Collective Reasons *via* Judgment Aggregation

1 Introduction

Judgment aggregation is the systematic study of how individual opinions can be aggregated into collective ones. [1] The theory is often taken to contribute to a model of collective intentional agency by identifying the limits and features of various models of collective opinion. In the individual case, however, beliefs do not stand in isolation from each other: we represent individuals as having beliefs that function as reasons (for other beliefs or for action). This raises the question whether groups can also be modeled as supporting beliefs with inferential reasons.[2] In this paper, I enrich the standard judgment aggregation framework so as to allow for a new and more comprehensive account of collective reasons.

Collective reasons are central to the very motivation of judgment aggregation theory. The theory itself only becomes complex and interesting when we aggregate judgments among sets of logically connected propositions.[3] But why should we aggregate beliefs on a set of logically connected propositions? A plausible answer is that sometimes we want a group to settle on more than just a Yes/No verdict on a proposition or a particular course of action. We want to identify that group's reasons for these verdicts, and reasons often (though not always) involve logical relations among propositions. One of the landmark papers in the area Pettit (2001) frames the central problem of Judgment Aggregation within the context of a discussion of deliberative democracy and gives a central place to reasons: "the problem in question is [...] tied [...] only to the enterprise of making

[1]See List and Pettit (2002) for the initial presentation of the judgment aggregation framework; see List and Puppe (2008) for a survey of the area.

[2]Talk of collective beliefs and collective reasons need not imply a commitment to a metaphysics of collective intentional states. For the question to be interesting it is enough that groups *sometimes* behave *functionally* in ways that are correctly and interestingly described as having beliefs and reasons.

[3]Arguments to the effect that judgment aggregation requires sets of logically connected propositions relate to analogues in judgment aggregation of May's celebrated characterization of the majority rule May (1952).

group judgments on the basis of reasons" (p. 272). Similarly, in motivating a new aggregation rule, Pigozzi (2008) stresses the importance of collective reasons:

> A verdict in a court is a public act. Not only, if convicted, has a defendant the right to know the reasons for which she has been convicted, but also these reasons will guide future decisions — they are patterns for future verdicts. In other words, the final decision must be supported and justified by reasons. (p.289)

Finally, Chapman (2002) criticizes one of the requirements imposed by List and Pettit (2002) because it fails to do justice to the nature and significance of collective reasons. All of these authors are concerned with reasons for collective decisions, but the same points apply to reasons for collective beliefs.

Strikingly, given how central collective reasons are in this area, the standard Judgment Aggregation framework lacks the resources to model it adequately (I argue for this in §3). At best, the standard framework can capture a relatively special case. The contribution of this paper is an expansion of that framework that is better suited to model collective reasons. The expansion is spelled out in §4. §5 operates within the technical confines of this framework and features a possibility result: I define an aggregation rule with a unique combination of properties. This rule is compatible with my account of collective reasons (though other rules are also compatible with it) and it is unavailable in less expressive frameworks.

I do not suggest that this rule is the unique normatively correct aggregation rule (rather: I accept some version of pluralism about aggregation rules, as well as the idea that considerations *pro* and *con* individual aggregation rules can be highly context-dependent). The key point of the paper is to show that an apparently small change in framework can open up new conceptual possibilities and even affect the informal interpretation of some theorems. §6 sharpens the theoretical tools of the framework further, by showing how certain apparent problems can be solved. Having shown the richness of the framework, in §7 I go on to show that the framework is not revisionary: it can capture standard ideas in the Judgment Aggregation literature about how to model collective reasons.

The possibility result of §5 bears (in ways that I will bring out below) on an important debate in judgment aggregation. A *localist* perspective in judgment aggregation maintains that, for every proposition q, the aggregated judgment on q depends *only* on the individual judgments on q itself (technically, this insight is captured by the Independence assumption: see §2). A *holist* perspective, by contrast, is motivated by the idea that aggregating judgments on any one proposition must be sensitive to the entire network of beliefs that can bear inferentially on that proposition.[4] I do not aim to lay out this debate in any systematic way. My possibility result bears on it by showing that, with the right sort of tools, we can satisfy the holist insight and at the same time retain much of what is attractive about the localist perspective (formally, we can abandon the Independence assumption, but retain the mathematical tractability it allows).

2 Basics.

First off, some clarifications concerning reasons. Philosophers contrast reasons for belief and reasons for action. It is an important philosophical question whether we should unify these, or, more strongly, reduce one to the other. In this paper, I essentially use the same model for both,[5] but I do not want that to signal my subscribing to a deep philosophical thesis in this respect. Judgment aggregation itself encourages this simplification by modeling both beliefs and actions using *sentences* drawn from a propositional language. I follow suit in modeling the *reasons-relation* as holding between a proposition q (the *target* proposition), a set Σ (the *reasons*) for an agent α.

The second contrast is between reasons in a psychological sense and reasons in a metaphysical/causal sense. In the latter sense, 'q is the reason for p' means something like 'q grounds the truth of p' or 'q causally explains p'. Obviously, this need not coincide with the reasons why any one agent believes p. My focus is on the psychological (but normatively constrained) notion.

