Comments on Caie, "Belief and Indeterminacy"

FORMAL EPISTEMOLOGY WORKSHOP
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HARTRY FIELD

I. Caie's paradox, stripped-down

Let γ be, or be uncontroversially equivalent to:

I don't believe $<\gamma>$.

Abbreviate this as: $\neg B\gamma$.

Suppose I'm *somewhat rational*, and *a very good introspector*.

Let $X \vDash Y$ here mean that Y follows from X together with (a precisification of) these assumptions about me.

The argument to paradox (1)

1a. $B\neg B\gamma \vDash B\gamma$

1b. $BB\gamma \vDash B\neg\gamma$

- (Weakened versions of what Caie gets using his "Evidence". The idea: since the equivalence of γ to $\neg B(<\gamma>)$ and hence by contraposition of $B(<\gamma>)$ to $\neg\gamma$ are simple theorems, failing to satisfy these would be a gross failure of rationality.)

2a. $B\gamma \vDash BB\gamma$

2b: $\neg B\gamma \vDash B\neg B\gamma$

- The transparency (introspection) assumptions.

3. $\neg B\gamma \vDash B\gamma$ by 2b and 1a

4. $B\gamma \vDash B\neg\gamma$, by 2a and 1b

5. $B\neg\gamma \vDash \neg B\gamma$ (a weakening of his consistency assumption).

6. $B\gamma \vDash \neg B\gamma$ by 4 and 5.

The argument to paradox (2)

To repeat:

3. $\neg B\gamma \vDash B\gamma$

6. $B\gamma \vDash \neg B\gamma$.

From 3 we get

$\neg B(<\gamma>) \vDash B(<\gamma>) \wedge \neg B(<\gamma>)$,

and from 6 we get

$B(<\gamma>) \vDash B(<\gamma>) \wedge \neg B(<\gamma>)$;

so reasoning by cases gives

$\mathbf{B(<\gamma>) \vee \neg B(<\gamma>) \vDash B(<\gamma>) \wedge \neg B(<\gamma>).}$

The law of excluded middle (LEM) thus leads to contradiction, given the transparency assumptions.

Morals

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If one doesn't question the transparency assumptions, the only obvious options are

(i) the dialetheic option of accepting some contradictions but limiting their impact by restricting explosion, and

(ii) saying that **LEM is not validly applied here.**

Caie prefers (ii). (As do I.)

CONSISTENCY and EVIDENCE a distraction?

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The advantage of this stripped down version is that it doesn't rely on Caie's full principles of CONSISTENCY and EVIDENCE.

I think an upholder of classical logic would be pretty much forced to focus his critique on the transparency assumptions.

That may well be possible, but I won't discuss them.

II. REJECTION v. INDETERMINACY

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REJECTION: If one ought to believe *A* indeterminate, one ought to reject both it and its negation (in a sense of rejection that precludes acceptance, i.e. belief).

INDETERMINACY: If one ought to believe *A* indeterminate, one ought to be such that it's indeterminate whether one believes A and indeterminate whether one believes its negation.

These are supposed to be the options both in the case when *A* is a Liar sentence and when it's a sentence like γ above.

Caie says that it would be *ad hoc* to treat the two cases differently.

A. Why assume γ "indeterminate"?

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REJECTION and INDETERMINACY are principles about sentences believed to be "indeterminate".

The conclusions about γ reached earlier were

(i) that the γ-instance of LEM leads to contradiction given certain natural assumptions, and so

(ii) we shouldn't assume that excluded middle can validly apply to γ.

There was no claim that γ is "indeterminate".

No super-determinacy predicate

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Caie is assuming the logic I've advocated. In it, we can introduce a notion of indeterminacy for which the Liar sentence can be declared indeterminate.

But lots of paradoxical sentences *can't* be declared not determinate, but only not determinately determinate, or not determinately determinately determinate, or not determinately$^{\omega \cdot \omega}$ determinate, ….

