{\mathrm{N}}\}$ and $\mathcal{D}_{\mathrm{V}} = \{\mathcal{D}'_{\mathrm{V}}, \mathcal{D}''_{\mathrm{V}}\}$, the rule $\mathbb{S}_{\mathrm{S}}$ is understood as the conjunction of rules $\mathbb{S}'_{\mathrm{S}}$ and $\mathbb{S}''_{\mathrm{S}}$, below:

$\mathbb{S}'_{\mathrm{S}}$. If $[\![R]\!]' \in \mathcal{D}'_{\mathrm{V}}$ and $[\![j]\!]' \in \mathcal{D}'_{\mathrm{N}}$, then $[\![R]\!]'([\![j]\!]') \in \mathcal{D}_{\mathrm{S}}$.

$\mathbb{S}''_{\mathrm{S}}$. If $[\![R]\!]'' \in \mathcal{D}''_{\mathrm{V}}$ and $[\![j]\!]'' \in \mathcal{D}''_{\mathrm{N}}$, then $[\![R]\!]''([\![j]\!]'') \in \mathcal{D}_{\mathrm{S}}$.

According to $\mathbb{S}'_{\mathrm{S}}$, the application of the first-order property $[\![\mathsf{runs}]\!]'$ to the denotation, 🯅, of John yields either truth (i.e. $[\![\mathsf{runs}]\!]'$(🯅) $= \mathbf{T}$) or falsity ($\mathbf{F}$). According to $\mathbb{S}''_{\mathrm{S}}$, the application of the higher-order property $[\![\mathsf{runs}]\!]''$ to the generalized quantifier, $[\![\mathsf{John}]\!]''$, denoted by John yields either truth (i.e. $[\![\mathsf{runs}]\!]''\big([\![\mathsf{John}]\!]''\big) = \mathbf{T}$) or falsity ($\mathbf{F}$).

2.2. **Montague's Theory and Intertheoretic Reduction.** Our exposition of the Montagovian framework (Sect. 2) has described the syntax-semantics pair as the instantiation of a specific type of reductive relation. The presented relation shares many properties of Nagelian reduction. Like the semantic correspondent of Nagelian bridge laws, the map $h : \mathcal{A}_{\mathrm{CG}} \to \mathcal{A}_{\mathrm{MS}}$ establishes a connection between the objects of our two theories. Further, it formalizes the derivability of propositions in one theory through the replacement of non-logical terms (e.g. semantic category names) in the relevant proposition from the other theory and, thus, allows the explanation of phenomena in the realm of the former by means of the latter.

The syntax-semantics map differs from Nagelian reduction with respect to the relation between different-theory objects and the logical form of propositions. In Section 1, above, we have mentioned Nagel's assumption of a bijective relation between (pairs of) objects in the two related theories. The semantic ambiguity of some syntactic categories in the Montagovian framework breaks this correspondence. Rather than associating every syntactic category with a single semantic counterpart, the homomorphism h sends some classes of expressions (e.g. the categories 'noun' and 'verb') to more than one referential domain, and certain propositions of one theory to a set (or conjunction) of propositions of the other theory. As a consequence, the relation between Categorial Grammar and Model-Theoretic Semantics is a directed relation: A change in the properties of an object in one theory (e.g. its 'lifting' from the level of individual objects to the level of generalized quantifiers) will not entail a change in the properties of its other-theory counterpart.[5]

[5]The multiple realizability of properties is often considered to undermine reduction. Common arguments include the collapse of the associating relation between different-theory objects, the disjunctive form (and possible open-endedness) of bridge laws, and the loss of the explanatory value of reduction, cf. (Fodor, 1974; Kim, 2008; Sober, 1999). For a presentation of these arguments, and a (neo-)Nagelian defense, the reader is referred to (Dijzadji-Bahmani et al., 2010a, Sect. 3.2 and 4). Our model of Integrative Reduction (developed in Sect. 5, 6) will further witness the possibility of reductive relations that accommodate cases of multiple realizability.

The directed dependency between Categorial Grammar and Model-Theoretic Semantics motivates our identification of Categorial Grammar (or 'syntax') with the reduced theory and of (Model-Theoretic) Semantics with the reducing theory. Figure 2 compares a simplified version of the syntax-semantics relation (right) to the Nagelian account of reduction (middle).

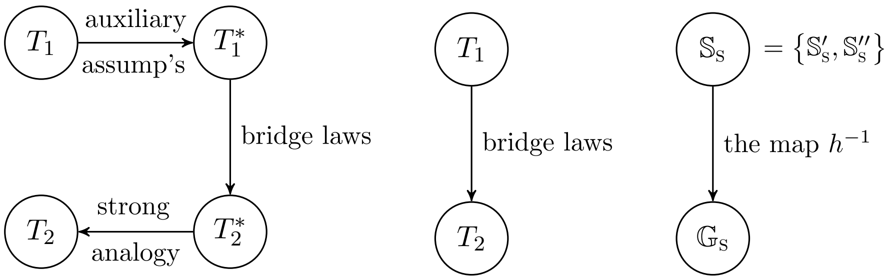

FIGURE 2. Schaffner's revised model (left), the Nagelian model (middle), and the 'Montagovian' model of reduction (right).

Admittedly, the admission of surjective relations between objects of the reducing and the reduced theory is nothing new. Schaffner's revised model of Nagelian reduction (Schaffner, 1967; 1974), as well as Nagel's reformulation of his original model (Nagel, 1977) (cf. the left diagram in Figure 2) accommodate cases of multiple realizability. However, while Schaffner introduces a dedicated level of 'corrected' propositions (whose auxiliary assumptions perform the necessary (dis-)ambiguation), our model of the syntax-semantic relation locates this faculty in the propositions themselves. Thus, the rule $\mathbb{S}_{\mathrm{S}}$ is analyzed as the set $\{\mathbb{S}'_{\mathrm{S}}, \mathbb{S}''_{\mathrm{S}}\}$ of alternative sentence-formation rules. We leave the incorporation of the 'Montagovian' model of reduction into a sophisticated variant of Schaffner's revised model for Section 6.

As is clear from the above, our model of the syntax-semantic relation instantiates only one particular type of an intertheoretic relation. There are many others, ranging from 'strict' Nagelian reduction via the 'weaker' Nagel-Schaffner reduction to undirected dependency relations, cf. (Darden and Maull, 1977; Hartmann, 1999; Mitchell, 2003). We expect that the relation between Categorial Grammar and Model-Theoretic Semantics be found in the mid-range of this spectrum. Significantly, the previously discussed Montagovian (or 'Montague'-) reduction (discussed, in more detail, in Sect. 4) may not be identified with the relation of Integrative Reduction, which will only be introduced in Section 5. Clearly, both models of the syntax-semantics relation share salient properties (e.g. homomorphic connections, directness). Notably, however, Montague reduction lacks the latter's defining property: intratheoretic connectability. In this sense, our Montagovian model of reduction constitutes only an intermediate step towards the development of the model of Integrative Reduction. This is not to deny that Montague reduction may be taken as an intertheoretic relation in its own right. We will see, however, that Integrative Reduction constitutes a considerable generalization and, in certain respects, an improvement of it.

We close the present subsection with a number of caveats about the syntax-semantics relation. Our previous considerations have identified Montague reduction as a weak, i.e. directed, variant of Nagelian reduction. Significantly, however, the presented intertheoretic relation is even weaker than has been previously established. This is due to the greater structural richness of Categorial Grammar, and attendant impossibility of providing a semantic account of all syntactic properties. Word order and agreement are a case in point: To obtain the 'right' complex expressions, Montague's syntactic rules specify the order of their constituent basic expressions, and the conditions for their agreement. Without this specification, we would concatenate the expressions John and run into the complex expression Run John rather than John runs. Other problems regard the denotation of the same semantic object by differently-formed expressions and the existence of purely syntactic well-formedness constraints. All of the above motivate our description of Montagovian (and also of Integrative) reduction as a distinct type of intertheoretic *relation*, rather than strong Nagelian *reduction*.

Our characterization of the syntax-semantics map as a (very weak) reductive relation requires a further clarification: The presented accounts of reduction (i.e. Nagel's original and Schaffner's revised model of reduction) assume the existence of the relevant theories' common domain of application. On this account, the reduced and the reducing theory both make (more-or-less) the same claims (e.g. about the behavior of a given physical system). This is not the case for our syntax-semantics pair. While Categorial Grammar accounts for the well-formedness of syntactic structures, Model-Theoretic Semantics accounts for their denotations' constructive properties. Admittedly, the denotation relation (formalized by the interpretation function I (cf. Sect. 2.1)) establishes a firm connection between the objects of the two theories. This does not change the fact, however, that their 'reductive achievement' will be comparatively weaker.

All of the above admonitions characterize our new type of intertheoretic relation. While they will be ignored in the rest of this paper, their neglect would distort our representation of the syntax-semantics relation. To enable the Bayesian analysis of its associated model, the following subsection discusses the use of probabilities in this part of linguistics. Section 3 gives a primer on Bayesian confirmation and network theory.

2.3. **Montagovian Rules and Probabilities.** Our presentation of Montague's theory has presupposed the existence of two sets of rules, $\mathbb{G}$, $\mathbb{S}$, for the formation of complex expressions and entities. Like hypotheses of any scientific theory, the latter are obtained by the scientific method (discussed, here, for the formulation of $\mathbb{G}_S$): Following the isolation of simple sentences in a given data-set (typically, an electronic text collection like the *British National Corpus*), linguists abstract information about the sentences' structural properties and propose a hypothesis (e.g. $\mathbb{G}_S$) about their formation. Hypotheses are tested through the analysis of other (new) corpora: A given string of expressions (e.g. the sentence John runs, Mary walks) is taken to support the hypothesis if its structure does, questions it if it does not reflect the assumed formation process (i.e. if it 'positively/negatively instantiates' $\mathbb{G}_S$).

To enable a Bayesian analysis of our model(s) of the syntax-semantics relation, we assign a probability to every syntactic or semantic rule. A rule's probability is informed by the frequentist data available at the time. Thus, the probability of the truth of the hypothesized rule $\mathbb{G}_S$ will be very high (low) if a very large (small) percentage of expressions of the described form instantiates $\mathbb{G}_S$.

Intuitively, a rule's frequentist probability will influence a linguist's psychological confidence in its descriptive adequacy. Consequently, if a very large (small) percentage of expressions of a given form instantiates the rule $\mathbb{G}_{\mathrm{S}}$, we expect the linguist's belief in the truth of $\mathbb{G}_{\mathrm{S}}$ to be similarly high (or low).

Our previous considerations have defined the probability of a given rule via their positive instantiations' frequency in a given sample. Notably, however, only syntactic rules are directly instantiated. This difference, which motivates our reductive endeavor, will recur in the two theories' pre-reductive confirmation (cf. Thm. 1, Sect. 4.1). The semantic rule $\mathbb{S}_{\mathrm{S}}$ derives its support from the linguistic support of $\mathbb{G}_{\mathrm{S}}$ via the assumption of the homomorphism h. While rules for the construction of more complex objects (e.g. the denotation of the sentence Mary walked for an hour or John runs fast) are directly supported by the established entailment relations and speakers' validity judgements, cf. (Dowty, 1979), the restriction of entailment to sentences prevents the direct semantic support of the simple rules $\mathbb{S}_{\mathrm{N}}$, $\mathbb{S}_{\mathrm{V}}$ and (by extension) $\mathbb{S}_{\mathrm{S}}$. Their probabilities are defined by the probabilities of their syntactic counterparts.

This concludes our discussion of the reductive and probabilistic aspects of Montague's theory. Before we move to our introduction to Bayesianism, however, one final caveat is in order: Importantly, our attribution of probabilities to Montagovian rules does not constitute a probabilistic extension of Montague Grammar. The central aim of our paper is methodological, not substantive. Consequently, we do not intend any revisions or additions to (our fragment of) Montague's theory. The attribution of probabilities is only a means to an end, i.e. the possibility of providing a Bayesian analysis of its associated model. For the latter, it will suffice to restrict ourselves to the use of probabilistic variables. While nothing prevents us from plugging in actual values, this is not necessary for the success of our analysis.

3. A Primer on Bayesianism

We analyze a rule's evidential support via Bayesian confirmation theory: Its central idea constitutes the interpretation of confirmation as probability-raising, and associated distinction between two notions of probability, relative to the receipt of a new piece of evidence: The initial, or prior, probability of a proposition H (for 'hypothesis') is the probability of H before, its final, or posterior, probability the probability after the evidence E is considered.