The next preliminary step is to set up the standard Judgment Aggregation framework. This is a familiar drill, but it's worth repeating:

> **The Basic Framework:** start with a *modeling language* $\mathcal{L}$ (a standard sentential logic language), generated by some atomic sentences $\mathcal{L}_0$.[6] Let $\mathcal{G}$ be a *group* of advisors/judges/voters (for simplicity, identify $\mathcal{G}$ with the set $\{1, ..., n\}$): n is typically assumed, for convenience, to be odd (until noted, I adopt this restriction). The *agenda* $\mathcal{I}$ is a subset of $\mathcal{L}$ that is closed under negation (i.e. if $p \in \mathcal{I}$, then $\sim p \in \mathcal{I}$). A *judgment set* j is a subset of $\mathcal{I}$. An *epistemic state* is a maximally consistent

[4] List (2008) makes a case for this sort of holist structure in the context of what he calls 'group deliberation'.

[5] It is a *very coarse* model, but it is intentionally designed so as to be neutral between more substantive views on the nature and logic of reasons.

[6] Notice that we use *sentences* of a formal language to stand for *propositions* (the contents of agent's beliefs). The potential confusions that this engenders are dodged by the fact that we are focusing on context-insensitive sentences and that typically aggregation rules treat logically equivalent sentences in exactly the same way.

judgment set. A *profile* $\langle j_1, ..., j_n\rangle$ is a vector of epistemic states (I often write this as $\vec{j}$). While referring to individuals, I sometimes say that i accepts (or rejects) p, meaning that $p \in j_i$ (or $p \notin j_i$). An *aggregation rule* $\mathcal{A}$ is a partial function from profiles to sets of sentences.

In a nutshell, one of the conclusions of this paper is that this framework's conception of epistemic state is too coarse to correctly model collective reasons. Getting there, however, requires some legwork.

The concept of an aggregation rule is often illustrated by providing examples (e.g. proposition-wise majority rule, proposition-wise consensus etc.). For my purposes, it is more useful to describe some general properties that aggregation rules may have or lack. The reader familiar with these definitions can no doubt skip to the next section. An aggregation rule $\mathcal{A}$ can be:

Universal: for every profile $\vec{j}$, $\mathcal{A}(\vec{j})$ is defined.

Complete (relative to $\mathcal{I}$): for every $\vec{j}$, and every $p \in \mathcal{I}$, $p \in \mathcal{A}(\vec{j})$ or $\sim p \in \mathcal{A}(\vec{j})$.

Consistent: for every $\vec{j}$, $\mathcal{A}(\vec{j})$ is logically consistent. [7]

Anonymous: for every permutation h of $\mathcal{G}$,

$$\mathcal{A}(j_1, ..., j_n) = \mathcal{A}(j_{h(1)}, ..., j_{h(n)})$$

Additional properties require further preliminary definitions. Let $\vec{j}[p]$ denote the vector of opinion on p alone. Say that $\vec{j}$ and $\vec{k}$ are:

q-**matching** iff $\vec{j}[q] = \vec{k}[q]$

q/r-**matching** iff $\vec{j}[q] = \vec{k}[r]$

With these additional concepts, $\mathcal{A}$ can be:

Independent: for every q and any two profiles $\vec{j}$ and $\vec{k}$ that are q-**matching**

$$q \in \mathcal{A}(\vec{j}) \Leftrightarrow q \in \mathcal{A}(\vec{k})$$

Systematic: for every $\mathcal{I}$ and every $q, r \in \mathcal{I}$ and any two profiles $\vec{j}$ and $\vec{k}$ that are q/r-matching

$$q \in \mathcal{A}(\vec{j}) \Leftrightarrow r \in \mathcal{A}(\vec{k})$$

Notice that we could get an even stronger version of Systematicity if we eliminated the quantification over agendas and allowed q and r to be drawn from

[7] Consistency must be understood as relative to some background knowledge. I specify the salient background when modeling particular examples, but leave it out when discussing general results that do not depend on particular choices of background knowledge.

different agendas (I do not do this, because I want to consider rules that behave somewhat differently as the agenda varies, but behave homogeneously within the same agenda). I also define:

$$\vec{k} \text{ is a } q\text{-superset of } \vec{j} \text{ iff } \{x \mid j_x \text{ accepts } q\} \subsetneq \{x \mid k_x \text{ accepts } q\}$$

Epistemic-state Monotonic: if $e = \mathcal{A}(\vec{j})$ and $\vec{k}$ is an e–superset of $\vec{j}$, then $e = \mathcal{A}(\vec{k})$.
Epistemic-state Paretan: if all judges accept the same epistemic state e, $e = \mathcal{A}(\vec{j})$.

3 Elusive Reasons.

What can one say about Collective Reasons in the standard aggregation framework? Let us start by looking at a non-controversial example involving the following propositions:

A: there will be a tornado in Chicago in 2012.
B: there will be an earthquake in San Francisco in 2012.
C: there will be a disaster in the US in 2012.