There's no weakest indeterminacy-type predicate.

And with vague sentences (and some paradoxical sentences too) there will typically not even be an α for which we can declare the sentence not determinately$^{\alpha}$ determinate. If we could, we'd be introducing sharp boundaries, making the restriction of LEM pointless.

γ not "indeterminate" for any such predicate

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Since a natural moral to draw from the failure of LEM for γ is that 'believes' is crucially vague, there's reason *not* to think that γ is literally indeterminate.

While this goes against Caie's official argument against REJECTION, it isn't so clear that his basic idea is undercut.

B. Rej-γ vs. Ind-γ

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Here's an argument that we should reject the specific claim γ:

Rej-γ: The LEM-instance $B\gamma \vee \neg B\gamma$ leads to contradiction. Indeed, given a logic with explosion, it leads to any absurdity one chooses. So we should reject that instance of LEM. But if we reject a disjunction, we should reject each disjunct. So we should reject $\neg B\gamma$. But that's equivalent to γ, so we should reject γ.

Here's an argument that we better not reject it:

Ind-γ: If we reject γ, we shouldn't believe it. But then we're back in the same paradox as before. So we better not reject γ (but just: not determinately accept it).

It's possible to question Ind-γ, but I'm inclined to agree with Caie that this isn't the best way to go.

Caie's resolution

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Caie's resolution is that there's an error in Rej-γ: the fact that $B\gamma \vee \neg B\gamma$ leads to absurdity doesn't show (he thinks) that we *should reject it*, but only that *we shouldn't determinately accept it*.

I'm tentatively inclined to agree, but one worry:

Consider a sentence γ* that is equivalent to "I don't determinately believe <γ*>".

An analog of the argument in Section 2 would show that the assumption

- Either I determinately believe γ* or I don't

leads to contradiction.

We can't resolve *this* by saying that I don't determinately believe γ*.

Response on Caie's behalf

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I assume that what Caie would say to the worry is this:

For γ, I don't determinately believe it.

*For γ**, I don't *determinately* determinately believe it.

And *for γ*** equivalent to "I don't determinately determinately believe <γ**>", the resolution is that I don't *determinately determinately* determinately believe it.

Pyrrhic victory?

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But then we need to have different attitudes for different paradoxical sentences.

That suggests that maybe for ordinary paradoxical sentences like the Liar, we might have the simpler attitude of not believing it?

Recall that Caie argues that it's *ad hoc* to have different attitudes in the case of Liar sentences and γ.

But if we already have different ones for γ, γ*, γ, ..., this argument loses its force.**

III. Are (REJECTION) and (INDETERMINACY) distinct?

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There's a verbal issue about how to talk about non-classical degrees of belief.

For logics like the one under consideration, my primary formulation tends to be in terms of nonstandard point-valued credences: nonstandard in that Cr(A) + Cr(¬A) will be less than 1 when excluded middle for A isn't accepted. (It can be as low as 0.)

Acceptance of A is credence over some high threshold T, rejection is credence below 1–T, so in cases where Cr(A) + Cr(¬A) is sufficiently small we can simultaneously reject A and ¬A.

Two re-descriptions?

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But I've often pointed out that there's an obviously-equivalent re-description: take the degree of belief in A to be not the point Cr(A), but the interval [Cr(A), 1 – Cr(¬A)].

And this suggests a third re-description: it is indeterminate what the agent's degree of belief in A is. (The legitimate candidates for it are the members of the interval [Cr(A), 1 – Cr(¬A)].)

If there's substance to the distinction between these three descriptions, I don't see it.

Notational variants?

And if not, then

- rejecting both A and its negation [in the sense of Cr(A) and Cr(¬A) both being 0 or close to it],

looks pretty much identical to

- having highly indeterminate degree of belief [i.e., having [Cr(A), 1 – Cr(¬A)] be the whole unit interval or close to it].

So are (REJECTION) and (INDETERMINACY) notational variants?