Cause and Chance

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Overview

- Propose a framework for a propensity theory of chance
- Will answer questions such as:

Overview (cont'd)

- When do you have a propensity?
- How do you get probabilities from propensities?
- How are propensities related to credences?
- What information is *inadmissible*?

Overview (cont'd)

Will not answer:

- What makes the propensity for heads on a coin toss (e.g.) take the specific value .51 rather than .49?

Overview (cont'd)

- Will motivate approach with some problems for propensity theories, as well as for Lewis's *Principal Principle*
- Make use of formal apparatus of causal Bayes nets

Propensity Interpretation of Chance

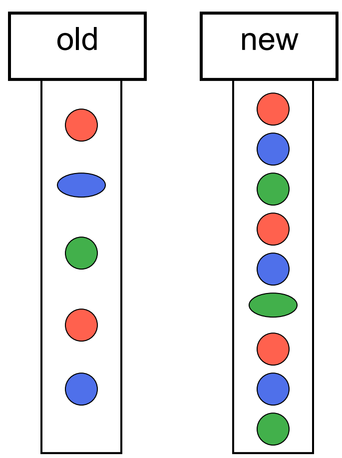
- Introduced by Popper; defended by Miller, Fetzer, Giere
- Propensities are measures of the strength of partial dispositions
- The disposition is grounded in a causal basis
- E.g. propensity of a coin to land heads; causal basis = constitution of coin, movement of thumb, constitution of landing surface, etc.

Three Problems for Propensities

- Humphreys' reversibility objection
- Identifying the causal basis
- The source of mathematical structure

Humphreys' Reversibility Objection

The frisbee factory



Humphreys' Reversibility Objection (cont'd)

Pr(f is defective | f is made by new machine) could be a propensity

Pr(f is made by new machine | f is defective) is not a propensity

Identifying the Causal Basis

Which sets of causes give rise to propensities?

Standard answer:
- Complete time-slice

Identifying the Causal Basis (cont'd)

Problems with the standard answer:

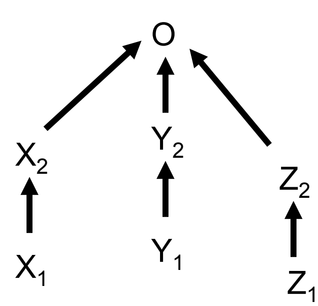
1. Too demanding:

   We rarely know the complete state at a time

2. Too restricitive:

   Non-simultaneous causes can confer a propensity

Identifying the Causal Basis (cont'd)



The Source of Structure

Why think that propensities have the mathematical structure of probability?

- Propensity theorists often compare propensities to physical parameters like mass, force, and charge
- It is an *empirical* matter that charge is discrete, has positive and negative values
- But we can't imagine discovering that chances can be negative or greater than 1

The Source of Structure (cont'd)

Three potential solutions

Propensities inherit their structure from:

1. Frequencies (early Popper, Gillies)
2. Ratios of possible outcomes (Giere, McCall)
3. Rational credences (Mellor, Skyrms, Lewis)

The Principal Principle

PP (Lewis)

Cr(A | Ch(A) = r & E) = r

- ‘Cr’ is the credence of a rational agent
- ‘Ch’ is chance
- ‘E’ is admissible relative to ‘A’ and ‘Ch’

The Principal Principle (cont’d)

PP tells us

- what it is for a rational agent to *take* ‘Ch’ to be chance
- how evidence bears on hypotheses about chance
- how beliefs about chance guide decisions

The Principal Principle (cont'd)

PP posits a *screening-off* relation:

Cr(A | Ch(A) = r & E) = r

'Ch(A) = r' screens off 'A' from admissible 'E'

The Principal Principle (cont'd)

Problem: when is 'E' admissible?

Why it matters:

- There is not one PP, but many
- Different objective probability concepts give rise to versions of PP
- They differ with respect to admissibility relations

The Principal Principle (cont'd)

Finite frequencies obey a PP-like principle:

- Knowing only that Pat is British, and 15.7% of Britons are aged 65 or older, what credence should I have that Pat is 65 or older?
- .157
- Many propositions are inadmissible – whether Pat plays bridge or listens to Lady Gaga

The Project

The problem of defining admissibility (for propensities) and finding appropriate causal bases are inter-related:

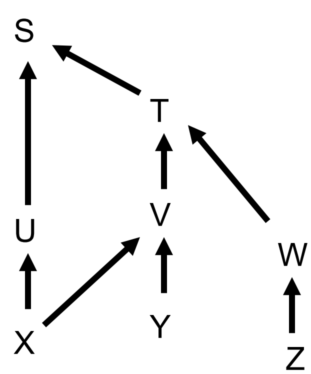
- A suitable causal basis will screen off inadmissible information
- Admissibility can be characterized causally

Causal Bayes Nets

A *causal structure* $S = \langle \mathbf{V}, \mathbf{G} \rangle$

- **V** is a set of variables
- **G** is a *directed acyclic graph* on **V**
- An arrow from *X* to *Y* represents that *X* has a causal influence on *Y* that is *direct* relative to **V**

Causal Bayes Nets (cont'd)



Causal Bayes Nets (cont'd)

Terminology:

X is a *parent* of *Y* if there is an arrow from *X* to *Y*

- **PA**(*X*) is the set of parents of *X*

Causal Bayes Nets (cont'd)

Y is a descendant of *X* if there is a *directed path* from *X* to *Y*

- Convenience: *X* is a descendant of *X*
- **DE**(*X*) is the set of descendants of *X*

Causal Bayes Nets (cont'd)

A *causal Bayes net* $\mathcal{N} = \langle \mathbf{V}, \mathbf{G}, Pr \rangle$

- $\langle \mathbf{V}, \mathbf{G} \rangle$ is a causal structure
- *Pr* is a probability measure on **V** that satisfies the *Markov condition*

Causal Bayes Nets (cont'd)

Markov condition:

For all $V \in \mathbf{V}$, all $\mathbf{W} \subseteq \mathbf{V} \backslash \mathbf{DE}(V)$:

$$Pr(V \mid \mathbf{PA}(V), \mathbf{W}) = Pr(V \mid \mathbf{PA}(V))$$

PA(*V*) screens off *V* from its non-descendants

Causal Bayes Nets (cont'd)

Fact: *Pr* satisfies the Markov condition on **V** = { $V_1,\ldots, V_n$ } iff *Pr* 'factorizes':

$\Pr(V_1,\ldots V_n) = \Pi_{i = 1,.., n} \Pr(V_i \mid \mathbf{PA}(V_i))$

Pr is uniquely determined by probabilities of the form:

$\Pr(V_i \mid \mathbf{PA}(V_i))$

Proposal

Reverse engineering:
want to end up with a causal Bayes net

Probabilities of the form
$\Pr(V = v \mid \mathbf{PA}(V) = \mathbf{p})$
are interpreted as propensities

Proposal (cont'd)

Pr can be uniquely constructed out of the probabilities that are directly interpreted as propensities

(follows from factorization)

Thus we get all of the probabilities, including backwards conditional probabilities, out of propensities

Proposal (cont'd)

$\mathbf{PA}(V) = \mathbf{p}$ is the causal basis for the propensity

$\Pr(V = v \mid \mathbf{PA}(V) = \mathbf{p})$

(We can liberalize this to allow other causal bases)

Proposal (cont'd)

Markov condition gives us a purely causal definition of admissibility

A proposition is admissible when it tells us only about variables that are not descendants of V

(This can be modified to allow for other kinds of causal basis)

The Construction

Doing things in the right order:

Propensity structure $\mathcal{P} = \langle \mathbf{V}, \mathbf{G}, \mathit{Prop} \rangle$

$\mathit{Prop}(\mathbf{PA}(V) = \mathbf{p}, V = v)$

the propensity for causal basis $\mathbf{PA}(V) = \mathbf{p}$ to yield $V = v$

The Construction (cont'd)

PP*:

$\mathrm{Cr}(V = v \mid \mathbf{PA}(V) = \mathbf{p} \ \& \ P \ \& \ E) = r$
$= \mathit{Prop}(\mathbf{PA}(V) = \mathbf{p},\ V = v)$

for all admissible propositions *E*,

where *P* is the proposition that $\mathcal{P}$ is an *appropriate* representation

The Construction (cont'd)

Define $Pr(\bullet) = Cr(\bullet|P)$

- *Pr* is a probability function
- *Pr* satisfies the Markov condition (from definition of admissibility)
- *Pr* is uniquely determined by *Prop* (by factorization)

$\langle \mathbf{V}, \mathbf{G}, Pr \rangle$ is a causal Bayes net

Liberalizing the Causal Basis

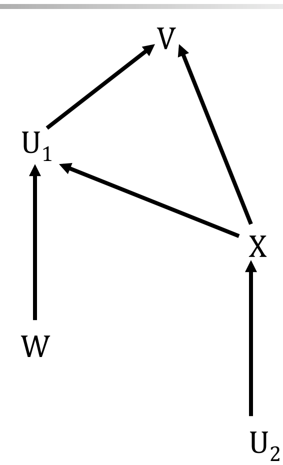
Allow sets of causes **U** other than **PA**(*V*) to be causal bases for propensities of *V*

U gives rise to a propensity for *V* = *v* if it has the right admissibility profile

Only information about variables that are causally downstream of **U** is inadmissible

Causal Bases

- $\{U_1, U_2\}$ does not form a causal basis for a propensity
- Because information about W is inadmissible
- {W, X} does form the causal basis for a propensity



From Framework to Theory

What determines the value of
$Prop(\mathbf{PA}(V) = \mathbf{p}, V = v)$?

What makes a propensity structure $\mathcal{P}$ an *appropriate* representation?

The framework is compatible with many possible answers

Unlikely to be one answer suitable for all causal systems

An Under-appreciated Option

Moderate anti-realism:

- The causal structure is real
- The causal basis is real
- Propensities are real; some are stronger than others
- But assigning them precise values is an idealization; like assigning precise values to degrees of belief

Moderate Anti-realism (cont'd)

PP* connects propensities with Bayesian confirmation theory and decision theory

There are facts about

- how well $\mathcal{P}$ fits the data
- whether $\mathcal{P}$ has been a good 'guide to life'

But (on this option) no facts about whether $\mathcal{P}$ is *true*