Let us suppose that our judges have agreed that A, B and C are related by the constraint $C \equiv (A \vee B)$, so that this is background knowledge for them. Consider two possible ways of distributing opinions on these propositions:

Example 1a

	A	B	C
1	Y	N	Y
2	Y	N	Y
3	N	N	N
$\mathcal{MA}$	Y	N	Y

Example 1b

	A	B	C
1	N	N	N
2	N	N	N
3	N	N	N
$\mathcal{MA}$	N	N	N

Suppose finally that we aggregate judgments by taking majority on each proposition (I refer to this rule as $\mathcal{MA}$), so that the last line in the tables represents the aggregated opinion of this group. The choice of $\mathcal{MA}$ is only for determinateness and does not affect the gist of my point. We need a general, non *ad-hoc* definition of what counts as a collective reason. In example 1a, we may want the definition to imply that A counts as a collective reason for C. Similarly, in example 1b, we may want the story about reasons to imply that $\sim A$ and $\sim B$ are (jointly) the reasons for the collective acceptance of $\sim C$.[8]

As far as I can see, the only principle that can deliver these verdicts within the current framework is:

> **Entailment-Reasons Link:** a set of propositions Σ collectively functions as a reason for p if and only if
>
> (i) Σ logically entails p
>
> (ii) the collective opinion of $\mathcal{G}$ (as determined by applying your favorite aggregation rule to $\vec{j}$) includes all of Σ and p.

No one to my knowledge explicitly advocates this principle. But refuting it does help the conceptual ground-clearing. The Entailment-Reasons Link is false in both directions. Right-to-left: one's reasons for p need not be propositions that logically entail p—e.g. inductive/statistical support may be enough. This worry is assuaged somewhat by the fact that I allow background knowledge to factor into entailment relations.[9] Be that as it may, the failure of the left-to-right direction is more worrisome and more directly significant to my argument. Consider a case involving only a single subject:

> **Little Kids:** Jodie is trying to figure out the features of a person she sees in the distance. She thinks that the person moves fast (F) and is small in size (S). On the basis of this, she infers that the person must be a child (C). Partly on the basis of this, and partly on the basis of other information, Jodie also comes to believe it's a boy (B). Finally, she accepts the following background claims: $(F \& S) \rightarrow C$, $C \equiv (B \vee G)$.

The Entailment-Reasons Link implies (correctly) that $F\&S$ are Jodie's reasons for C, but it also implies (incorrectly) that B is a reason for C (since B entails (modulo background information) C). Generalizing, entailments can hold not only *downstream* with respect to the reasons relation, but also *upstream* (i.e. from a proposition to a subset of the reasons that support it).[10] The Entailment-Reasons Link

[8] These stipulations are not meant to play a role in the argument, but just to fix ideas. I will in fact argue that any such stipulation is , in some sense, misguided.

[9] Even then, depending on what 'background knowledge' means, one might still quibble with the requirement that, for p to be supported by reasons Σ, Σ must entail it relative to background knowledge.

[10] I find the same problems with another possible approach, based on some interesting results in Dietrich (forthcoming). Dietrich considers a variation on the basic framework: instead of creating a language $\mathcal{L}$, by taking the boolean closure of $\mathcal{L}_0$, first expand $\mathcal{L}_0$ to a negation-closed

implies that entailments can only run downstream with respect to the reasons relation (i.e. from the reasons to their conclusions).

The standard solution to these problems is to enhance the expressive capacity of the framework. Instead of thinking that all propositions in the agenda have equal roles, one could think that some propositions have the *reasons*-role (they function as reasons for others) and some others have the *target*-roles (they are the sorts of things that reasons are given for or against). This idea is developed by designating a set of pairwise logically independent sentences as *premises*. Designate a further sentence as the *conclusion*. Additionally, require that any distribution of truth-values on the premises settle by entailment (relative to background knowledge) the truth-value of the conclusion. A popular way to aggregate judgments in this enriched framework is *Premise-Based Majority*: take majority on the premises and propagate by entailment to the conclusion.

In this framework, one can give this account of collective reasons:

> **Premise-Reasons Link:** the collective reasons for accepting a conclusion q are the collectively accepted premises.

On this view, collective reasons depend on an underlying assignment of roles to propositions. In Example 1, one could designate A and B as premises and C as conclusion. The upshot is that, in 1a, $\{A, \sim B\}$ would be the reasons for accepting $A \vee B$; in 1b, $\{\sim A, \sim B\}$ are the reasons for rejecting it. The fact that, on our current account, $\sim B$ is part of the reason for accepting $A \vee B$ is a bit of an embarassment, but perhaps one can complicate the criterion so as to circumvent this problem.

I won't attempt to solve this problem on behalf of the Premise-Reasons Link, since I am about to point to some further, independent, problems. First, the account's domain of applicability is rather limited: we need pairwise logically independent premises and a conclusion that can be settled by *any* distribution of truth-value on the premises. The fact that the Premise-Reasons Link is limited in this way factors negatively into a complete assessment of the proposal.[11]

set $\mathcal{L}_0^+$, then expand once again to a language $\mathcal{L}_1$ whose primitive symbols are the same as those of $\mathcal{L}_0^+$ plus a conditional $\Box\!\!\rightarrow$; the formation rules of $\mathcal{L}_1$ allow you to create conditionals linking any sentences of $\mathcal{L}_0^+$ (but not embedded conditionals) and negations of such conditionals. Provided that the conditional is interpreted subjunctively, Dietrich proves that some *quota* rules are possible (quota rules are generalizations of the majority rule in which each proposition is assigned to a threshold $t \geqslant 1/2$ that's necessary and sufficient for acceptance). In this framework one could interpret the conditional $\Box\!\!\rightarrow$ as representing the *reasons relation*. I think that would involve a mistake akin to the one I just ascribed to the Entailment-Reasons Link: just like entailments, the subjunctive conditionals one entertains do not need to track the reasons and can easily run 'upstream' from the reasons relation.