Bayesian conditionalization on E requires an update of the prior probability, $\boldsymbol{P}(\mathrm{H})$, to the posterior probability, $\boldsymbol{P}'(\mathrm{H})$, of H, where $\boldsymbol{P}'(\mathrm{H})$ is typically expressed in terms of the original probability measure, i.e. $\boldsymbol{P}'(\mathrm{H}) = \boldsymbol{P}(\mathrm{H}|\mathrm{E})$, provided that $\boldsymbol{P}(\mathrm{E}) > 0$. Our use of Bayes' Theorem, a result from probability theory, yields the following expression for the posterior probability of H:

$$\begin{aligned}
\boldsymbol{P}(\mathrm{H}|\mathrm{E}) &= \frac{\boldsymbol{P}(\mathrm{E}|\mathrm{H})\boldsymbol{P}(\mathrm{H})}{\boldsymbol{P}(\mathrm{E})} \\
&= \frac{\boldsymbol{P}(\mathrm{E}|\mathrm{H})\boldsymbol{P}(\mathrm{H})}{\boldsymbol{P}(\mathrm{E}|\mathrm{H})\boldsymbol{P}(\mathrm{H}) + \boldsymbol{P}(\mathrm{E}|\neg\mathrm{H})\boldsymbol{P}(\neg\mathrm{H})} \\
&= \frac{\boldsymbol{P}(\mathrm{H})}{\boldsymbol{P}(\mathrm{H}) + \boldsymbol{P}(\neg\mathrm{H})\, x}\,.
\end{aligned} \tag{1}$$

In the above, the expression $x := \boldsymbol{P}(\mathrm{E}|\neg\mathrm{H})/\boldsymbol{P}(\mathrm{E}|\mathrm{H})$ is the likelihood ratio.

According to Bayesian confirmation theory, a piece of evidence E confirms a hypothesis H if the posterior probability of H (given E) is greater than the prior probability of H, i.e. if $\boldsymbol{P}(\mathrm{H}|\mathrm{E}) > \boldsymbol{P}(\mathrm{H})$. The piece of evidence, E, disconfirms H if $\boldsymbol{P}(\mathrm{H}|\mathrm{E}) < \boldsymbol{P}(\mathrm{H})$, and is irrelevant for H if $\boldsymbol{P}(\mathrm{H}|\mathrm{E}) = \boldsymbol{P}(\mathrm{H})$.[6]

While the case of two propositions is easy to compute, the confirmatory situation is often much more complicated. This is due to the fact that the respective hypothesis may have a fine structure, and that there may be different pieces of evidence that stand in certain probabilistic relations to each other. As we will see is due course, the relation between syntax and semantics, upon which we focus in this paper, exhibits a similarly high degree of complexity.

Bayesian networks prove to be a highly efficient tool for the computation of the above-described scenarios.[7] A Bayesian network is a directed acyclical graph whose nodes represent propositional variables and whose arrows encode the conditional independence relations that hold between the variables. The rest of this paragraph introduces some useful vocabulary: Thus, *Parent nodes* are nodes with outgoing arrows; *child nodes* nodes with incoming arrows. *Root nodes* (marked in grey) are unparented nodes; *descendant nodes* are child nodes, or the child of a child node, etc.

By the special choice of graph, paths of arrows may not lead back to themselves (thus motivating the graph's acyclicity). Variables at each node can take different numerical values, assigned by the probability function $\boldsymbol{P}$. Thus, Bayesian networks do not only provide a direct visualization of the probabilistic dependency relations between variables, but come along with a set of efficient algorithms for the computation of whichever conditional or unconditional probability over a (sub-)set of the variables involved we are interested in.

We illustrate the use of Bayesian networks by framing the confirmatory relation between H and E. To do so, we first introduce two binary propositional variables, H and E (printed in italic script). Each of them has two values (printed in roman script): H or ¬H (i.e. 'the hypothesized rule is true/false'), and E or ¬E ('the evidence obtains/does not obtain'), respectively. The relation between E and H can then be represented in the graph in Figure 3.

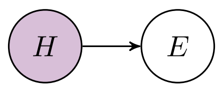

FIGURE 3. Bayesian network representation of the dependence between E and H.

The arrow from H to E denotes a direct influence of the variable in the parent to the variable in the child node. The truth or falsity of the hypothesis affects the probability of the obtaining of E.

[6] Bayesianism is presented and critically discussed in (Howson and Urbach, 2005) and (Earman, 1992). These texts also discuss Jeffrey conditionalization, which is an alternative updating rule. For an introduction to Bayesian epistemology, see (Hájek and Hartmann, 2010) and (Hartmann and Sprenger, 2010).

[7] For an introduction to Bayesian networks, see (Neapolitan, 2003; Pearl, 1988). (Bovens and Hartmann, 2003) discusses applications from epistemology and the philosophy of science, and provides a short introduction to the theory of Bayesian networks.

To turn the graph from Figure 3 into a Bayesian network, we further require the marginal probability distributions for each variable in a root node (i.e. the prior probability, $\boldsymbol{P}(\mathrm{H})$, of H), and the conditional probability distributions for every variable in a child node, given its parents. In the present case, the latter involves fixing the likelihoods $\boldsymbol{P}(\mathrm{E}|\mathrm{H})$ and $\boldsymbol{P}(\mathrm{E}|\neg\mathrm{H})$. Using the machinery of Bayesian networks, we can then obtain all other probabilities. As will be relevant below, the graph's probability distribution respects the Parental Markov Condition (PMC): A variable represented by a node in a Bayesian network is independent of all variables represented by its non-descendant nodes in the Bayesian network, conditional on all variables represented by its parent nodes.

4. Reduction and Confirmation I: Nagelian Pairwise Reduction

To enable a Bayesian analysis of Montagovian reduction, we hereafter focus on propositions in $\mathbb{G}$ and $\mathbb{S}$. The ostensible exclusion of expression- and entity classes (cf. the two leftmost pairs of nodes in Figure 1) from our probabilistic considerations does not hamper the success of our proposed model of reduction. This is due to the strong correspondence between basic-type objects and their corresponding (syntactic or semantic) rules (cf. Sect. 2.1). The reductive relation between syntax and semantics can be represented via the Bayesian network in Figure 4:

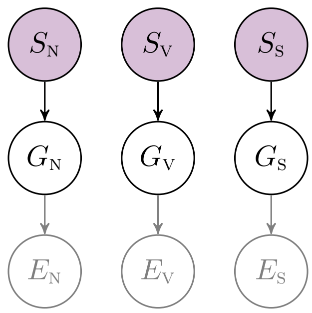

Figure 4. Post-reductive relations between pairs of propositions $\langle S_k, G_k \rangle$.

For simplicity, we assume that every rule in $\mathbb{G}$ is supported by exactly one piece of evidence. As specified in Section 2.3, we take the latter to be an intuitively well-formed expression whose structure reflects the rule's assumed formation process. The inversion of the direction of arrows with respect to Figure 1 is motivated by the directedness of the syntax-semantics relation (cf. Sect. 2.2), and attendant dependence of the probability of the truth of syntactic on the probability of the truth of semantic rules. Moreover, the conditional dependency of syntactic on semantic rules (cf. Fig. 4) enables us to obtain an aligned chain of arrows, and, thus, to represent a flow of evidence from the syntactic to the semantic theory.

Using Bayesian networks, Dizadji-Bahmani et al. (2010b) have recently shown that the establishment of an intertheoretic reduction relation has a positive confirmational and epistemic impact on the two theories involved. We aim to show the same for the Integrative Reduction of syntax to semantics. While pieces of evidence E_k initially confirm (or disconfirm) only syntactic propositions G_k, the establishment of reductive relations effects a boost of the joint probabilities of, and flow of confirmation between, the syntactic and semantic theory. The independence of proposition pairs $\langle S_k, G_k \rangle$ (cf. the lack of horizontal arrows between propositional nodes in Figure 4) justifies our preliminary restriction to the single-proposition case. Correspondingly, we abbreviate S_{S} as S, G_{S} as G, and E_{S} as E. Figures 5, 6 display the graphs associated with the pre- and post-reductive dependence relations between S, G and E:

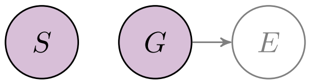

FIGURE 5. Pre-reductive dependence relations between S, G, and E.

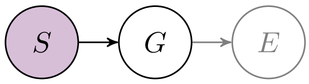

FIGURE 6. Post-reductive dependence relations between S, G, and E.

We determine the confirmation of S and G via their relevant probabilities, beginning with the pre-reductive situation (Fig. 5).

4.1. **Pre-Reductive Confirmation.** Let $\boldsymbol{P}_1(\mathrm{S})$ and $\boldsymbol{P}_1(\mathrm{G})$ be the marginal probabilities of the root nodes S, G, with $\boldsymbol{P}_1$ the corresponding probability measure. Let $\boldsymbol{P}_1(\mathrm{E|G})$ and $\boldsymbol{P}_1(\mathrm{E|\neg G})$ be the conditional probabilities of the child node E. For convenience, we use the following abbreviation scheme:[8]

$$\begin{aligned} \boldsymbol{P}_1(\mathrm{S}) = \sigma \quad &, \quad \boldsymbol{P}_1(\mathrm{G}) = \gamma \,, \\ \boldsymbol{P}_1(\mathrm{E|G}) = \pi \quad &, \quad \boldsymbol{P}_1(\mathrm{E|\neg G}) = \rho \,. \end{aligned} \tag{2}$$

We assume a positive confirmatory relation between E and G, such that $\pi > \rho$.

From the network structure in Figure 5, we can read off the conditional and unconditional independences $E \perp\!\!\!\perp S | G$ resp. $S \perp\!\!\!\perp G$ such that $\boldsymbol{P}_1(\mathrm{S|E}) = \boldsymbol{P}_1(\mathrm{S})$. Evidence E does not confirm (or disconfirm) S. Hence, there is no flow of confirmation from the syntactic to the semantic theory. In the absence of the map $h : \mathbb{G} \to \mathbb{S}$ (or related bridge laws), the variables G and S are probabilistically independent before the reduction:

$$\boldsymbol{P}_1(\mathrm{S}, \mathrm{G}) = \boldsymbol{P}_1(\mathrm{S})\, \boldsymbol{P}_1(\mathrm{G}) = \gamma\, \sigma \,. \tag{3}$$

By equation (3), the prior probability of the conjunction of S and G equals the product of the marginal probabilities of the positive instantiations of their root nodes. Using the methodology of (Bovens and Hartmann, 2003), we obtain their posterior probability as follows:

$$\begin{aligned} \boldsymbol{P}_1^* \;:=\; \frac{\boldsymbol{P}_1(\mathrm{S}, \mathrm{G}, \mathrm{E})}{\boldsymbol{P}_1(\mathrm{E})} \;&=\; \frac{\boldsymbol{P}_1(\mathrm{S}, \mathrm{G}, \mathrm{E})}{\sum_{S,G}(S, G, \mathrm{E})} \\ =\; \frac{\gamma\, \pi\, \sigma}{\gamma\, \pi + \bar{\gamma}\, \rho} \,, \end{aligned} \tag{4}$$

[8]To prevent the equivocation of probabilistic and (subsequently introduced) type variables, we denote numerical values by lowercase Greek letters.

where the denominator of the fraction in the final line is a convex combination of π and ρ weighed by γ, and where $\bar{\gamma} := 1 - \gamma$.[9]

We close the present subsection by assessing the degree of confirmation of the conjunction of S and G. To this aim, we use the difference measure d, cf. (Carnap, 1950), defined for our case as follows:[10]

$$d_1 := \boldsymbol{P}_1(\mathrm{S}, \mathrm{G}|\mathrm{E}) - \boldsymbol{P}_1(\mathrm{S}, \mathrm{G})\,. \tag{5}$$

Thus, E confirms G if its consideration raises the probability of the conjunction of S and G. By calculating d_1, we show that the latter is indeed the case:

$$d_1 = \frac{\gamma\,\bar{\gamma}\,\sigma\,(\pi - \rho)}{\gamma\,\pi + \bar{\gamma}\,\rho}\,. \tag{6}$$

Assuming that γ, π, ρ, and σ lie in the open interval $(0,1)$ with $\pi > \rho$, the above fraction is always strictly positive. We summarize our observation in the following theorem:

Theorem 1. *E confirms S and G iff E confirms G.*

We next investigate the joint probabilities of S, G in the post-reductive situation.

4.2. **Post-Reductive Confirmation.** The consideration of Montague's inter-theory mapping (cf. the arrow from S to G in Fig. 4, 6) requires a restatement of the above probabilities. Since G is no longer a root node in Figure 6 (and is, thus, not assigned a prior probability), we replace the second equation in (2) by (7), below, with $\boldsymbol{P}_2$ the new probability measure:

$$\boldsymbol{P}_2(\mathrm{G}|\mathrm{S}) = 1 \quad , \quad \boldsymbol{P}_2(\mathrm{G}|\neg\mathrm{S}) = 0\,. \tag{7}$$

Equation (7) is warranted by Montague's homomorphism h. All other assignments are as for $\boldsymbol{P}_1$. Our introduction of the new measure $\boldsymbol{P}_2$ is motivated by the move to a different probabilistic situation, and attendant need to assign the received Montagovian propositions possibly distinct probabilistic values. Equality statements of the form $\boldsymbol{P}_2(\mathrm{S}) = \boldsymbol{P}_1(\mathrm{S})$ ensure the possibility of comparing the respective propositions' confirmation in different scenarios.

