Collective Reasons *via* Judgment Aggregation.

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Introduction

- Judgment Aggregation investigates rules that determine collective judgments on the basis of the individual judgments of the members of a group.
- (Possible) Applications:
  - *Group Organization*: how should a group organize its own deliberations to meet logical and epistemological desiderata?
  - *Group Deference:* Suppose n (seemingly equally competent) people give me conflicting testimony on whether the bus for Denver has already left. What should I believe?
- Simple examples of aggregation rules:
  - (M) $\mathcal{G}$(roup) accepts p iff a majority of $\mathcal{G}$'s members accepts p.
  - (U) $\mathcal{G}$ accepts p iff all of $\mathcal{G}$'s members accept p.

My Project (Generally)

- To investigate how *reasons* should enter the picture.
  1. Can we model *collective reasons*, alongside *collective judgments*?
  2. Should collective judgments be sensitive to more than individual judgments (e.g. should they be sensitive to individual reasons)?

The Argument to Come:

Need:
The cases that motivate Judgment Aggregation theory also motivate an account of collective reasons.

Problem:
The standard framework lacks room for such an account.

Proposal:
So we need a somewhat different framework.

The Need

- Aggregation problems are only interesting when we aggregate judgments on sets of logically connected propositions.

Example 1

	p	q	$p \vee q$
1	Y	N	Y
2	N	Y	Y
3	N	N	N
(M)	N	N	Y

But why care about these cases?

Pettit: "the problem in question is [...] tied [...] only to the enterprise of making group judgments on the basis of reasons"

Pigozzi: " A verdict in a court is a public act. Not only, if convicted, has a defendant the right to know the reasons for which she has been convicted, but also these reasons will guide future decisions [...]. In other words, the final decision must be supported and justified by reasons."

The Standard Framework.

- A *modeling language* $\mathcal{L}$
- $\mathcal{G}$: a finite (and odd-sized) set of judges
- The *agenda* $\mathcal{I}$ is a subset of $\mathcal{L}$ that is closed under negation
- A *judgment set* j is a non-empty subset of $\mathcal{I}$
- An *epistemic state* is a maximally consistent (relative to $\mathcal{I}$) judgment set
- A *profile* $\vec{j} = \langle j_1, ..., j_n \rangle$ is a vector of epistemic states
- An *aggregation rule* $\mathcal{A}$ is a partial function from profiles to judgment sets

Some Properties of Aggregation Rules.

Universal: for every $\vec{j}$, $\mathcal{A}(\vec{j})$ is defined.

Consistency-Preserving: for every $\vec{j}$, $\mathcal{A}(\vec{j})$ is logically consistent.

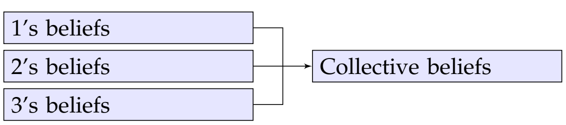
Anonymous: for every permutation h of $\mathcal{G}$, $\mathcal{A}(j_1, ..., j_n) = \mathcal{A}(j_{h(1)}, ..., j_{h(n)})$

Complete (relative to $\mathcal{I}$): for every $\vec{j}$, and every $p \in \mathcal{I}$, $p \in \mathcal{A}(\vec{j})$ or $\sim p \in \mathcal{A}(\vec{j})$.

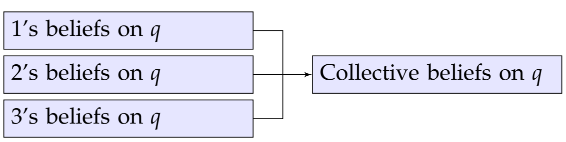
Independent: whether $p \in \mathcal{A}(\vec{j})$ depends only on the pattern of individual opinion on p.

Supervenience Patterns.

On the standard picture, aggregation rules describe supervenience patterns of collective beliefs on individual beliefs:



For *independent* rules, the supervenience pattern is even tighter.



1st try: Entailment-Reasons Link

Suppose the group accepts some salient propositions $p_1, ..., p_n, q$,

(ER) $p_1, ..., p_n$ count as a collective reason for q iff $p_1, ..., p_n$ entail q.

- (ER) is pretty clearly false in both directions.
  - $\Rightarrow$ inductive support can be enough to give you a reason (also related: reasons are generally assumed to be non-monotonic).
  - $\Leftarrow$ sometimes, entailment can run from a conclusion to some of the reasons that support it.
- You can't read reasons off of entailment patterns.

Premises

Some enrich the framework in this way:

- designate some propositions as *premises*
  - the premises must be logically independent.
- designate some others as *conclusions*
  - the conclusion must be settled by any distribution of truth-value on the premises.
- You can define rules like:

  PB If p is a premise, then p is collectively accepted iff a majority supports p.
  If p is a conclusion, first figure out the collective judgments on the premises, then settle by entailment.

2nd try: Premise-Reasons Link

Suppose $\mathcal{G}$ accepts some salient propositions $p_1, ..., p_n, q$,

(PR) $p_1, ..., p_n$ count as collectives reason for q iff $p_1, ..., p_n$ are premises and q is a conclusion.

Example:

Suppose the agenda consists of $\{p, p \equiv q, q, negations\}$.

Suppose you designate p and $p \equiv q$ as premises and q as a conclusion.

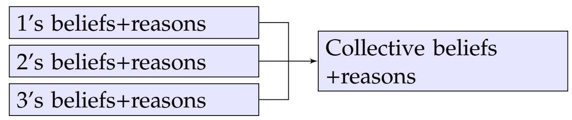
Then whatever the pattern of acceptance on p and $p \equiv q$ will determine the reasons for the verdict on q.