[11]This point is effectively pressed, among others, by Nehring and Puppe (2010), who also use it to motivate some new aggregation rules that are not premise based but that, like the rules I

Second, and more significant, the Premise-Reasons Link assumes that we can, in general, fix reason-roles and target-roles for propositions with respect to an agent independently of what else that agent believes. Generally speaking, this is false—both for individuals and for groups. Consider:

> **Hard Decisions:** A university administration must decide whether to start a basketball program (B) and whether to start a volleyball program (V). The teams cannot train in the same gym. So they also need to decide whether to buy a new space (N). They all agree that they will buy the new space if they decide to start both programs ($B \,\&\, V \rightarrow N$)–but they are contemplating buying the new space regardless of the outcome of the vote on the program.

In cases like this, it would make sense for one judge to take B and V as reasons for N, while for another judge to take B and $\sim N$ as a reason for $\sim V$, and for yet another to take $\sim N$ and V as a reason for $\sim B$. No independently fixed pattern of premisehood can capture this.

Once this point is acknowledged, it is easy to see that things are more complicated than they appeared: not even the *yes/no* judgments on the propositions in the agenda can fix the *reasons-relation*. So, in Example 1b it seems permissible to treat $\{\sim A, \sim B\}$ as reasons for $\sim C$ just as much as it seems permissible to treat $\sim C$ as a reason for each of $\sim A$ and $\sim B$ (imagine being told by an infallible oracle that there wouldn't be major disasters in the US in 2012 and deducing $\sim A$ and $\sim B$ from that). The upshot of this discussion is this: not only is it a mistake to read the *reasons-relation* off of the pattern of entailments among the propositions one accepts, but it is more generally a mistake to fix that relation *externally*[12]—as if it was disjointed from agents' epistemic states.

For the same reason, I am also inclined to avoid addressing the problem within another generalization of the framework, also due to Dietrich. Dietrich (2008) studies a framework in which, in addition to all the standard tools, we single out a relation of dependence ◀ holding among propositions in the agenda. Dietrich does not impose a particular interpretation on ◀, suggesting that it could vary from application to application. He does remark that, given some structural constraints, it could be interpreted as a generalization of the *premise/conclusion* dichotomy. If that is meant to suggest that it can help model the *reason-relation*

present below, require that a collectively accepted proposition be justified, when possible. Another important study, from the same symposium is Dietrich and Mongin (2010) which takes up Premise-Based rules more generally.

[12] I am using 'external' in its intuitive sense: I am not trying to make contact with how the terms are used in philosophical debates on *externalism/internalism*.

in the sense I have been discussing, I offer exactly the same criticism: $\blacktriangleleft$ is fixed externally and the *reasons-relation* is not.

4 Internal Collective Reasons.

Collective reasons should derive from the epistemic states of the individual agents. To make room for this, I adopt a richer representation of individual epistemic states. Instead of representing agents' epistemic states as maximally consistent judgment sets, I represent them as a pair of one such set *together* with a *reasons*-relation $\hookrightarrow$. In the epistemic case, for each agent i, $\hookrightarrow_i$ records how i bases beliefs onto other beliefs—it is in other words what epistemologists call a *basing* relation. The nature of the basing relation is, of course, a subject of deep controversy; our account of it, however, need only track its formal features.[13] A complete story about collective reasons should embed an account of reasons that captures more of their structure and dynamics—e.g. the *default theories* discussed in Horty (2007a,b). However, for my current purposes, such a structured level of analysis is not necessary.

So, when i inferentially bases a belief in q on Σ, I write $\Sigma \hookrightarrow_i q$ (when completely unambiguous, I drop the subscript to ease reading). Of course, i can accept q non-inferentially: in that case, I write $\emptyset \hookrightarrow_i q$ (to avoid notational clutter, I generally omit representing this explicitly: if no basing facts are recorded for a proposition q that i accepts, $\emptyset \hookrightarrow_i q$ is implicitly understood). Furthermore, i can accept a proposition for multiple independent reasons. For example, one may believe a disjunction $p \vee q$ on the basis of independent beliefs in the disjuncts: in these cases, I write $\{p\} \hookrightarrow p \vee q$ and $\{q\} \hookrightarrow p \vee q$.[14]

At the individual level, the reasons-relation is then represented by $\hookrightarrow$. I require very little of $\hookrightarrow$:

Acceptance: If $\Sigma \hookrightarrow_i p$, then i accepts p and every member of Σ.

Notice that **Acceptance** together with the requirement of logical consistency on individual opinion implies that $\Sigma \hookrightarrow_i p$ only if $\Sigma \cup \{p\}$ is logically consistent. Notice also that logical entailment is neither sufficient nor necessary for $\hookrightarrow_i$ to hold. Neither is logical complexity a guide to one's pattern of reasons: atomic sentences have no more of a right to be called 'reasons' than complex ones. Call the enriched framework I just described $\mathcal{R}$.