We interrupt our presentation of post-reductive confirmation by an observation on Montague's syntax-semantics map. As we have argued in Section 2.2, the latter assumes the function of bridge laws, establishing connections between objects and propositions of the two theories. Naturally, linguists who question the stipulated relation between certain pairs of objects (assuming, e.g., that sentences sometimes denote propositions, not truth-values) may assign the relevant syntactic rule a lower conditional probability than the one in (7). This does not threaten the success of our enterprise: While a lower probability of G decreased the flow of confirmation to S (with the latter's strength varying with the value of $\mathbf{P}_2(\mathrm{G}|\mathrm{S})$), the reduction would still be epistemically valuable (unless, of course, $\boldsymbol{P}_2(\mathrm{G}|\mathrm{S}) = 0$).

The possibility of denying the perfect correspondence between pairs in $\mathcal{E}_k$ and $\mathcal{D}_k$ marks a presupposition on the equations in (7). Thus, propositions h_{N}, h_{V}, and h_{S} (below) are implicit in the relevant probability measures:

[9] We will hereafter always abbreviate $1 - x$ as $\bar{x}$.

[10] As discussed in (Fitelson, 1999; Eells and Fitelson, 2000), results may depend on our choice of confirmation measure. Whether (and to what extent) they do, will be a question for future research.

h_{N}. Grammatical nouns denote (i.e. are interpreted in the domain of) objects.
h_{V}. Grammatical verbs denote properties.
h_{S}. Sentences denote truth-values.

Following the introduction of semantic types, we will revisit the question of the content and epistemology of bridge laws in Sections 5.1, 5.2.

Let us return to the dependencies between, and confirmation of, S, G, and E. As encoded by the arrow from S to G, Montague's mapping effects a flow of evidence from syntax to semantics. The confirmation of the relevant semantic rule is defined simply as follows:

Theorem 2. *E confirms S iff* $\pi > \rho$.

According to the above theorem, our piece of evidence confirms the semantic proposition if, as assumed in Section 4.1, E supports G. Equations (7) ensure a positive flow of confirmation from G to S.

The prior and posterior probabilities of the conjunction of S and G are as follows (all calculations are in the Appendix):

$$\boldsymbol{P}_2(\mathrm{S},\mathrm{G}) = \sigma\,, \tag{8}$$

$$\boldsymbol{P}_2^* := \boldsymbol{P}_2(\mathrm{S},\mathrm{G}|\mathrm{E}) = \frac{\pi\,\sigma}{\pi\,\sigma + \rho\,\bar{\sigma}}\,. \tag{9}$$

The degree of confirmation of the conjunction of S and G under $\boldsymbol{P}_2$ is recorded below:

$$d_2 := \boldsymbol{P}_2(\mathrm{S},\mathrm{G}|\mathrm{E}) - \boldsymbol{P}_2(\mathrm{S},\mathrm{G}) = \frac{\sigma\,\bar{\sigma}\,(\pi-\rho)}{\pi\,\sigma + \rho\,\bar{\sigma}}\,. \tag{10}$$

To show the epistemic value of Montagovian reduction, we next compare the conjunction's probabilities and degree of confirmation in both scenarios. We accept a reduction if it raises the conjunction's probabilities or evidential support.

4.3. **Comparing Situations.** We begin by comparing the conjunction's prior probabilities, $\boldsymbol{P}_1(\mathrm{S},\mathrm{G})$ and $\boldsymbol{P}_2(\mathrm{S},\mathrm{G})$. While the propositional variables S, G are independent before, they have become dependent after the reduction. This is due to the fact that G is no longer a root node in Figure 6. In order to compare the joint probabilities of S and G, we assume the identity of $\boldsymbol{P}_2(\mathrm{G})$ and $\boldsymbol{P}_1(\mathrm{G})$, and $\boldsymbol{P}_2(\mathrm{E}|\mathrm{G})$ and $\boldsymbol{P}_1(\mathrm{E}|\mathrm{G})$, respectively. By the first equality in (7), we further assume the equality in (11)

$$\boldsymbol{P}_2(\mathrm{G}) = \boldsymbol{P}_2(\mathrm{G}|\mathrm{S})\,\boldsymbol{P}_2(\mathrm{S}) = \sigma \tag{11}$$

such that $\gamma = \sigma$.

Using the above, we calculate the difference, Δ_0, between the conjunction's pre- and post-reductive prior probabilities and obtain

$$\Delta_0 := \boldsymbol{P}_2(\mathrm{S},\mathrm{G}) - \boldsymbol{P}_1(\mathrm{S},\mathrm{G}) = \sigma\,\bar{\sigma}\,. \tag{12}$$

Intuitively, the reduction is epistemically valuable if the conjunction's prior probability is higher post- than pre-reduction, i.e. if $\Delta_0 > 0$. Since we assume all non-h-based probabilities to be non-extreme, we know that the latter is indeed the case. Theorem 3 captures this requirement:

Theorem 3. $\Delta_0 = 0$ *iff* $\sigma = 0$ *or* 1; $\Delta_0 > 0$ *iff* $\sigma \in (0,1)$.

The difference between the conjunction's posterior probabilities is also strictly positive:

$$\Delta_1 := \boldsymbol{P}_2(\mathrm{S},\mathrm{G}|\mathrm{E}) - \boldsymbol{P}_1(\mathrm{S},\mathrm{G}|\mathrm{E}) = \frac{\pi\,\sigma\,\bar{\sigma}}{\pi\,\sigma + \rho\,\bar{\sigma}}\,. \tag{13}$$

We show this via the above assumptions, together with the fact that $\pi > \rho$.

Our propositions' confirmation witnesses a similar increase. To establish this, we calculate the difference between their conjunction's pre- and post-reductive degree of confirmation under the difference measure and obtain

$$\Delta_2 := d_2 - d_1 = \frac{\sigma\,\bar{\sigma}^2\,(\pi - \rho)}{\pi\,\sigma + \rho\,\bar{\sigma}}\,. \tag{14}$$

As can be read off from the expression in (14), the positivity of Δ_2, and attendant epistemic value of Montague reduction, is conditional on the following requirements:

Theorem 4. $\Delta_2 > 0$ *iff* $\sigma \in (0,1)$ *and iff* $\pi > \rho$.

We have seen that Montagovian (or Nagel-style) reduction increases the probabilities and evidential support of the relevant conjunction. One problem remains: While Montague's syntax-semantics map renders E probabilistically relevant for S, the stipulation of independent morphisms between all pairs $\langle S_k, G_k\rangle$ does not assign the reduction an optimal epistemic value. This is due to the fact that the probability of syntax reduced to semantics will correspond to the product of the probabilities of all proposition pairs:[11]

$$\begin{aligned} \boldsymbol{P}_2\big(\bigcap_k \langle \mathrm{S}_k, \mathrm{G}_k\rangle\big) &= \boldsymbol{P}_2(\mathrm{S}_{\mathrm{N}}, \mathrm{G}_{\mathrm{N}})\, \boldsymbol{P}_2(\mathrm{S}_{\mathrm{V}}, \mathrm{G}_{\mathrm{V}})\, \boldsymbol{P}_2(\mathrm{S}_{\mathrm{S}}, \mathrm{G}_{\mathrm{S}}) && (15)\\ &= \boldsymbol{P}_2(\mathrm{S}_{\mathrm{N}})\, \boldsymbol{P}_2(\mathrm{S}_{\mathrm{V}})\, \boldsymbol{P}_2(\mathrm{S}_{\mathrm{S}})\,, && \text{(by (7))} \end{aligned}$$

respectively

$$\begin{aligned} \boldsymbol{P}_2\big(\bigcap_k \langle \mathrm{S}_k, \mathrm{G}_k|\mathrm{E}_k\rangle\big) &= \boldsymbol{P}_2(\mathrm{S}_{\mathrm{N}}, \mathrm{G}_{\mathrm{N}}|\mathrm{E}_{\mathrm{N}})\, \boldsymbol{P}_2(\mathrm{S}_{\mathrm{V}}, \mathrm{G}_{\mathrm{V}}|\mathrm{E}_{\mathrm{V}})\, \boldsymbol{P}_2(\mathrm{S}_{\mathrm{S}}, \mathrm{G}_{\mathrm{S}}|\mathrm{E}_{\mathrm{S}})\\ &= \boldsymbol{P}_2(\mathrm{S}_{\mathrm{N}}|\mathrm{E}_{\mathrm{N}})\, \boldsymbol{P}_2(\mathrm{S}_{\mathrm{V}}|\mathrm{E}_{\mathrm{V}})\, \boldsymbol{P}_2(\mathrm{S}_{\mathrm{S}}|\mathrm{E}_{\mathrm{S}})\,. && (16) \end{aligned}$$

The probability of the conjunction decreases in inverse proportion to the number of its conjuncts. Contrary to (Dijzadji-Bahmani et al., 2010b), the optimal generalization of the network in Figure 6 to theories with multiple propositional elements (cf. Fig. 4) is not conceptually straightforward, but requires insight into the mutual dependencies between same-theory propositions or proposition-reducing principles.

5. Reduction and Confirmation II: Integrative Reduction

Integrative Reduction accounts for such intratheoretical connections. Its model (presented in Sect. 5.2 and Sect. 6) is developed in abstraction from a sophisticated version of Montague's 'two theories' theory (introduced above). To increase the perspicuity of the rule-connecting mechanism, Montague (1973) stipulates a third level of *types*, i.e., logico-semantic rôles which mediate between syntactic expressions and their semantic referents, cf. (Russell, 1908; Church, 1940). Every syntactic category k is thus correlated with a semantic type α, whose referential domain, $\mathcal{D}_\alpha$, constitutes the familiar denotation set of all expressions in $\mathcal{E}_\alpha$. Figure 7 (next page) schematizes the use of types on the level of objects:

[11]We assume that $\boldsymbol{P}(\{X\}) = \boldsymbol{P}(X)$ s.t. $\boldsymbol{P}_2(\bigcap_k \langle \mathrm{S}_k, \mathrm{G}_k\rangle) = \boldsymbol{P}_2(\langle \mathrm{S}_{\mathrm{N}}, \mathrm{G}_{\mathrm{N}}\rangle, \langle \mathrm{S}_{\mathrm{V}}, \mathrm{G}_{\mathrm{V}}\rangle, \langle \mathrm{S}_{\mathrm{S}}, \mathrm{G}_{\mathrm{S}}\rangle) = \boldsymbol{P}_2(\mathrm{S}_{\mathrm{N}}, \mathrm{S}_{\mathrm{V}}, \mathrm{S}_{\mathrm{S}}, \mathrm{G}_{\mathrm{N}}, \mathrm{G}_{\mathrm{V}}, \mathrm{G}_{\mathrm{S}})$.

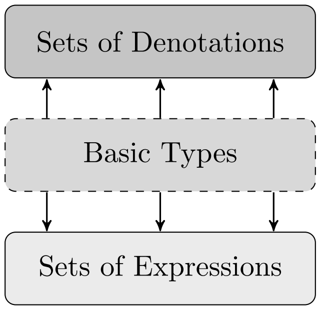

FIGURE 7. The type basis of CG- and MS-categories.

To demonstrate the requirements on the use of an intermediate type-level, we present three different cases, stipulating the existence of multiple, two, or a single basic type for the reduction of syntactic to semantic propositions. We begin with a discussion of the multi-type case.

5.1. **Case 1: Separate Types.** The assumption of a separate type for each category pair does not improve upon the above-observed independence. Let the types $e, p, t, (p \to t)$, and $((p \to t) \to t)$ be associated with individual objects ('entities'), properties, truth-values, generalized quantifiers, and properties of generalized quantifiers, respectively. Assume that the type rule $\mathbb{T}_{\mathrm{S}}$, associated with the construction of sentence-denotations, is defined as follows:

$\mathbb{T}_{\mathrm{S}}$. If $\boldsymbol{R} \in X_{\mathrm{V}}$ and $\boldsymbol{j} \in X_{\mathrm{N}}$, then $\boldsymbol{R} * \boldsymbol{j} \in X_{\mathrm{S}}$, and
if $\boldsymbol{R} \in X_{((p\to t)\to t)}$ and $\boldsymbol{j} \in X_{(p\to t)}$, then $\boldsymbol{R} * \boldsymbol{j} \in X_{\mathrm{S}}$

where X is neutral between the notation for expression sets, $\mathcal{E}$, and referential domains, $\mathcal{D}$, and where $*$ is neutral between the concatenation/agreement operator, and the designation of functional application. Note the ambiguity of the rule $\mathbb{T}_{\mathrm{S}}$. In contrast to the semantic rule $\mathbb{S}_{\mathrm{S}}$ (cf. Sect. 2.1), the type rule $\mathbb{T}_{\mathrm{S}}$ makes the different possibilities of obtaining sentence-denotations explicit.

