A Generalised Sorites

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Outline

The Garden Variety (Conditional) Sorites

Consider a vague predicate Φ and a well-ordered set of subject expressions $\{a_1, a_2, a_3, ..., a_n\}$ such that Φa_1 and $\neg\Phi a_n$. Because Φ is vague, we have it that $\Phi a_{i-1} \rightarrow \Phi a_i$. But:

$$\frac{\begin{array}{c}\Phi a_1 \\ \Phi a_{i-1} \rightarrow \Phi a_i\end{array}}{\Phi a_n}$$

Contradiction.

The Continuous Sorites

Consider a vague predicate Φ mapped onto a real-number interval $[0, 1]$, exhaustively partitioned into two non-empty sets,

$$\begin{aligned} A &= \{x \in [0,1] : \Phi(x)\}, \\ B &= \{x \in [0,1] : \neg\Phi(x)\}, \end{aligned}$$

with $a < b$ for all $a \in A, b \in B$.
We assume that $\Phi(0)$ and $\neg\Phi(1)$, and monotonicity: If some number is not Φ, then no numbers after it are Φ either.
Thus A is the left side of the interval and B is the right. A has least upper bound: $supA$. Points vanishingly close to $supA$ are Φ, and Φ is vague, so $\Phi(supA)$. By a symmetrical argument, $\neg\Phi(infB)$.

The Continuous Sorites

By the linear order on $\mathbb{R}$, one of the following must be true:

$$\begin{aligned} & supA < infB \\ \text{or} \quad & infB < supA \\ \text{or} \quad & supA = infB. \end{aligned}$$

Since the reals are dense, we have the following contradiction: if *supA* and *infB* are different, then there is some z between them, $supA < z < infB$ or $infB < z < supA$. But then Φz and $\neg\Phi z$, since by definition anything less than *infB* is Φ but anything greater than *supA* is not. If $supA = infB$, we have $\Phi supA$ and $\neg\Phi supA$. This exhausts all the cases. Therefore there is a point both Φ and $\neg\Phi$. Contradiction. (Chase, unpublished)

Further Generalisations

- We invoked the continuity and well-orderedness of the reals rather than the recursiveness of the naturals.
- I take it that this is clearly a sorites paradox.
- We can generalise further to a topological version of the sorites:
  - If a space X is connected, all locally-constant functions on it are globally constant
  - If a predicate Φ is vague, its characteristic function is locally constant
  - If there is some $a \in X$ such that Φa, then all $a \in X$ are Φ.
- The latter invokes the connectedness of the space in question and that's all.
- Topological versions are useful for representing family resemblance paradoxes.

What is the Principle of Uniform Solution?

Principle of Uniform Solution: Whenever we face the same kind of paradox, we should invoke the same kind of solution.

Some Questions

- What counts as the same kind of paradox? At what level of abstraction (see Nick Smith (2000) and Graham Priest (1994))?
- What counts as the same kind of solution? At what level of abstraction?
- Why should we buy such a principle? Why shouldn't we be content to just have solutions to the various paradoxes?

Motivation for the Principle of Uniform Solution

- We don't want to deal with the paradox of the heap by invoking a multi-valued logic, $Ł_\infty$, say, and the paradox of the bald man with a supervaluational logic.
- Despite the former being about sand and the latter about hair, they are of a kind and should be dealt with the same way.
- What really underwrites the principle of uniform solution is the concern that without it we might be treating the symptoms but missing the underlying disease.
- So we clearly buy some version of the Principle of Uniform Solution, but the exact form and its application depends on how we answer the other questions about levels of abstraction.

Similarity of Paradoxes

- What do we say about:
  - the Liar and Russell's paradox?
  - the Liar and Curry's paradox?
  - the Liar and the Strengthened Liar?
  - the Liar and Liar Cycles?
  - the Liar and Yablo's paradox?
  - the numerical (total-ordered) sorites and the non-numerical (multi-dimensional) sorites?
  - the sorites and continuous and topological versions of the sorites
  - the Liar and the Sorites paradox? *

The Inclosure Schema

- Following Russell (and Priest) we can show that a number of paradoxes satisfy the *inclosure schema*.
- *Inclosure Schema*: There are two properties φ and ψ, and a function δ such that
  1. [Existence] $\Omega = \{y | \varphi(y)\}$ exists, and $\psi(\Omega)$
  2. If x is a subset of Ω such that $\psi(x)$, then
     - (a) [Transcendence] $\delta(x) \notin x$, and
     - (b) [Closure] $\delta(x) \in \Omega$.
- According to Priest we are obliged to treat the large class of paradoxes captured by this schema by the same means.