[13] For a useful survey piece on the basing relation see Korcz (2002).

[14] Notice that, on the current interpretation, $\{p, q\} \hookrightarrow p \vee q$ would mean something different—namely, that each of p and q is only a *part* of the agent's reasons for $p \vee q$.

$\mathcal{R}$ represents two dimensions of an agent's epistemic state—their judgments and the basing relations that connect those judgments. In this connection, I make two assumptions:

> (i) basing relations are not themselves propositions—they are rather relational features of an agent's epistemic state. A specific consequence is that we should not think of them as being aggregated in the same way in which propositions are aggregated.
> (ii) I do not aggregate the two dimensions as if they were separate compartments. That is to say, I do not aggregate the judgments, on the one hand, and then *separately* aggregate the individual basing relations.[15]

The interesting aggregation rules in $\mathcal{R}$ are holistic in the following sense: on the basis of the individuals' epistemic states as a whole, they produce both a collective opinion and a collective basing relation. I focus on these.

The expressive power of $\mathcal{R}$ allows us to draw distinctions that would otherwise prove elusive. One of the most significant complaints against the Independence assumption is that the individual pattern of acceptance on q is too thin a basis to settle whether or not q should be collectively accepted. In particular, it makes a difference if the judges that support q are, as I say, *cohesive* (that is, support q for the same reason, or compatible reasons, or mutually reinforcing reasons) or if they support q *non-cohesively* (for incompatible or mutually undermining reasons).

At the very least, non-cohesiveness should *sometimes* undermine apparent agreement. If that's the case, the Independence assumption cannot be maintained across the board. I strongly sympathize with this complaint, although spelling it out into a full argument would take us further afield than I want to. The upshot of rejecting Independence is sometimes taken to be that something like a premise-based approach would be preferable. This is in a nutshell the suggestion in List and Pettit (2002).[16] As I argued, however, premise-based approaches cannot be the whole story about how reasons interact with collective opinions. A key goal of this paper is precisely to block the inference from a rejection of Independence to the adoption of premise-based approaches.

In $\mathcal{R}$ we can try a different strategy. In particular, we can explicitly define what it is for a group of judges to support a proposition cohesively.

[15] One problem with that course is that aggregating graphs is not particularly easier than aggregating propositions, as shown, in a different context, by Bradley *et al.* (2006).

[16] After presenting their now well-known impossibility result, they point-out prioritizing proposition is a possible escape route from their now famous impossibility theorem.

Definition 1 (Strong Cohesiveness). *M strongly cohesively supports p iff there is a set Σ of propositions such that:*

(i) every member of M accepts every member of Σ as well as p and

(ii) every member i of M endorses $\Sigma \hookrightarrow_i p$.

I call this notion *strong* cohesiveness, because, as I understand it, the intuitive notion of cohesiveness that this definition explicates does not require that every judge accept p for precisely the *same* reasons (arguably, something weaker is enough—namely that the reasons not be mutually undermining). In §6, I explain how to accomodate this point by producing a more liberal definition of cohesiveness that does not lose any of its important features. For now, it will be easier to proceed with this simplified definition.

I now define a simple aggregation rule that is responsive to the difference between cohesive and non-cohesive inputs. Let me introduce a new piece of notation: given a set of judges P and a proposition q, let $P[q]$ be the subset of P consisting of all and only the members that accept q (for whatever reason).

Definition 2 (Cohesive Majority). *$p \in \mathcal{CM}(\vec{j})$ iff there is a set of judges $S \subseteq \mathcal{G}$, such that S strongly cohesively supports p and*

$$\frac{|S|}{|S| + |\mathcal{G}[\sim p]|} > 1/2$$

Even though this rule will eventually turn out to be problematic, it helps illustrate some important points. As the name suggests, if a cohesive majority supports p, then a majority supports p, but the converse is not true. In fact, $\mathcal{CM}$ violates the completeness assumption by design.

Example 2

	p	q	$p \vee q$	reasons-relation
1	Y	N	Y	$\{p\} \hookrightarrow_1 p \vee q$
2	Y	Y	Y	$\{q\} \hookrightarrow_2 p \vee q$
3	N	N	N	
$\mathcal{CM}$	Y	N	–	
$\mathcal{MA}$	Y	N	Y	

It also violates Independence by design. We can change the collective outcome on $p \vee q$ in Example 2 merely by intervening on the reasons-relation and adding $\{p\} \hookrightarrow_2 p \vee q$.

Crucially, however, $\mathcal{CM}$ satisfies a weaker version of Independence.

Definition 3. *$\vec{j}$ and $\vec{k}$ are* deeply q-matching *iff for every judge i,*
(i) $j_i[q] = k_i[q]$ and

(ii) for every set of propositions Σ, $\Sigma \hookrightarrow_i q$ holds in $\vec{j}$ iff it holds in $\vec{k}$.