Following the notational convention from the beginning of Section 3, we denote the values of variables $T_{\mathrm{S}}, T_{\mathrm{N}}$, and T_{V} by $\mathrm{T_S}, \neg\mathrm{T_S}, \mathrm{T_N}, \neg\mathrm{T_N}$, and $\mathrm{T_V}, \neg\mathrm{T_V}$, respectively. The graph in Figure 8 encodes the dependencies of propositional variables after the newly introduced reduction:
As in the untyped case (cf. Sect. 4.3), the independence of triples $\langle S_k, T_k, G_k\rangle$ warrants the derivation of their joint probabilities via the product of their individual probabilities. For this reason, we initially restrict ourselves to the prior and posterior probabilities of the conjunction of $\mathrm{T_S}, \mathrm{S_S}$, and $\mathrm{G_S}$. To emphasize our model's connection with the network from Figure 6, we use a similar abbreviation scheme, with $\boldsymbol{P}_3$ the new probability measure:

$$\begin{aligned} \boldsymbol{P}_3(\mathrm{T_S}) &= \tau \quad , \\ \boldsymbol{P}_3(\mathrm{S_S}|\mathrm{T_S}) &= 1 \quad , \quad \boldsymbol{P}_3(\mathrm{S_S}|\neg\mathrm{T_S}) = 0\,, \\ \boldsymbol{P}_3(\mathrm{G_S}|\mathrm{T_S}) &= 1 \quad , \quad \boldsymbol{P}_3(\mathrm{G_S}|\neg\mathrm{T_S}) = 0\,, \\ \boldsymbol{P}_3(\mathrm{E_S}|\mathrm{G_S}) &= \pi \quad , \quad \boldsymbol{P}_3(\mathrm{E_S}|\neg\mathrm{G_S}) = \rho\,. \end{aligned} \tag{17}$$

The equations in the last line are as for Figures 5, 6, above. The identities in lines two and three are necessitated by the replacement of S_{S} and G_{S} by T_{S} as

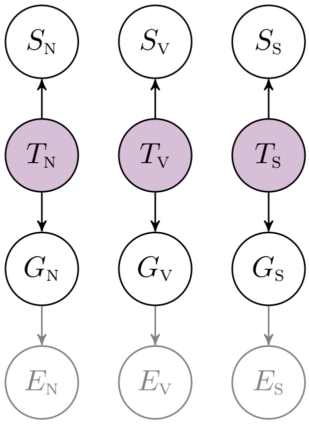

FIGURE 8. Case 1: Separate types.

root nodes. Since, thus,

$$\boldsymbol{P}_3(\mathrm{S_S}) = \boldsymbol{P}_3(\mathrm{T_S}) = \boldsymbol{P}_3(\mathrm{G_S})\,, \tag{18}$$

Theorem 2 also holds in the new model. This is due to the strong dependence of syntactic and semantic on type rules, and assumption of a positive confirmatory relation between E and G.

Admittedly, the attribution of our type rules' probabilities seems less intuitive than the assignment of probabilities to their syntactic or semantic counterparts. This is amended by the equalities in the second line of (17). Thus, the probabilities of rules $\mathrm{T_S}$, $\mathrm{T_N}$, $\mathrm{T_V}$ can be obtained via the probabilities of their semantic (or syntactic) correlates $\mathrm{S_S}$, $\mathrm{S_N}$, $\mathrm{S_V}$ ($\mathrm{G_S}$, $\mathrm{G_N}$, $\mathrm{G_V}$).

The close association of semantic and type rules prompts a general remark: Our introduction to this paper (cf. Sect. 1) has announed the development of a new type of intertheoretic relation on the model of Montague's characterization of the syntax-semantics relation. As we will show at the end of Section 5.2.2, the introduction of a separate level of types only serves to elucidate the relation between same-theory objects and propositions. Given the establishment of their constructive relations, and attendant identification of propositional interdependencies, the set of types (and associated type-propositions) is dispensable.

The introduction of semantic types requires a revision of Montague's entity-connecting principles. In the context of Section 2.2, we have identified the map $h : \mathcal{E} \to \mathcal{D}$ as the Montagovian counterpart of bridge laws. Logical types refine the previously presented mechanism: Instead of merely associating different syntactic and semantic categories, types provide a fully-fledged interface for their interaction. Propositions f_k, g_k (below) express the content of Montagovian maps $g : \mathrm{TY} \to \mathcal{E}$ and $f : \mathrm{TY} \to \mathcal{D}$, where $h = g^{-1} \circ f$:

g_{N}. Nouns are expr's of type e.	f_{N}. Type-e expr's are interpreted in $\mathcal{D}_{\mathrm{N}}$.
g_{V}. Verbs are expr's of type p.	f_{V}. Type-p expr's are interpreted in $\mathcal{D}_{\mathrm{V}}$.
g_{S}. Sentences are of type t.	f_{S}. Type-t expr's are interpreted in $\mathcal{D}_{\mathrm{S}}$.

We will leave the discussion of propositional interdefineability, together with the epistemology of types, for the following subsection.

Let us proceed to the confirmation of the conjunction of $\mathrm{T_S}, \mathrm{S_S}$, and $\mathrm{G_S}$. The prior and posterior probability of the above propositions are as follows:

$$\boldsymbol{P}_3(\mathrm{T_S}, \mathrm{S_S}, \mathrm{G_S}) \;=\; \tau \tag{19}$$

$$\boldsymbol{P}_3^* := \boldsymbol{P}_3(\mathrm{T_S}, \mathrm{S_S}, \mathrm{G_S}|\mathrm{E_S}) \;=\; \frac{\pi\,\tau}{\pi\,\tau + \rho\,\bar{\tau}}\,. \tag{20}$$

The probabilistic equivalence of the separately typed and untyped model is obvious: Given the equalities $\boldsymbol{P}_3(\mathrm{S_S}) = \boldsymbol{P}_2(\mathrm{S})$, $\boldsymbol{P}_3(\mathrm{G_S}) = \boldsymbol{P}_2(\mathrm{G})$, and $\boldsymbol{P}_3(\mathrm{E_S}|\mathrm{G_S}) = \boldsymbol{P}_2(\mathrm{E}|\mathrm{G})$ such that $\tau = \sigma$, it is easy to see the identity between the prior and posterior probabilities of the tuples $\langle \mathrm{T_S}, \mathrm{S_S}, \mathrm{G_S}\rangle$ and $\langle \mathrm{S}, \mathrm{G}\rangle$. Like the joint probability of the latter, the joint probability of former form converges to 0 as their number increases.

The difference, Δ_3, of their associated degrees of confirmation witnesses confirmation stasis. Consequently, the conjunction of $\mathrm{S_S}$ and $\mathrm{G_S}$ is not better confirmed than its untyped competitor in the post-reductive situation. We summarize our findings in the following theorem, where $|\mathrm{CAT}|$ and $|\mathrm{TY}|$ denote the number of basic (syntactic or semantic) categories and types, respectively:

Theorem 5. *If* $|\mathrm{TY}| = |\mathrm{CAT}|$*, then* $\bigcap_k \langle \mathrm{T}_k, \mathrm{S}_k, \mathrm{G}_k\rangle$ *has the same prior and posterior probability and is confirmed to the same degree as* $\bigcap_k \langle \mathrm{S}_k, \mathrm{G}_k\rangle$ *under the difference measure.*

5.2. **Case 2: Two Types.** The desired increase in confirmation requires the identification of connections between same-theory propositions. Montague's framework provides this link: Rather than taking different semantic or syntactic categories and rules to be structurally independent, Montague observes a strong connection in their constructive properties: Thus, the assumption of basic types e, t enables the construction of complex (i.e. derived) types for the representation of all other referential domains (paradigmatically, the domain of properties). This is due to the fact that functions $\mathcal{D}_{\mathrm{N}} \to \{\mathbf{T}, \mathbf{F}\}$ represent the set of entities of which a given property is true (false). In a world w, that is inhabited by John, Mary, and Fido, the property $[\![\textsf{is a dog}]\!]$ is, thus, identified with the set $\{x \in \mathcal{D}_{\mathrm{N}} \mid [\![\textsf{is a dog}]\!](x) = \mathbf{T}\} = \big\{[\![\textsf{Fido}]\!]\big\}$.[12]

While types e and t (or their associated rules) are directly involved in the reduction of rules $\mathbb{S}_{\mathrm{S}}$, $\mathbb{S}_{\mathrm{N}}$ and $\mathbb{G}_{\mathrm{S}}$, $\mathbb{G}_{\mathrm{N}}$, they only serve as 'building blocks' in the formulation of rules $\mathbb{S}_{\mathrm{V}}, \mathbb{G}_{\mathrm{V}}$. The above-described mechanism leaves open two possibilities for the construction of derived-type rules: While rules for the behavior of complex expressions or objects can be directly formulated in terms of derived types (cf. the graph in Fig. 9), their statement can, alternatively, involve rules for the obtaining of basic expressions and objects (cf. the graph in Fig. 10). As we will see in due course, both formulations yield the same probabilities.

5.2.1. *Case 2.i: Two Types Direct.* We begin by determining the joint probabilities and confirmation of directly typed propositions (cf. Fig. 9, next page).

Let $\boldsymbol{P}_4(\mathrm{T_S}) = \tau$ and $\boldsymbol{P}_4(\mathrm{T_N}) = \tau'$ be the marginal probabilities of $\mathrm{T_S}, \mathrm{T_N}$, respectively. We specify conditional probabilities via the following scheme:

[12] The latter assumption underlies our description of generalized quantifiers and their properties as objects of type $(p \to t)$ and type $((p \to t) \to t)$, respectively.

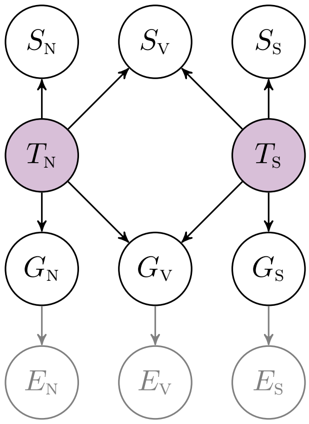

FIGURE 9. Case 2.i: Two types direct.

$$
\begin{array}{rcl}
\boldsymbol{P}_4(\mathrm{S_V}|\mathrm{T_N},\mathrm{T_S}) = 1 & , & \boldsymbol{P}_4(\mathrm{S_V}|\neg\mathrm{T_N},\mathrm{T_S}) = 0\,, \\
\boldsymbol{P}_4(\mathrm{S_V}|\mathrm{T_N},\neg\mathrm{T_S}) = 0 & , & \boldsymbol{P}_4(\mathrm{S_V}|\neg\mathrm{T_N},\neg\mathrm{T_S}) = 0\,, \\
\boldsymbol{P}_4(\mathrm{G_V}|\mathrm{T_N},\mathrm{T_S}) = 1 & , & \boldsymbol{P}_4(\mathrm{G_V}|\neg\mathrm{T_N},\mathrm{T_S}) = 0\,, \\
\boldsymbol{P}_4(\mathrm{G_V}|\mathrm{T_N},\neg\mathrm{T_S}) = 0 & , & \boldsymbol{P}_4(\mathrm{G_V}|\neg\mathrm{T_N},\neg\mathrm{T_S}) = 0\,, \\
\boldsymbol{P}_4(\mathrm{E_V}|\mathrm{G_V}) = \pi'' & , & \boldsymbol{P}_4(\mathrm{E_V}|\neg\mathrm{G_V}) = \rho''\,.
\end{array}
\tag{21}
$$

The probabilities of $\mathrm{S_S}, \mathrm{G_S}, \mathrm{E_S}$ resp. $\mathrm{S_N}, \mathrm{G_N}, \mathrm{E_N}$ are as in (17). Their dependencies (cf. ll. 1–4) are justified by propositions $f_{\mathrm{S}}, g_{\mathrm{S}}$ and $f_{\mathrm{N}}, g_{\mathrm{N}}$, above. The equations in lines 3–4 and 1–2 ensure a positive flow of confirmation from G to T and from T to S, respectively.