Inclosing the Sorites

- Briefly, the diagonaliser here is the supremum and infemum functions.
- In a bit more detail:
  - Existence is satisfied since the extension of the vague predicate in question (and its complement) is non-empty.
  - Transcendence is satisfied because the limit points of the extension of the vague predicate and its complement cannot both be in the relevant sets.
  - Closure is satisfied by the indiscernability (or tolerance) of the vague predicate (and its complement).
- We thus have the continuous sorites as an inclosure paradox. (Chase, unpublished).

The Liar and the Sorites

- Field, McGee, and Tappenden have all suggested gappy approaches to both on the grounds that the truth predicate is indeterminate and thereby analogous to vague predicates. But I think we can do better than this.
- The Liar and the Sorites both have strengthened forms that raise problems for gappy approaches.
- They both respond to glutty approaches.
- This suggests that the two might have something in common and the Principle of Uniform Solution might be thought to apply.
- The case is strengthened by the fact that the continuous and topological sorites (at least) are inclosure paradoxes.

The Sorites Family | Principle of Uniform Solution | The Sorites and the Liar are of a Kind | Defining 'Vagueness' | Summary

Standard Accounts

- A predicate is vague iff it permits borderline cases, where a borderline case is a gap.
- This begs the question against glutty approaches to vagueness.
- A predicate is vague iff it is tolerant.
- Here 'tolerance' is understood as: whenever the predicate holds in some case, it holds of the next case.
- Or better: whenever the predicate holds in some case, it holds in nearby cases.
- This is nearly right.

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A New Definition

- A predicate is vague iff it supports a generalised sorites argument.
- Apart from the word 'generalised', this is not really new—several people have advanced such a definition (but for the wrong reasons).
- Being more explicit about what the generalised sorites is (i.e. the topological version) we get:
- A predicate is vague iff its characteristic function is locally constant but not globally constant.
- This definition is a version of the tolerance definition given previously, but with a topological spin on tolerance.

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Why the Definition Matters

- Without a decent definition we may be begging the question against some legitimate treatments.
- We may miss the essential feature of vagueness.
- E.g., thinking of vagueness in well-ordered discrete cases suggests that the well-ordering or the conditional are at the heart of the problem.
- But if we're right about the generalised forms, the new definition of vagueness is the right one and we see that the conditional is irrelevant.
- Tolerance, in the sense of locally constant but not globally constant characteristic functions, is where the action is.
- Finally, this matters in practical applications, such as in conservation biology, where vagueness is topological (e.g. endangered species).

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Summary

- We generalised the sorites paradox to continuous and topological spaces.
- In light of this, some of the standard treatments are seen to fail to engage with what really drives the sorites.
- A case can be made that the generalised versions of the sorites is of a kind with the liar.
- This, in turn, pushes for a uniform treatment of the sorites and the liar.
- Standard definitions of 'vagueness' are inadequate; the generalised sorites motivates a new definition in terms of locally-constant characteristic functions.
- This new definition is welcome in many applications.

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Selected Bibliography I

- Beall, JC and Colyvan, M. 2001. 'Heaps of Gluts and Hyde-ing the Sorites', *Mind*, 110: 401–8.
- Bueno, O. and Colyvan, M. 2012. 'Just What is Vagueness?', *Ratio*, 25: forthcoming.
- Colyvan, M. 2008. 'Vagueness and Truth', in H. Dyke (ed.), *From Truth to Reality*, Routledge, pp. 29–40.
- Hyde, D. 1997. 'From Heaps and Gaps to Heaps of Gluts', *Mind*, 106: 641–60.
- Hyde, D. 2001. 'Reply to Beall and Colyvan', *Mind*, 110: 409–11.
- Hyde, D. and Colyvan, M. 2008: 'Paraconsistent Vagueness: Why Not?', *Australasian Journal of Logic*, 6: 107–21.
- Priest, G. 1994. 'The Structure of the Paradoxes of Self Reference', *Mind*, 103: 25–34.
- Priest, G. 2010. 'Inclosures, Vagueness, and Self-Reference', *Notre Dame Journal of Formal Logic*, 51: 69–84.
- Smith, N.J.J. 2000. 'The Principle of Uniform Solution (of the Paradoxes of Self-Reference)', *Mind*, 109: 117–22.
- Weber, Z. 2010. 'A Paraconsistent Model of Vagueness', *Mind*, 119: 1025–45.
- Weber, Z. and Colyvan, M. 2010. 'A Topological Sorites', *Journal of Philosophy*, 107: 311–25.