Definition 4 (Weak Independence). *$\mathcal{A}$ is weakly independent iff for every proposition q and any two profiles $\vec{j}$ and $\vec{k}$ that are deeply q-matching, $q \in \mathcal{A}(\vec{j}) \Leftrightarrow q \in \mathcal{A}(\vec{k})$.*

Proposition 1. *$\mathcal{CM}$ is weakly independent.*

> *Proof:* Suppose that q is accepted by $\mathcal{CM}$ on profile $\vec{j}$. Since q was accepted in $\vec{j}$, there must be a strongly cohesive set S that supports it and has larger size than the set of advisors who oppose it. Both of these sets must be preserved in any deeply q-matching profile. □

This is significant: standard independence is sometimes glossed as the claim that the aggregated outcome on q depends only on the individual opinions on q *and not on opinions on any other proposition p*. There is something persuasive about the idea that a judgment on q shouldn't be sensitive to irrelevant information. However, nothing in the standard definition of Independence characterizes 'relevant information'. If interpreted as ruling out 'irrelevant information', the independence principle would imply the absurd claim that no other propositions can be relevant to p (a point also highlighted by Dietrich 2008). Our slogan should then be that the aggregated outcome on q should depend only on the opinions on q and on whatever other propositions individual agents take to be salient to q.

The distinctive feature of $\mathcal{R}$ is that it involves a *subjective* interpretation of 'relevance': what is relevant to a proposition q is simply what individual judges base beliefs for or against q on. The distinctive *advantage* of $\mathcal{R}$ is more abstract: almost all of the informative theorems in the Judgment Aggregation literature require the Independence assumption.[17] As Douven and Romeijn (2007) point out, this is no doubt due to the fact that the prospects of saying something substantive and general about aggregation rules in the absence of Independence conditions may appear bleak. $\mathcal{R}$ provides a terrain on which a range of non-independent aggregation rules can be mapped out and studied precisely.

I have elaborated on the mechanics of *acceptance* by $\mathcal{CM}$. But aggregation rules in my framework must also produce, where available, a pattern of collective reasons. $\mathcal{CM}$ allows a natural account of collective reasons. Notice that $\mathcal{CM}$

[17]With some exceptions: Douven and Romeijn (2007), Cariani, Pauly and Snyder (2008).

requires for acceptance of q that there be a comparatively large *cohesive* group of q-supporters. Letting $\hookrightarrow_c$ be the collective reasons relation, define:

Definition 5 (Collective Reasons). $\Sigma \hookrightarrow_c q \in \mathcal{CM}(\vec{j})$ *iff*

(i) $q \in \mathcal{CM}(\vec{j})$

(ii) $\Sigma \subseteq \mathcal{CM}(\vec{j})$

(iii) there are no ties for "largest subset of $\mathcal{G}$ that cohesively supports q".

(iv) for every j in the largest subset of $\mathcal{G}$ that cohesively supports q endorses $\Sigma \hookrightarrow_j q$.

The first two conditions require that the appropriate propositions belong to the aggregated opinion. In this way, group reasons satisfy the Acceptance requirement. Why require (iii)? Suppose that p and q are incompatible propositions. There may be cases where we may want $p \vee q$ to be accepted, but collective reasons cannot be produced. Consider:

Example 3

	p	q	$p \vee q$	reasons-relation
1	Y	N	Y	$\{p\} \hookrightarrow_1 p \vee q$
2	N	Y	Y	$\{q\} \hookrightarrow_2 p \vee q$

We have here a situation in which two advisors unanimously accept $p \vee q$, but do so for incompatible reasons. In cases like Example 3, I see two possible stances:

(a) If one agrees with $\mathcal{CM}$'s verdict that $p \vee q$ should be accepted given the input, the natural position is to think that neither p nor q should be part of the collective reason. It may be necessary, sometimes, to model a group as if they accept a proposition but refrain from modeling them as having a reason in support of it. (iii) is an attempt to capture this—more sophisticated ones are possible.[18]

(b) On the other hand, if one disagrees with $\mathcal{CM}$'s verdict that $p \vee q$ should be accepted, something like (iii) needs to be imposed as a condition of collective acceptance.[19] That is, the aggregation rule should not merely require a certain amount of cohesive support, but it should also require that there be

[18] In particular, (iii) implicitly assumes that all judges have the same weight.

[19] There need not be anything implausible in this standpoint. From the perspective of a defender of a Premise Based approach p and q could count as premises for $p \vee q$, so the fact that neither ends up being collectively accepted implies that the conclusion $p \vee q$ should also not be collectively accepted.

collective reasons. One consequence here is that one should be willing to forego the Unanimity requirement. Be that as it may, once one makes this move, it does no harm (except for triviality) to include condition (iii) as a requirement for collective reasons.

Nonetheless, $\mathcal{CM}$ is problematic. If every judge i accepts every proposition non-inferentially, $\mathcal{CM}$ collapses onto the Majority rule. Since the Majority rule is not guaranteed to be consistent, neither is $\mathcal{CM}$. Consider for instance a version of the discursive dilemma in which judges have empty reasons relations:

Example 4

	p	q	$p \vee q$	reasons-relation
1	Y	N	Y	
2	N	Y	Y	
3	N	N	N	
$\mathcal{MA}$, $\mathcal{CM}$	N	N	Y	

Since everything is grounded non-inferentially, $\mathcal{MA}$ and $\mathcal{CM}$ coincide. Of course, it is hard to envision a case in which p, q and $p \vee q$ are all releveant and yet individuals completely ignore the logical relationships between them. If there is a way of ruling these cases out, then perhaps $\mathcal{CM}$ can be rehabilited.