The prior and posterior probabilities of the conjunction of the variables' positive instantiations are as follows:

$$\boldsymbol{P}_4(\mathrm{T_S},\mathrm{T_N},\mathrm{S_S},\mathrm{S_N},\mathrm{S_V},\mathrm{G_S},\mathrm{G_N},\mathrm{G_V}) = \tau\,\tau'\,, \tag{22}$$

$$\boldsymbol{P}_4^* \;:=\; \boldsymbol{P}_4(\mathrm{T_S},\mathrm{T_N},\mathrm{S_S},\mathrm{S_N},\mathrm{S_V},\mathrm{G_S},\mathrm{G_N},\mathrm{G_V}|\mathrm{E_S},\mathrm{E_N},\mathrm{E_V})\,. \tag{23}$$

Its degree of confirmation under the difference measure, d_4, is positive under the conditions from Theorem 6:

Theorem 6. *If $\pi > \rho, \pi' > \rho', \pi'' > \rho''$ and if τ and $\tau' \in (0,1)$, then $d_4 > 0$.*

To show the epistemic value of the two- over the three-typed case, we must first specify the probabilities of the (initially neglected) rules for entity- and property-types: The values of $\boldsymbol{P}_3(\mathrm{S_N}), \boldsymbol{P}_3(\mathrm{G_N})$, $\boldsymbol{P}_3(\mathrm{E_N})$ are as for $\boldsymbol{P}_4$. The conditional probabilities of $\mathrm{S_V}, \mathrm{G_V}$ and $\mathrm{E_V}$ are analogous to their entity- and truth-value correlates. We state the marginal probability of $\mathrm{T_V}$ below:

$$\boldsymbol{P}_3(\mathrm{T_V}) = \tau''. \tag{24}$$

By the above-observed independence (Sect. 5.1, cf. Sect. 4.3), the prior and posterior probability of $(\mathrm{T_S},\mathrm{T_N},\mathrm{T_V},\mathrm{S_S},\mathrm{S_N},\mathrm{S_V},\mathrm{G_S},\mathrm{G_N},\mathrm{G_V})$, cf. (25) and (26), below, amount to the product of their respective probabilities.

$$\boldsymbol{P}_3(\mathrm{T_S},\mathrm{T_N},\mathrm{T_V},\mathrm{S_S},\mathrm{S_N},\mathrm{S_V},\mathrm{G_S},\mathrm{G_N},\mathrm{G_V}) \;=\; \tau\,\tau'\,\tau''\,, \tag{25}$$

$$\begin{aligned}(\boldsymbol{P}_3^*)' \quad &:= \quad \boldsymbol{P}_3(\mathrm{T_S}, \mathrm{T_N}, \mathrm{T_V}, \mathrm{S_S}, \mathrm{S_N}, \mathrm{S_V}, \mathrm{G_S}, \mathrm{G_N}, \mathrm{G_V} | \mathrm{E_S}, \mathrm{E_N}, \mathrm{E_V}) \\ &= \quad \left(\frac{\pi\,\tau}{\pi\,\tau + \rho\,\bar{\tau}}\right)\left(\frac{\pi'\,\tau'}{\pi'\,\tau' + \rho'\,\bar{\tau}'}\right)\left(\frac{\pi''\,\tau''}{\pi''\,\tau'' + \rho''\,\bar{\tau}''}\right).\end{aligned} \tag{26}$$

By the positivity of the difference measure d_2 (cf. Sect. 4.2), the degree of confirmation of the above conjunction, d_5, is positive under the conditions from Theorem 7:

Theorem 7. *If τ, τ', or $\tau'' = 0$, then $d_5 = 0$. If $\pi < \rho, \pi' < \rho'$, and $\pi'' < \rho''$, then $d_5 < 0$. If τ, τ', τ'' and $\pi, \pi', \pi'' \in (0, 1)$ and if $\pi > \rho, \pi' > \rho', \pi'' > \rho''$, then $d_5 > 0$.*

For comparability, we assume equalities between $\boldsymbol{P}_4(\mathrm{S}_k|\mathrm{T}_k)$ and $\boldsymbol{P}_3(\mathrm{S}_k|\mathrm{T}_k)$, $\boldsymbol{P}_4(\mathrm{G}_k|\ \mathrm{T}_k)$ and $\boldsymbol{P}_3(\mathrm{G}_k|\mathrm{T}_k)$, and $\boldsymbol{P}_4(\mathrm{E}_k|\mathrm{G}_k)$ and $\boldsymbol{P}_3(\mathrm{E}_k|\mathrm{G}_k)$ (with $(e \to t) \equiv p \in$ TY). We begin by comparing the conjunction's prior probabilities, reflected in the difference Δ_4:

$$\begin{aligned}\Delta_4 \quad &:= \quad \boldsymbol{P}_4(\mathrm{T_S}, \mathrm{T_N}, \mathrm{S_S}, \mathrm{S_N}, \mathrm{S_V}, \mathrm{G_S}, \mathrm{G_N}, \mathrm{G_V}) - \boldsymbol{P}_3\Big(\bigcap_k \big\langle \mathrm{T}_k, \mathrm{S}_k, \mathrm{G}_k \big\rangle\Big) \\ &= \quad \tau\,\tau'\,\bar{\tau}''\end{aligned} \tag{27}$$

As is clear from the relevant term in (27), the positivity of Δ_4 depends in particular on the non-certainty of $\mathrm{T_V}$ such that $\tau'' \neq 1$. Theorem 8, below, summarizes the positivity conditions for Δ_4:

Theorem 8. $\Delta_4 = 0$ *iff either (i) $\tau = 0$, (ii) $\tau' = 0$, or (iii) $\tau'' = 1$. $\Delta_4 > 0$ iff τ, τ', and $\tau'' \in (0, 1)$.*

The establishment of the positivity of the difference, $\Delta_5 := \boldsymbol{P}_4^* - (\boldsymbol{P}_3^*)'$, of the conjunction's posterior probabilities is more involved. Theorem 9, below, captures the conditions for $\Delta_5 > 0$. Its proof is included in the Appendix.

Theorem 9. $\Delta_5 > 0$ *if $\pi'' > \rho''$.*

Notably, the positivity condition from Theorem 9 is only sufficient, not necessary. However, since our rules' confirmation by the relevant piece(s) of evidence constitutes one of our permanent assumptions (cf. Sect. 4.1), we content ourselves with this criterion.

The conditions for a higher degree of confirmation are motivated by our previous observations: Given the difference $\Delta_6 := d_4 - d_5$, the replacement of three- by two-typed propositions of the syntactic and the semantic theory will increase the flow of confirmation between the two theories only if $\Delta_6 > 0$, i.e. if

$$\Delta_6 \quad = \quad \Big(\boldsymbol{P}_4^* - \boldsymbol{P}_4\Big(\bigcap_k \big\langle \mathrm{T}_k, \mathrm{S}_k, \mathrm{G}_k \big\rangle\Big)\Big) - \Big((\boldsymbol{P}_3^*)' - \boldsymbol{P}_3\Big(\bigcap_k \big\langle \mathrm{T}_k, \mathrm{S}_k, \mathrm{G}_k \big\rangle\Big)\Big) \tag{28}$$

Hence, $\Delta_6 > 0$ iff $\Delta_5 > \tau\,\tau'\,\bar{\tau}''$.

As can be seen from (28), it is 'easier' to raise the posterior probability of Montague's theories by establishing a relation between different syntactic and semantic objects than it is to increase their degree of confirmation. Especially if $(\boldsymbol{P}_3^*)'$ is (comparatively) high, the confirmation may not be greater after the reduction.

Our findings are captured in the following theorem, where TY_2 and TY_n are the basic-type sets associated with theories of two- and n-typed syntax/semantics, with $\mathrm{TY}_2 \subseteq \mathrm{TY}_n$.

Theorem 10. *If $|\mathrm{TY}_2| < |\mathrm{TY}_n|$, then the conjunction of two-typed propositions has a higher prior and posterior probability and is better confirmed under the difference measure than the conjunction of their n-typed counterparts if the following holds:*

i. The marginal probability of the truth of the propositions for members of TY_n is non-extreme.

ii. For every proposition $\mathbb{T}_i$ associated with a member, i, of $\mathrm{TY}_n \backslash \mathrm{TY}_2$, the likelihood of T_i on G_i is higher than the likelihood of $\neg\mathrm{T}_i$ on G_i.

iii. The difference between the posterior probability of the conjunction of two- and n-typed propositions is greater than the product of the marginal probability of the truth of the propositions for members of TY_2 and the probability of the falsity of the propositions for members of $\mathrm{TY}_n \backslash \mathrm{TY}_2$.

5.2.2. *Case 2.ii: Two Types Indirect.* To compare the probabilities of the directly with those of the indirectly typed propositions, we next consider the probabilities of the network in Figure 10.

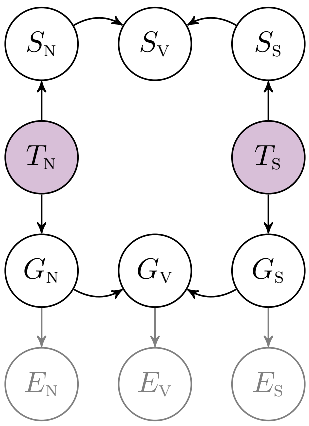

FIGURE 10. Case 2.ii: Two types indirect.

The mediated formulation of derived-type rules (cf. the chains of arrows from T_{N} and T_{S} via $S_{\mathrm{N}}, S_{\mathrm{S}}$ to S_{V}) requires the replacement of the first two lines in (21) by the equalities in (29), below (with $\boldsymbol{P}_5$ the new probability measure):

$$\begin{aligned} \boldsymbol{P}_5(\mathrm{S}_{\mathrm{V}}|\mathrm{S}_{\mathrm{N}},\mathrm{S}_{\mathrm{S}}) = 1 \quad &, \quad \boldsymbol{P}_5(\mathrm{S}_{\mathrm{V}}|\neg\mathrm{S}_{\mathrm{N}},\mathrm{S}_{\mathrm{S}}) = 0\,, \\ \boldsymbol{P}_5(\mathrm{S}_{\mathrm{V}}|\mathrm{S}_{\mathrm{N}},\neg\mathrm{S}_{\mathrm{S}}) = 0 \quad &, \quad \boldsymbol{P}_5(\mathrm{S}_{\mathrm{V}}|\neg\mathrm{S}_{\mathrm{N}},\neg\mathrm{S}_{\mathrm{S}}) = 0\,. \end{aligned} \tag{29}$$

The conditional probability of G_{V} is similarly defined: Rather than depending only on the probabilities of T_{N} and T_{S}, the probability of the truth of S_{V} is now also dependent on the probabilities of $\mathrm{S}_{\mathrm{N}}, \mathrm{S}_{\mathrm{S}}$. This is not to claim a fundamental difference between the presently and previously introduced models: Notably, our choice of different type-rule formulations does not impact the theories' probabilities and confirmation. This is due to the probabilistic equivalence of chains of arrows $(T_{\mathrm{S}} \to S_{\mathrm{S}}) \circ (S_{\mathrm{S}} \to S_{\mathrm{V}})$, $(T_{\mathrm{S}} \to S_{\mathrm{V}})$, and corresponding identities $\boldsymbol{P}_5(\mathrm{S}_{\mathrm{V}}) = \boldsymbol{P}_4(\mathrm{S}_{\mathrm{V}})$, $\boldsymbol{P}_5(\mathrm{G}_{\mathrm{V}}) = \boldsymbol{P}_4(\mathrm{G}_{\mathrm{V}})$. The confirmation of indirectly typed propositions is thus the same as that of directly typed propositions.

We have motivated the presentation of the model of our new type of intertheoretic reduction by the need to identify dependencies between same-theory objects (and, thus, propositions). Our investigation into the probabilistic impact of different rule-formulations yields further insight into the latter requirement: While the use of types (as a surrogate for term-connecting bridge laws) increases the perspicuity of the effected reduction, the improvement of our theories' confirmation is not conditional on the introduction of an intermediate type level. This is warranted by the identity of $\boldsymbol{P}_5(\mathrm{S_S}), \boldsymbol{P}_5(\mathrm{T_S}), \boldsymbol{P}_5(\mathrm{G_S})$ and $\boldsymbol{P}_5(\mathrm{S_N}), \boldsymbol{P}_5(\mathrm{T_N}), \boldsymbol{P}_4(\mathrm{G_N})$, respectively.[13] The only requirement lies in the establishment of definitional connections between same-theory objects.

The latter constitute the core feature of Integrative Reduction. We define Integrative Reduction as a weak variant of Nagelian reduction that differs from the latter in its directedness and the establishment of constructive relations between same-theory objects and propositions. Consequently, a separately-typed directed variant of the Nagelian model (along the lines of Sect. 5.1) does not qualify as a *proper* model of Integrative Reduction.[14] This is due to the probabilistic equivalence of their associated propositions, and attendant collapse of the separately-typed model of Integrative Reduction into a variant of the Nagelian model. We conclude this section with considerations about the optimal number of basic types (or primitive semantic objects).

5.3. **Case 3: One Type.** Our previous findings suggest an inverse proportionality between the theories' probabilities, or degree of confirmation, and the number of basic types: As the latter decreases, the former rises. To check this hypothesis, and identify possible constraints, we now turn to the last case. Figure 11 displays a graph associated with the assumption of a single type, e, for the formulation of syntactic and semantic rules. By the results from Section 5.2, our type choice does not influence the confirmation of Montagovian propositions.

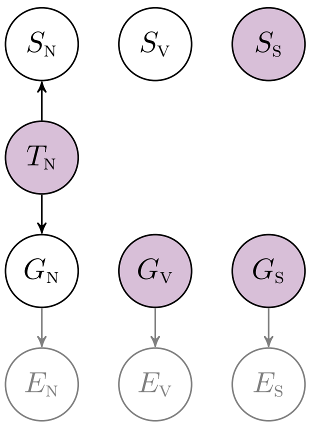

FIGURE 11. Case 3: One type.

[13]This constitutes the probabilistic basis for linguists' choice between 'direct' and 'indirect' interpretations of natural language into set-theoretic models, cf. (Partee, 1997).

[14]Note, however, the possibility of treating Integrative Reductions of any kind (including non-proper reductions) as a generalization of Nagelian reduction.