Two Objections

1. The domain of applicability of the proposal is too restricted.
   It only works if the premises are logically independent, and the conclusion is settled by any distribution of truth-value on the premises. It also requires that every proposition in the agenda be either a premise or a conclusion.
2. In general, reasons cannot be fixed as "external" to the epistemic state.
   Some judges can take p and $p \equiv q$ as reasons for q; others can take q and $p \equiv q$ as reasons for p.
   That's keeping the *very same judgments*. If you consider the general case (judges with different opinion) the implausibility of PR is even more pervasive (think: *one man's Modus Ponens...*).

Generalized Framework

- In order to model collective reasons we need finer inputs.
- Instead of representing individual states as maximally consistent sets of propositions, I represent them as pairs consisting of one such set and a basing relation $\hookrightarrow$.
- Each advisor i *can* accept a proposition p on the basis of a set $\{q_1, ..., q_n\}$ (we write: $\{q_1, ..., q_n\} \hookrightarrow_i p$).
- Of course, some propositions may be supported non-inferentially.



Sketch

1. Illustrate some new rules for collective acceptance.
2. Show how with these rules there is a viable notion of collective reason to be defined.

Cohesiveness

Definition (Strong Cohesiveness)

M strongly cohesively supports p iff there is a set Σ of propositions such that:

(i) every member of M accepts every member of Σ as well as p and
(ii) every member i of M accepts $\Sigma \hookrightarrow_i p$.

Cohesive Majority.

Let $\mathcal{G}[q]$ denote the set of group members that accepts q.

Definition (Cohesive Majority)

$p \in \mathcal{CM}(\vec{j})$ iff there is a set of judges $S \subseteq \mathcal{G}$, such that S strongly cohesively supports p and $|S| > |\mathcal{G}[\sim p]|$

A useful equivalent formulation of the last condition is:

$$\frac{|S|}{|S| + |\mathcal{G}[\sim p]|} > 1/2$$

$\mathcal{CM}$, informal algorithm version

For each proposition q,

1. find the largest cohesive group of q supporters (or one that is tied for largest).
2. Now discount from $\mathcal{G}$ all of the judges that support q for other reasons–thus creating a subset of $\mathcal{G}$ we can call $\mathcal{G}^-$.
3. Finally apply an aggregation rule (in this case majority) drawn from the standard framework to $\mathcal{G}^-$.

Example

Suppose p and q are incompatible.

Example 2

	p	q	$p \vee q$	reasons-relation
1	Y	N	Y	$\{p\} \hookrightarrow_1 p \vee q$
2	N	Y	Y	$\{q\} \hookrightarrow_2 p \vee q$
3	N	N	N	$\{\sim p, \sim q\} \hookrightarrow_3 \sim(p \vee q)$
CM	N	N	—	—

$p \vee q$ cannot be Y because the only cohesive sets that support it are {1} and {2} but neither of these outnumbers {3}.

It also cannot be N because there is a minority of N.

Properties of Cohesive Majority.

- CM is not *complete* and not *independent*.
- It does satisfy a weakening of independence:
  - **Weak Independence** The collective opinion on p depends only on: (i) individual opinions on p and (ii) individual opinions on any other propositions that judges consider relevant to p.
- It does not always preserve consistency.

  This can be fixed.

Cohesive Supermajority

In the standard framework, there is an easy fix to the consistency problem [Pettit (2006), List (2007)].

Definition (Cohesive Supermajority)

$p \in \mathcal{CSM}(\vec{j})$ iff there is a set of judges $S \subseteq \mathcal{G}$, such that S strongly cohesively supports p and

$$\frac{|S|}{|S| + |\mathcal{G}[\sim p]|} > t_I$$

t_I can be picked as a function of some logical properties of the agenda so as to guarantee consistency regardless of the input.

[List (2007): $t_I = (x-1)/x$, where x is the size of the largest minimally inconsistent subset of the agenda]

Collective Reasons.

We define the collective reasons relation $\hookrightarrow_c$ as follows:

Definition (Collective Reasons)

$\Sigma \hookrightarrow_c q \in \mathcal{CSM}(\vec{j})$ iff

(i) $q \in \mathcal{CSM}(\vec{j})$ and $\Sigma \subseteq \mathcal{CSM}(\vec{j})$

(ii) there are no ties for "largest subset of $\mathcal{G}$ that cohesively supports q".

(iii) Judges in this largest subset of $\mathcal{G}$ accept $\Sigma \hookrightarrow q$.

Gaps in Collective Reasons

Under this definition, collective reasons won't always exist.

Suppose p and q are incompatible.

Example 3

	p	q	$p \vee q$	reasons-relation
1	Y	N	Y	$\{p\} \hookrightarrow_1 p \vee q$
2	N	Y	Y	$\{q\} \hookrightarrow_2 p \vee q$
CSM	—	—	Y	—

I don't think this is a problem. Groups, just like individuals, can accept propositions without supporting them with inferential reasons.

Bugs & Fixes

(I) *Strong Cohesiveness is too Strong*. "The intuitive notion of cohesiveness does not require that judges believe q for *exactly* the same reasons. It's enough if they do it for reasons that are not mutually undermining."

A: Give a suitably more liberal definition of Cohesiveness (but doing so complicates the account of collective reasons).

(II) *Strong Cohesiveness is too Weak*. "Suppose you and I believe q because we believe p, but believe p for mutually undermining reasons. Should we really count as cohesive w.r.t. q?"

A: define Cohesiveness with respect to the total 'inferential ancestry' of q (which is exactly what the objector has in mind). In the paper I call this the 'cone above q' and give a precise definition of it.

In the applications I described at the beginning, (I) and (II) are rarely problematic.