I take a different line here. The next section aims to study whether we can obtain consistent rules that allow the natural account of collective reasons I just sketched while at the same time having an interesting profile of properties.

5 A Special Rule.

Let me start this section by delimiting the space of aggregation rules to focus on. I am interested in rules that share some significant structural features with $\mathcal{CM}$. Thinking of $\mathcal{CM}$ as an algorithm helps extrapolate these features:

> ($\mathcal{CM}$, informal algorithm version) For each proposition q, first find the largest cohesive group of q supporters (or the one that is tied for largest). Now discount from $\mathcal{G}$ all of the judges that support q for other reasons–thus creating a subset of $\mathcal{G}$ we can call $\mathcal{G}^-$. Finally apply an aggregation rule (call this the *local rule*) drawn from the standard framework to the opinions of this (typically) reduced group of judges.

I used the Majority rule as a local rule, but that choice isn't forced. I am going to propose an alternative local rule, to be pinned down through the standard axiomatic method of social choice theory. In my framework, this methodology can be enriched in one respect: we can impose conditions both on the local rule (the rule that is applied after we check for cohesive groups) and the global one (the overall algorithm). This does not affect the domain what is possible, but it certainly affects the plausibility of the axioms: a condition can fail to be globally desirable while at the same time being locally acceptable. This is especially useful with respect to the Independence principle. If the only intuitive violations of Independence are due to failures of cohesiveness, avoiding Independence is only mandated at the global level—not at the local one.

I assume that the local rule is: anonymous, consistent, systematic, epistemic-state monotonic (henceforth simply *monotonic*) and epistemic state paretan (henceforth simply *paretan*). For example, a Unanimity rule (although rather demanding) satisfies all of these conditions. Let us call the rules that satisfy these properties C-rules. It is well known that there are no complete $C - rules$.[20] Moreover, every C-rule is a threshold rule of sort: collective acceptance of a proposition requires acceptance by a certain percentage of individuals (precise statement and proof will follow shortly). It will turn out that there is a $C - rule$ whose threshold is lowest among all the C-rules. This rule is maximal, among C-rules in the following sense:

Definition 6. *$\mathcal{A}$ is maximal in C iff $\mathcal{A}$ is a C-rule and for every $\mathcal{I}$, $p \in \mathcal{I}$ and $\vec{j}$,*

$$p \in \mathcal{A}(\vec{j}) \iff \text{there is a rule } \mathcal{B} \text{ in } C,\ p \in \mathcal{B}(\vec{j}).$$

It turns out that a maximal C-rule exists, the next few remarks characterize it.

Definition 7 (Threshold Rules)**.** *$\mathcal{A}$ is a threshold rule iff there is a threshold $t \geqslant 1/2$ such that for every q and $\vec{j}$,*

$$q \in \mathcal{A}(\vec{j}) \Leftrightarrow \frac{|\mathcal{G}[q]|}{|\mathcal{G}|} \geqslant t$$

Proposition 2. *Every C-rule is a threshold rule.*

> *Proof.* Suppose that $\mathcal{A}$ is a C-rule. By Independence $\mathcal{A}$'s verdict on q depends only on the sequence of Yes/No judgments on q. By anonymity any two such sequences that are permutations of each other produce the same outcome. In other words, the aggregated outcome on q is a function h of the ratio of votes for q (regardless of who threw

[20] Some of the considerations to follow also serve as a proof of this fact. See Proposition 2 below.

those votes) over total votes. By neutrality, this function is identical for all propositions within the same agenda. Since $\mathcal{A}$ is paretan and independent, if n/n of the judges support q in $\vec{j}$, then $q \in \mathcal{A}(\vec{j})$. If $0/n$ support q, then since judges' opinions are complete, $\sim q \in \mathcal{A}(\vec{j})$. Consider a $k < n$: by monotonicity, if k/n is sufficient for acceptance of q in $\vec{j}$, then so is $k + 1/n$. So, there must be a k with the following properties: (i) k/n is sufficient for $q \in \mathcal{A}(\vec{j})$ (ii) for all $y > k$, y/n is sufficient for $q \in \mathcal{A}(\vec{j})$ (iii) for no $y < k$, y/n is sufficient for acceptance. In other words k/n is a threshold. □

Pettit (2006) describes informally a threshold rule that is guaranteed to produce a consistent output. This is a supermajority rule in which the acceptance threshold varies with some global logical properties of the agenda. List (2007) analyzes this insight formally and observes that the threshold $t_{\mathcal{I}}$ must be at least $x - 1/x$ where x is the size of the largest minimally inconsistent subset of the agenda. Let us call this rule $\mathcal{P}$. For any thresholds below this value we can construct profiles on which the appropriate supermajority rules would be inconsistent.