A glance at the graph in Figure 11 reveals the large number of root nodes and conditional independencies. This is due to the impossibility of constructing the remaining types (e.g. $(e \to t)$, t) from a single base type (here, e), and related need to separately introduce their associated syntactic and semantic rules.

The abbreviation scheme, below, contains the marginal and conditional probabilities of all nodes in the Bayesian network in Figure 11:

$$\begin{array}{rcccl} \boldsymbol{P}_6(\mathrm{T_N}) = \tau' & , & \boldsymbol{P}_6(\mathrm{S_S}) = \sigma & , & \boldsymbol{P}_6(\mathrm{S_V}) = \sigma'' \\ \boldsymbol{P}_6(\mathrm{G_S}) = \gamma & , & \boldsymbol{P}_6(\mathrm{G_V}) = \gamma'' & , & \boldsymbol{P}_6(\mathrm{S_N}|\mathrm{T_N}) = 1 \\ \boldsymbol{P}_6(\mathrm{S_N}|\neg\mathrm{T_N}) = 0 & , & \boldsymbol{P}_6(\mathrm{G_N}|\mathrm{T_N}) = 1 & , & \boldsymbol{P}_6(\mathrm{G_N}|\neg\mathrm{T_N}) = 0 \\ \boldsymbol{P}_6(\mathrm{E_N}|\mathrm{G_N}) = \pi' & , & \boldsymbol{P}_6(\mathrm{E_N}|\neg\mathrm{G_N}) = \rho' & , & \boldsymbol{P}_6(\mathrm{E_S}|\mathrm{G_S}) = \pi \\ \boldsymbol{P}_6(\mathrm{E_S}|\neg\mathrm{G_S}) = \rho & , & \boldsymbol{P}_6(\mathrm{E_V}|\mathrm{G_V}) = \pi'' & , & \boldsymbol{P}_6(\mathrm{E_V}|\neg\mathrm{G_V}) = \rho'' \end{array} \tag{30}$$

The probabilities of $\mathrm{T_N}, \mathrm{S_N}, \mathrm{G_N}$, and $\mathrm{E_N}$ are as in (17). The other values in the first, second, ultimate and penultimate lines correspond to those from (2). By the absence of property- or truth-value types, the positive flow of confirmation between $\mathrm{G_N}, \mathrm{T_N}$, and $\mathrm{S_N}$ (cf. ll. 4, 5) is disabled at the verbal and sentential level.

The independence of tuples $\langle T_{\mathrm{N}}, S_{\mathrm{N}}, G_{\mathrm{N}}\rangle$, $\langle S_{\mathrm{S}}, G_{\mathrm{S}}\rangle$, $\langle S_{\mathrm{V}}, G_{\mathrm{V}}\rangle$ facilitates the comparative assessment of our theories' probabilities and confirmation. While the prior and posterior probability of the conjunction $(\mathrm{T_N}, \mathrm{S_N}, \mathrm{E_N})$ correspond to the probabilities of the separately typed case in Section 5.1 (granted the usual comparability conditions), the probabilities of $(\mathrm{S_S}, \mathrm{G_S})$ and $(\mathrm{S_V}, \mathrm{G_V})$ are parallel to those of the pre-reductive untyped case (cf. Sect. 4.1; Fig. 4). Their multiplication yields the following prior probability:

$$\boldsymbol{P}_6(\mathrm{T_N}, \mathrm{S_N}, \mathrm{S_S}, \mathrm{S_V}, \mathrm{G_N}, \mathrm{G_S}, \mathrm{G_V}) \quad = \quad \boldsymbol{P}_6(\mathrm{T_N}, \mathrm{S_N}, \mathrm{G_N})\boldsymbol{P}_6(\mathrm{S_S}, \mathrm{G_S})\boldsymbol{P}_6(\mathrm{S_V}, \mathrm{G_V}) \tag{31}$$

By the above argument, (31) is greater than the prior probability of the conjunction of untyped propositions in the pre-reductive, but smaller than the conjunction of separately typed propositions in the post-reductive situation. The same holds, by an argument from $\boldsymbol{P}_2^*$ and $(\boldsymbol{P}_3^*)'$, of the conjunction's posterior probability:

$$\boldsymbol{P}_6^* \quad := \quad \boldsymbol{P}_6(\mathrm{T_N}, \mathrm{S_N}, \mathrm{S_S}, \mathrm{S_V}, \mathrm{G_N}, \mathrm{G_S}, \mathrm{G_V}|\mathrm{E_N}, \mathrm{E_S}, \mathrm{E_V})\,. \tag{32}$$

We assess the conjunction's evidential support via the measure d_6 and observe that the positivity conditions for $d_6 > 0$ are similar to the positivity conditions for d_5 (cf. Thm. 7):

Theorem 11. *If $\gamma, \gamma'', \sigma, \sigma''$, or $\tau' = 0$, then $d_6 = 0$. If $\pi < \rho, \pi' < \rho'$, and $\pi'' < \rho''$, then $d_6 < 0$. If $\gamma, \gamma'', \sigma, \sigma'', \tau'$ and $\pi, \pi', \pi'' \in (0, 1)$ and if $\pi > \rho, \pi' > \rho', \pi'' > \rho''$, then $d_6 > 0$.*

To compare our theories' degree of confirmation with the support of the separately typed model, we calculate $\Delta_7 := d_6 - d_5$ and obtain the following theorem:

Theorem 12. *If $\pi > \rho, \pi' > \rho', \pi'' > \rho''$ and $\gamma, \gamma'', \sigma, \sigma''$, and $\tau' \in (0, 1)$, then $\Delta_7 < 0$.*

The concession of a Montagovian map between the elements in $\langle S_{\mathrm{S}}, G_{\mathrm{S}}\rangle$ and $\langle S_{\mathrm{V}}, G_{\mathrm{V}}\rangle$ (cf. Sect. 4.2) hardly improves this situation: While the homomorphism h cancels some of the above-observed independencies – requiring a restatement of the relevant probabilities in (33), below – the theories' probabilities and degree of confirmation do not exceed that of the separately typed model. To mark the move to a different probabilistic situation, we introduce the new probability measure $\boldsymbol{P}_7$. Significantly,

$$\begin{aligned} \boldsymbol{P}_7(\mathrm{G_S}|\mathrm{S_S}) = 1 \quad &, \quad \boldsymbol{P}_7(\mathrm{G_S}|\neg\mathrm{S_S}) = 0\,, \\ \boldsymbol{P}_7(\mathrm{G_V}|\mathrm{S_V}) = 1 \quad &, \quad \boldsymbol{P}_7(\mathrm{G_V}|\neg\mathrm{S_V}) = 0\,, \end{aligned} \tag{33}$$

(cf. Section 4.2, (7)). All other assignments are as above.

Since tuples $\big\langle(\mathrm{T}_k,)\,\mathrm{S}_k, \mathrm{G}_k\big\rangle$ remain independent, we calculate their joint probabilities via the mechanism, above. The prior and posterior probability of the conjunction are

$$\boldsymbol{P}_7(\mathrm{T_N}, \mathrm{S_N}, \mathrm{S_S}, \mathrm{S_V}, \mathrm{G_N}, \mathrm{G_S}, \mathrm{G_V}) = \sigma\,\sigma''\,\tau'\,, \tag{34}$$

and

$$\boldsymbol{P}_7^* = \left(\frac{\pi\,\sigma}{\pi\,\sigma + \rho\,\bar{\sigma}}\right)\left(\frac{\pi''\,\sigma''}{\pi''\,\sigma'' + \rho''\,\bar{\sigma}''}\right)\left(\frac{\pi'\,\tau'}{\pi'\,\tau' + \rho'\,\bar{\tau}'}\right). \tag{35}$$

It is easy to see that, granted the above requirements, the conjunction's prior and posterior probabilities under $\boldsymbol{P}_7$ are exactly the probabilities of (25) and (26).

We summarize the results of our investigation of the separate-, the two-, and the single-type case in the following theorem, where TY_m and TY_n are different basic-type sets such that $\mathrm{TY}^m \subseteq \mathrm{TY}^n$:

Theorem 13. *If* TY_m *enables the construction of all linguistically relevant types, then, granted the conditions from Theorem 10,* $\bigcap_m \langle \mathrm{S}_m, \mathrm{G}_m\rangle$ *has a higher prior and posterior probability and is better confirmed under the difference measure than* $\bigcap_n \langle \mathrm{S}_n, \mathrm{G}_n\rangle$*.*

Given the derivability of all syntactic or semantic propositions, the minimal number of basic types yields the highest probabilities and effects a maximal flow of confirmation between the two theories.

6. Extended Models of Integrative Reduction

The past two sections have identified a new kind of intertheoretic relation inspired by Montague's theory of syntax and semantics. Its model differs from existing models of reduction with respect to the relation between elements from different theories, and the dependency of elements from the same theory: Nagelian bridge laws are replaced by a designated map between propositions of the reduced and the reducing theory; connections between same-theory objects are captured through mediating types. In contrast to the Nagelian model of reduction – and like Schaffner's revised model –, our model of Integrative Reduction accommodates cases of multiple realizability. Thus, a single object (or type of object) in the reduced theory may be related (via bridge laws or the map h) to different (types of) objects in the reducing theory. While our model locates the origin of this (dis-)ambiguity directly in the propositions of the reducing theory, Schaffner's revised model ascribes its origin to auxiliary assumptions, whose consideration motivates the introduction of 'corrected' versions of propositions from both theories, cf. (Schaffner, 1974; Nagel, 1977). The present section formulates a Schaffner-style variant of our previously developed model. For convenience, we first review Schaffner's revised Nagelian model, as presented in (Dijzadji-Bahmani et al., 2010b).

Like Nagel's model and our integrative model of intertheoretic reduction, Schaffner's revised model assumes two theories, $\mathcal{T}_1$ (i.e. the reducing theory) and $\mathcal{T}_2$ (i.e. the reducing theory). With each of the two theories is associated a set of empirical propositions, $\{T_1^1, \ldots, T_1^n\}$ resp. $\{T_2^1, \ldots, T_2^n\}$ (with $n \in \mathbb{N}$), that constitute the laws of $\mathcal{T}_1$, respectively $\mathcal{T}_2$. The theory $\mathcal{T}_2$ is then reduced to the theory $\mathcal{T}_1$ by first deriving a corrected version, T_1^*, of every proposition, T_1, in $\mathcal{T}_1$ (step 1), obtaining its reduced theory counterpart, T_2^*, via bridge laws (step 2), and showing a strong analogy between T_2^* and the corresponding proposition, T_2, in $\mathcal{T}_2$ (step 3). Figure 12 illustrates Schaffner's revised model of reduction for theories $\mathcal{T}_1$ and $\mathcal{T}_2$.

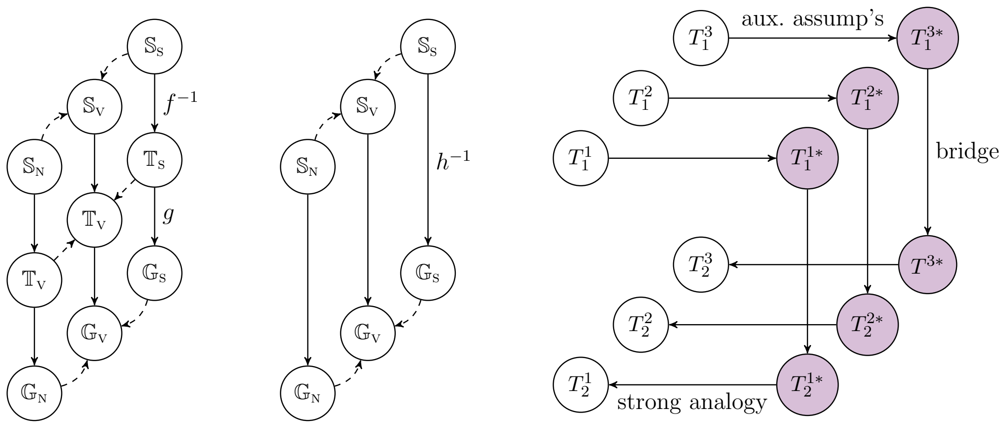

FIGURE 12. The integrative model (left, middle) and Schaffner's revised model of reduction for three proposition-pairs (right).

For comparison, Schaffner's model is printed next to a diagram of our model of Integrative Reduction with mediating types (left) and without types (middle).

To transform our model of Integrative Reduction into a variant of Schaffner's revised model, we only need to introduce a corrected version, $\mathbb{S}_k^*$ and $\mathbb{G}_k^*$, of every proposition, $\mathbb{S}_k$ and $\mathbb{G}_k$, in $\mathbb{S}$ and $\mathbb{G}$ (i.e. add steps 1, 3). The latter, which themselves take the form of (dis-)ambiguating propositions, remove the ambiguity from every semantic rule, $\mathbb{S}_k$, in $\mathbb{S}$. For illustration, we state the corrected version, $\mathbb{S}_{\mathrm{S}}^*$, of the sentence-formation rule $\mathbb{S}_{\mathrm{S}}$:

$\mathbb{S}_{\mathrm{S}}^*$. If $[\![R]\!]' \in \mathcal{D}'_{\mathrm{V}}$ and $[\![j]\!]' \in \mathcal{D}'_{\mathrm{N}}$, then $[\![R]\!]'([\![j]\!]') \in \mathcal{D}_{\mathrm{S}}$, and
if $[\![R]\!]'' \in \mathcal{D}''_{\mathrm{V}}$ and $[\![j]\!]'' \in \mathcal{D}''_{\mathrm{N}}$, then $[\![R]\!]''([\![j]\!]'') \in \mathcal{D}_{\mathrm{S}}$.

The rule $\mathbb{S}_{\mathrm{S}}^*$ makes $\mathbb{S}_{\mathrm{S}}$'s ambiguity between the rules $\mathbb{S}'_{\mathrm{S}}$ and $\mathbb{S}''_{\mathrm{S}}$ explicit. Consequently, the resulting model of Schaffner-style Integrative Reduction will exhibit all properties of Schaffner's revised model of Nagelian reduction (including the 'origin' of multiple realizability). In contrast to Schaffner's model (cf. Fig. 12, right), however, our Schaffner-style variant of Integrative Reduction accommodates intratheoretical propositional dependencies. The latter is represented by either of the diagrams in Figure 13 (next page).

In Figure 13, relations between same-theory propositions are represented by dashed arrows. The diagram on the right identifies entity-connecting types with

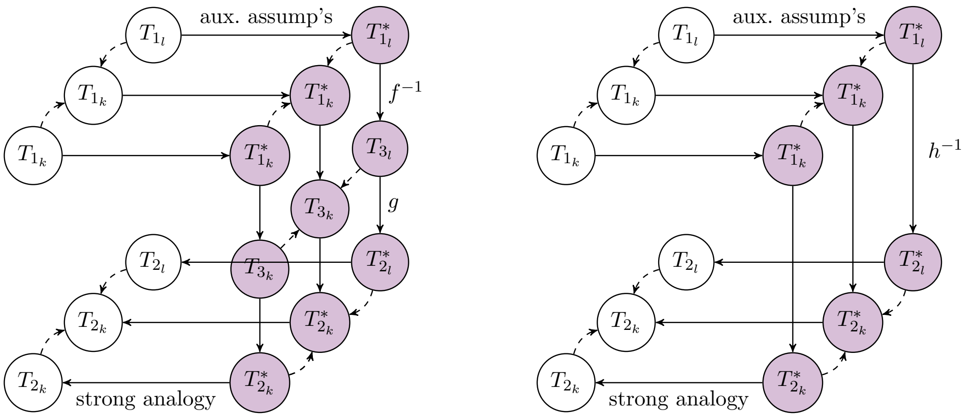

FIGURE 13. Integrative GNS-Reduction with types (right) and without types (left) for three proposition-pairs.

primitive objects in the reducing theory; the diagram on the left preserves their separate status. In the former case, the role of types will be taken over by entities that are neutral between objects of both theories.

Significantly, the extended model of Integrative Reduction (above) behaves towards our original model of Integrative Reduction (cf. Sect. 5) just like Schaffner's revised model of reduction behaves towards Nagel's original model. This is no surprise: Like Schaffner's revised model, our extended model of Integrative Reduction aims to accommodate cases of multiple realizability, while leaving the remaining features of the model unchanged. In contrast to the Nagelian model of reduction, however, our model of Integrative Reduction already assumes the relation's directedness. While Schaffner's extension of the Nagelian model thus serves principally to introduce ambiguating relations into the model, the Schaffner-style extension of our model only moves the locus of ambiguation from the level of propositions (i.e. syntactic and semantic rules) to their corrected versions. We leave the detailed study of the extended model of Integrative Reduction, and its relation to other models of intertheoretic reduction for another occasion.

7. Conclusion

In this paper, we have identified a new type of intertheoretic relation, Integrative Reduction, that is instantiated in linguistic syntax and semantics. We have shown its commonalities with Nagelian reduction, and established their salient differences. To assess the epistemic value of Integrative Reduction, we have given its analysis in the framework of Bayesian confirmation theory. We have shown that the Integrative Reduction of syntax to semantics is epistemically advantageous over its Nagelian reduction in three respects: It raises the prior and posterior probabilities and (given certain conditions) the degree of confirmation of the conjunction of syntactic and semantic propositions under the difference measure. This is achieved through the establishment of dependency relations between same-theory objects and propositions, and attendant confirmation of some syntactic (or semantic) propositions by the supporting evidence of other (syntactic or semantic) propositions.

Appendix

We have calculated the pre-reductive probabilities of the conjunction of S, G in Section 4.1. The joint distribution, $\boldsymbol{P}_2(S, G, E)$, of the (post-reductive) graph in Figure 6 is given by the expression

$$\boldsymbol{P}_2(S)\,\boldsymbol{P}_2(G)\,\boldsymbol{P}_2(E|G).$$

Using the methodology employed in (Bovens and Hartmann, 2003), the prior probability of the conjunction of S and G is obtained as follows:

$$\boldsymbol{P}_2(\mathrm{S}, \mathrm{G}) = \sum_E \boldsymbol{P}_2(\mathrm{S}, \mathrm{G}, E) = \pi\,\sigma + \bar{\pi}\,\sigma = \sigma\,. \tag{36}$$

We yield the posterior probability, $\boldsymbol{P}_2^* := \boldsymbol{P}_2(\mathrm{S}, \mathrm{G}|\mathrm{E})$, thus:

$$\boldsymbol{P}_2^* = \frac{\boldsymbol{P}_2(\mathrm{S}, \mathrm{G}, \mathrm{E})}{\boldsymbol{P}_2(\mathrm{E})} = \frac{\pi\,\sigma}{\pi\,\sigma + \rho\,\bar{\sigma}}\,. \tag{37}$$

To obtain the difference Δ_0, we calculate

$$\boldsymbol{P}_2(\mathrm{S}, \mathrm{G}) - \boldsymbol{P}_1(\mathrm{S}, \mathrm{G}) = \sigma - \sigma^2 = \sigma\,\bar{\sigma}\,.$$

This proves Theorem 3. The difference Δ_1 is obtained as follows:

$$\boldsymbol{P}_2^* - \boldsymbol{P}_1^* = \frac{\pi\,\sigma - \pi\,\sigma^2}{\pi\,\sigma + \rho\,\bar{\sigma}} = \frac{\pi\,\sigma\,\bar{\sigma}}{\pi\,\sigma + \rho\,\bar{\sigma}}\,. \tag{38}$$

From the difference measure

$$d_2 := \boldsymbol{P}_2(\mathrm{S}, \mathrm{G}|\mathrm{E}) - \boldsymbol{P}_2(\mathrm{S}, \mathrm{G}) = \frac{\sigma\,\bar{\sigma}\,(\pi - \rho)}{\pi\,\sigma + \rho\,\bar{\sigma}}\,, \tag{39}$$

we calculate Δ_2 as follows:

$$d_2 - d_1 = \frac{\sigma\,\bar{\sigma}\,(\pi - \rho) - \sigma^2\,\bar{\sigma}\,(\pi - \rho)}{\pi\,\sigma + \rho\,\bar{\sigma}} = \frac{\sigma\,\bar{\sigma}^2\,(\pi - \rho)}{\pi\,\sigma + \rho\,\bar{\sigma}}\,. \tag{40}$$

Let us consider the confirmation of the conjunction for the separate-type case (1). The joint distribution $\boldsymbol{P}_3(T_{\mathrm{S}}, S_{\mathrm{S}}, G_{\mathrm{S}}, E_{\mathrm{S}})$ is given by the expression

$$\boldsymbol{P}_3(T_{\mathrm{S}})\boldsymbol{P}_3(S_{\mathrm{S}}|T_{\mathrm{S}})\boldsymbol{P}_3(G_{\mathrm{S}}|T_{\mathrm{S}})\boldsymbol{P}_3(E_{\mathrm{S}}|G_{\mathrm{S}}).$$

To obtain the prior probability of the conjunction of $\mathrm{T}_{\mathrm{S}}, \mathrm{S}_{\mathrm{S}}$, and G_{S}, we calculate

$$\boldsymbol{P}_3(\mathrm{T}_{\mathrm{S}}, \mathrm{S}_{\mathrm{S}}, \mathrm{G}_{\mathrm{S}}) = \sum_E \boldsymbol{P}_3(\mathrm{T}_{\mathrm{S}}, \mathrm{S}_{\mathrm{S}}, \mathrm{G}_{\mathrm{S}}, E_{\mathrm{S}}) = \tau\,. \tag{41}$$

The posterior probability, $\boldsymbol{P}_3^* := \boldsymbol{P}_3(\mathrm{T}_{\mathrm{S}}, \mathrm{S}_{\mathrm{S}}, \mathrm{G}_{\mathrm{S}}|\mathrm{E}_{\mathrm{S}})$, is obtained as follows:

$$\boldsymbol{P}_3^* = \frac{\boldsymbol{P}_3(\mathrm{T}_{\mathrm{S}}, \mathrm{S}_{\mathrm{S}}, \mathrm{G}_{\mathrm{S}}, \mathrm{E}_{\mathrm{S}})}{\boldsymbol{P}_3(\mathrm{E}_{\mathrm{S}})} = \frac{\pi\,\tau}{\pi\,\tau + \rho\,\bar{\tau}}\,. \tag{42}$$

The difference, Δ_3, between the degree of confirmation of the separately typed and the untyped proposition witnesses confirmation stasis:

$$\Delta_3 := d_3 - d_2 = 0\,, \tag{43}$$

where d_2 is as above, and $d_3 = d_2$.

We next discuss the probabilities and degree of confirmation of the two-type case (2). The joint distribution $\boldsymbol{P}_4(T_{\mathrm{S}}, T_{\mathrm{N}}, S_{\mathrm{S}}, S_{\mathrm{N}}, S_{\mathrm{V}}, G_{\mathrm{S}}, G_{\mathrm{N}}, G_{\mathrm{V}}, E_{\mathrm{S}}, E_{\mathrm{N}}, E_{\mathrm{V}})$ is given by the expression

$$\begin{aligned} &\boldsymbol{P}_4(T_{\mathrm{S}})\boldsymbol{P}_4(T_{\mathrm{N}})\boldsymbol{P}_4(S_{\mathrm{S}}|T_{\mathrm{S}})\boldsymbol{P}_4(S_{\mathrm{N}}|T_{\mathrm{N}})\boldsymbol{P}_4(S_{\mathrm{V}}|T_{\mathrm{N}}, T_{\mathrm{S}})\boldsymbol{P}_4(G_{\mathrm{S}}|T_{\mathrm{S}}) \\ &\boldsymbol{P}_4(G_{\mathrm{N}}|T_{\mathrm{N}})\boldsymbol{P}_4(G_{\mathrm{V}}|T_{\mathrm{N}}, T_{\mathrm{S}})\boldsymbol{P}_4(E_{\mathrm{S}}|G_{\mathrm{S}})\boldsymbol{P}_4(E_{\mathrm{N}}|G_{\mathrm{N}})\boldsymbol{P}_4(E_{\mathrm{V}}|G_{\mathrm{V}})\,. \end{aligned} \tag{44}$$

The prior probability of the conjunction of positive instantations of the above variables is as follows:

$$\boldsymbol{P}_4(\mathrm{T_S}, \mathrm{T_N}, \mathrm{S_S}, \mathrm{S_N}, \mathrm{S_V}, \mathrm{G_S}, \mathrm{G_N}, \mathrm{G_V}) = \tau\,\tau' \,. \tag{45}$$

Their posterior probability, $\boldsymbol{P}_4^* := \boldsymbol{P}_4(\mathrm{T_S}, \mathrm{T_N}, \mathrm{S_S}, \mathrm{S_N}, \mathrm{S_V}, \mathrm{G_S}, \mathrm{G_N}, \mathrm{G_V} | \mathrm{E_S}, \mathrm{E_N}, \mathrm{E_V})$, is obtained thus:

$$\begin{aligned} \boldsymbol{P}_4^* &= \frac{\boldsymbol{P}_4(\mathrm{T_S}, \mathrm{T_N}, \mathrm{S_S}, \mathrm{S_N}, \mathrm{S_V}, \mathrm{G_S}, \mathrm{G_N}, \mathrm{G_V}, \mathrm{E_S}, \mathrm{E_N}, \mathrm{E_V})}{\boldsymbol{P}_4(\mathrm{E_S}, \mathrm{E_N}, \mathrm{E_V})} \\ &= \frac{\pi\,\pi'\,\pi''\,\tau\,\tau'}{\pi\,\pi'\,\pi''\,\tau\,\tau' + \rho\,\rho'\,\rho''\,\bar{\tau}\,\bar{\tau}'} \,. \end{aligned} \tag{46}$$

Rather than calculating all 2^8 possibilities, we use the equalities in (17)–(18) and (21) to isolate the significant, i.e. non-zero, cases. Since the non-uniform (i.e. positive or negative) instantiation of T_k, S_k, and G_k renders the product in (44) zero, we restrict our attention to the following two cases:

i. $\mathrm{T_S}, \mathrm{T_N}, \mathrm{S_S}, \mathrm{S_N}, \mathrm{S_V}, \mathrm{G_S}, \mathrm{G_N}, \mathrm{G_V}$;
ii. $\neg\mathrm{T_S}, \neg\mathrm{T_N}, \neg\mathrm{S_S}, \neg\mathrm{S_N}, \neg\mathrm{S_V}, \neg\mathrm{G_S}, \neg\mathrm{G_N}, \neg\mathrm{G_V}$.

The degree of confirmation of the conjunction of propositions under the difference measure is as follows:

$$\begin{aligned} d_4 &:= \boldsymbol{P}_4^* - \boldsymbol{P}_4(\mathrm{T_S}, \mathrm{T_N}, \mathrm{S_S}, \mathrm{S_N}, \mathrm{S_V}, \mathrm{G_S}, \mathrm{G_N}, \mathrm{G_V}) \\ &= \frac{\tau\,\tau'\,\bar{\tau}\,\bar{\tau}'\,(\pi\,\pi'\,\pi'' - \rho\,\rho'\,\rho'')}{\pi\,\pi'\,\pi''\,\tau\,\tau' + \rho\,\rho'\,\rho''\,\bar{\tau}\,\bar{\tau}'} \,. \end{aligned} \tag{47}$$

To obtain the difference Δ_4, we calculate

$$\begin{aligned} &\boldsymbol{P}_4(\mathrm{T_S}, \mathrm{T_N}, \mathrm{S_S}, \mathrm{S_N}, \mathrm{S_V}, \mathrm{G_S}, \mathrm{G_N}, \mathrm{G_V}) - \boldsymbol{P}_3(\mathrm{T_S}, \mathrm{T_N}, \mathrm{T_V}, \mathrm{S_S}, \mathrm{S_N}, \mathrm{S_V}, \mathrm{G_S}, \mathrm{G_N}, \mathrm{G_V}) \\ =\;& \tau\,\tau' - \tau\,\tau'\,\tau'' = \tau\,\tau'\,\bar{\tau}'' \,. \end{aligned} \tag{48}$$

The difference Δ_5 is obtained as follows:

$$\Delta_5 := \boldsymbol{P}_4^* - (\boldsymbol{P}_3^*)', \tag{49}$$

with $\boldsymbol{P}_4^*$ as above and

$$(\boldsymbol{P}_3^*)' = \left(\frac{\pi\,\tau}{\pi\,\tau + \rho\,\bar{\tau}}\right)\left(\frac{\pi'\,\tau'}{\pi'\,\tau' + \rho'\,\bar{\tau}'}\right)\left(\frac{\pi''\,\tau''}{\pi''\,\tau'' + \rho''\,\bar{\tau}''}\right) \,. \tag{50}$$

To show that $\Delta_5 > 0$, we first observe that the function

$$f(\tau'') := \frac{\pi''\,\tau''}{\pi''\,\tau'' + \rho''\,\bar{\tau}''} \tag{51}$$

is strictly monotonically increasing in τ''. Consequently,

$$f(\tau'') \leq f(1) = 1 \,. \tag{52}$$

By the assumption that $\pi, \pi', \rho, \rho', \tau, \tau' \in (0, 1)$, it thus holds that

$$(\boldsymbol{P}_3^*)' \leq \left(\frac{\pi\,\tau}{\pi\,\tau + \rho\,\bar{\tau}}\right)\left(\frac{\pi'\,\tau'}{\pi'\,\tau' + \rho'\,\bar{\tau}'}\right) \,.$$

Then,

$$\begin{aligned} \Delta_5 &\geq X \cdot \left(\frac{1}{\pi\,\pi'\,\pi''\,\tau\,\tau' + \rho\,\rho'\,\rho''\,\bar{\tau}\,\bar{\tau}'} - \frac{1}{(\pi\,\tau + \rho\,\bar{\tau})(\pi'\,\tau' + \rho'\,\bar{\tau}')\,\pi''}\right) \\ &= X' \cdot \left((\pi\,\tau + \rho\,\bar{\tau})(\pi'\,\tau' + \rho'\,\bar{\tau}')\,\pi'' - (\pi\,\pi'\,\pi''\,\tau\,\tau' + \rho\,\rho'\,\rho''\,\bar{\tau}\,\bar{\tau}')\right) \\ &= X' \cdot \left(\pi\,\pi''\,\rho'\,\tau\,\bar{\tau}' + \pi'\,\pi''\,\rho\,\bar{\tau}\,\tau' + \rho\,\rho'\,\bar{\tau}\,\bar{\tau}' \cdot (\pi'' - \rho'')\right) \,. \end{aligned} \tag{53}$$

with

$$\begin{aligned} X &:= \pi\,\pi'\,\pi''\,\tau\,\tau' \\ X' &:= \frac{\pi\,\pi'\,\pi''\,\tau\,\tau'}{(\pi\,\tau+\rho\,\bar\tau)(\pi'\,\tau'+\rho'\,\bar\tau')\pi''\cdot(\pi\,\pi'\,\pi''\,\tau\,\tau'+\rho\,\rho'\,\rho''\,\bar\tau\,\bar\tau')}\,. \end{aligned}$$

The expression in the final line of (53) is greater than 0 if $\pi'' > \rho''$. Note that this is a sufficient, not a necessary condition. This completes the proof that $\Delta_5 > 0$.

To assess the confirmatory status of the direct two-typed case, we first identify the measures d_4, d_5, with

$$\begin{aligned} d_4 &:= \boldsymbol{P}_4^* - \boldsymbol{P}_4(\mathrm{T_S},\mathrm{T_N},\mathrm{S_S},\mathrm{S_N},\mathrm{S_V},\mathrm{G_S},\mathrm{G_N},\mathrm{G_V}) \\ &= \frac{\tau\,\tau'\,\bar\tau\,\bar\tau'\,(\pi\,\pi'\,\pi''-\rho\,\rho'\,\rho'')}{\pi\,\pi'\,\pi''\,\tau\,\tau'+\rho\,\rho'\,\rho''\,\bar\tau\,\bar\tau'} \end{aligned} \tag{54}$$

and

$$\begin{aligned} d_5 &:= (\boldsymbol{P}_3^*)' - \boldsymbol{P}_3(\mathrm{T_N},\mathrm{T_S},\mathrm{T_V},\mathrm{S_N},\mathrm{S_S},\mathrm{S_V},\mathrm{G_N},\mathrm{G_S},\mathrm{G_V}) \\ &= \left(\frac{\pi\,\tau}{\pi\,\tau+\rho\,\bar\tau}\right)\left(\frac{\pi'\,\tau'}{\pi'\,\tau'+\rho'\,\bar\tau'}\right)\left(\frac{\pi''\,\tau''}{\pi''\,\tau''+\rho''\,\bar\tau''}\right) - \tau\,\tau'\,\tau''\,. \end{aligned} \tag{55}$$

Their difference, $\Delta_6 := d_4 - d_5$, is easily obtained though the use of the fact that $d_i - d_k = (\boldsymbol{P}_i^* - \boldsymbol{P}_i(\dots)) - (\boldsymbol{P}_k^* - \boldsymbol{P}_k(\dots)) = (\boldsymbol{P}_i^* - \boldsymbol{P}_k^*) - (\boldsymbol{P}_i(\dots) - \boldsymbol{P}_k(\dots))$ such that

$$\Delta_6 \;=\; \Delta_5 - \tau\,\tau'\,\bar\tau''. \tag{56}$$

We close by considering the confirmation of the conjunction in the single-type case (3). The joint distribution $\boldsymbol{P}_6(T_\mathrm{N},S_\mathrm{S},S_\mathrm{N},S_\mathrm{V},G_\mathrm{S},G_\mathrm{N},G_\mathrm{V},E_\mathrm{S},E_\mathrm{N},E_\mathrm{V})$ is given by the expression

$$\begin{gathered} \boldsymbol{P}_6(T_\mathrm{N})\boldsymbol{P}_6(S_\mathrm{S})\boldsymbol{P}_6(S_\mathrm{V})\boldsymbol{P}_6(G_\mathrm{S})\boldsymbol{P}_6(G_\mathrm{V}) \\ \boldsymbol{P}_6(S_\mathrm{N}|T_\mathrm{N})\boldsymbol{P}_6(G_\mathrm{N}|T_\mathrm{N})\boldsymbol{P}_6(E_\mathrm{N}|G_\mathrm{N})\boldsymbol{P}_6(E_\mathrm{S}|G_\mathrm{S})\boldsymbol{P}_6(E_\mathrm{V}|G_\mathrm{V})\,. \end{gathered} \tag{57}$$

Our calculation of the conjunction's prior and posterior probabilities exploit the independence of pairs, $\langle S_k, G_k\rangle$, together with the results from Sections 4.1, 4.2, 5.1 such that

$$\begin{aligned} \boldsymbol{P}_6(\mathrm{T_N},\mathrm{S_N},\mathrm{S_S},\mathrm{S_V},\mathrm{G_N},\mathrm{G_S},\mathrm{G_V}) &= \boldsymbol{P}_6(\mathrm{T_N},\mathrm{S_N},\mathrm{G_N})\boldsymbol{P}_6(\mathrm{S_S},\mathrm{G_S})\boldsymbol{P}_6(\mathrm{S_V},\mathrm{G_V}) \\ &= \gamma\gamma''\,\sigma\sigma''\,\tau'\,, \end{aligned} \tag{58}$$

and

$$\begin{aligned} \boldsymbol{P}_6^* &:= \boldsymbol{P}_6(\mathrm{T_N},\mathrm{S_N},\mathrm{S_S},\mathrm{S_V},\mathrm{G_N},\mathrm{G_S},\mathrm{G_V}|\mathrm{E_N},\mathrm{E_S},\mathrm{E_V}) \\ &= \left(\frac{\pi'\,\tau'}{\pi'\,\tau'+\rho'\,\bar\tau'}\right)\left(\frac{\gamma\,\pi\,\sigma}{\gamma\,\pi+\bar\gamma\,\rho}\right)\left(\frac{\gamma''\,\pi''\,\sigma''}{\gamma''\,\pi''+\bar\gamma''\,\rho''}\right). \end{aligned} \tag{59}$$

We assess the conjunction's evidential support via the measure d_6 and observe that, under the positivity conditions from d_1 and d_5, the difference d_6 is also positive:

$$\begin{aligned} d_6 &:= \boldsymbol{P}_6^* - \boldsymbol{P}_6(\mathrm{T_N},\mathrm{S_N},\mathrm{S_S},\mathrm{S_V},\mathrm{G_N},\mathrm{G_S},\mathrm{G_V}) \\ &= \left(\frac{\pi'\,\tau'}{\pi'\,\tau'+\rho'\,\bar\tau'}\right)\left(\frac{\gamma\,\pi\,\sigma}{\gamma\,\pi+\bar\gamma\,\rho}\right)\left(\frac{\gamma''\,\pi''\,\sigma''}{\gamma''\,\pi''+\bar\gamma''\,\rho''}\right) - \gamma\,\gamma''\,\sigma\,\sigma''\,\tau'\,. \end{aligned} \tag{60}$$

From the measures d_5 and d_6, above, we obtain Δ_7 as follows:

$$\begin{aligned} \Delta_7 &= \left(\frac{\pi'\,\tau'}{\pi'\,\tau' + \rho'\,\bar{\tau}'}\right)\left(\frac{\gamma\,\pi\,\sigma}{\gamma\,\pi + \bar{\gamma}\,\rho}\right)\left(\frac{\gamma''\,\pi''\,\sigma''}{\gamma''\,\pi'' + \bar{\gamma}''\,\rho''}\right) - \gamma\,\gamma''\,\sigma\,\sigma''\,\tau' \\ &\quad - \left(\left(\frac{\pi\,\tau}{\pi\,\tau + \rho\,\bar{\tau}}\right)\left(\frac{\pi'\,\tau'}{\pi'\,\tau' + \rho'\,\bar{\tau}'}\right)\left(\frac{\pi''\,\tau''}{\pi''\,\tau'' + \rho''\,\bar{\tau}''}\right) - \tau\,\tau'\,\tau''\right) \\ &= -\,\sigma\,\sigma''\,\tau'\,(1 - \gamma\gamma'')\left(\left(\frac{\pi}{\pi\,\tau + \rho\,\bar{\tau}}\right)\left(\frac{\pi'}{\pi'\,\tau' + \rho'\,\bar{\tau}'}\right)\left(\frac{\pi''}{\pi''\,\tau'' + \rho''\,\bar{\tau}''}\right) - 1\right). \end{aligned} \tag{61}$$

Since expressions of the form $\pi/(\pi\,\tau + \rho\,\bar{\tau})$ are greater than 1 for every π, ρ, τ of the same type if $\pi > \rho$ and $\tau \in (0, 1)$, the difference Δ_7 is always negative. This completes our calculations.

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