Proposition 3. *The threshold in a C-rule, relative to $\mathcal{I}$ must be at least as high as t_I.*

Proof: by the above considerations about the tightness of the threshold and the fact that C-rules must be consistent. □

Incidentally, this means that no $C - rule$ can be complete, since, in general, $t_I > 1/2$, and majority is the only complete and consistent threshold rule. Our final and key observation is now immediate:

Proposition 4. *$\mathcal{P}$ is the only maximal $C - rule$.*

Proof: (⇒) Trivial, because $\mathcal{P}$ is itself a C-rule.
(⇐) Suppose $\mathcal{B}$ is a C-rule: by Lemma 2, $\mathcal{B}$ must have a threshold y at least as high as t_i, so if $p \in \mathcal{B}(\vec{j})$, p must clear this threshold. So p must clear $t_{\mathcal{I}}$ itself (because $y > t_{\mathcal{I}}$). □

It is very easy to plug $\mathcal{P}$ as a local rule into the same structure that gave us $C\mathcal{M}$:

Definition 8 (Cohesive Supermajority). *$p \in C\mathcal{P}(\vec{j})$ iff there is a set of judges $S \subseteq \mathcal{G}$, such that S strongly cohesively supports p and*

$$\frac{|S|}{|S| + |\mathcal{G}[\sim p]|} > t_{\mathcal{I}}$$

$\mathcal{CP}$ is the rule we aimed for. It is cohesive, weakly independent, consistent and inherits from $\mathcal{CM}$ the attractive account of collective reasons I described in the previous section.

$\mathcal{CP}$ is, of course, not complete, but, even granting the pluralism I mentioned, completeness does not seem on a par with the other assumptions. For one thing, we must drop completeness once we make harmless generalizations such as:

(a) dropping the restriction that the numbers of judges must be odd and

(b) dropping the irrealistic assumption that individual judges must have complete beliefs (relative to the agenda).

For another, requiring cohesiveness, as I do here, introduces points of incompleteness anyway.[21] Dietrich and List (2008) argue that relaxing completeness cannot be the entire solution to all the complexities of judgment aggregation: they show that a core of impossibility results is still available if we relax completeness to some requirement of deductive closure. Recall, however, that I only accept weak Independence and all of the results in Dietrich and List (2008) turn on Independence: they do not hold if we relax it to the weaker assumption I have laid out in the previous section.

Having mentioned deductive closure, it is worth remarking on it. Unless we make further structural assumptions relating $\hookrightarrow$ and entailment, deductive closure fails for both $\mathcal{CM}$ and $\mathcal{CP}$, if it is defined in the obvious way:

Definition 9 (Deductive Closure). *If* $\mathcal{A}(\vec{j}) \vDash p$, *then* $p \in \mathcal{A}(\vec{j})$.

It's possible for p and q to be cohesively supported by k judges with $k > t_I$, but $p\&q$ may only be supported by fewer than than t_I judges (of course, the same holds of any other non-trivial treshold). However, $\mathcal{CP}$ satisfies a more normatively significant closure principle:

Definition 10 (Reason Closure). *If* $\Sigma \subseteq \mathcal{CP}(\vec{j})$ *and* $\Sigma \hookrightarrow_c q$, *then* $q \in \mathcal{CP}(\vec{j})$.

Informally, collective opinion is closed under the collective reasons relation.

To repeat a point made in the introduction, I do not claim that $\mathcal{CP}$ is in an absolute sense better than other rules. For example, whether or not we should require that an aggregation rule be anonymous essentially depends on the modeling application. What I claim, rather, is that $\mathcal{CP}$ sits at the intersection of several important properties; that these properties can be simultaneously desirable; that it provides us with a natural story about collective reasons; and, finally, that the possibility of rules like $\mathcal{CP}$ should induce us to be careful in interpreting some impossibility results proven in less expressive frameworks.

[21] See Gärdenfors (2006) for further resistance against Completeness in judgment aggregation.

5.1 Comparison with distance-based rules.

It is useful to compare $\mathcal{CP}$ with some similarly inspired rules, in particular the *distance-based* account of Pigozzi (2008). One invisible drawback is that distance-based rules presuppose something akin to a *premise-conclusion* distinction. First, represent each complete judgment set by the valuation v over the atomic sentences $\mathcal{L}_0$ of $\mathcal{L}$. Second, define a distance measure between pairs of valuations v and w: for example

$$d(v, w) = \sum_{A \in \mathcal{L}_0} |v(A) - w(A)|$$

where $v(A)$ denotes the truth-value of A in v. Third, assign to each valuation v a score relative to a sequence $\langle w_1, ...w_z \rangle$ of valuations:

$$\sum_{1 \geqslant i \geqslant z} d(v, w_i)$$

Fourth, say that $\mathcal{DB}$ accepts a valuation v at a profile $\vec{j}$ of judgments just in case (i) v is consistent with background knowledge and (ii) v minimizes score relative to the sequence of valuations inducted by $\vec{j}$.

Definition 11 (Distance Based Rule). *$p \in \mathcal{DB}(\vec{j})$ iff for all valuations v s.t. $\mathcal{DB}$ accepts v given $\vec{j}$, $v(p) = 1$*

Like $\mathcal{CP}$, $\mathcal{DB}$ has a holistic inspiration: whether or not a proposition is accepted by it, depends on global features of the collective opinion.

However, here are some reasons to prefer $\mathcal{CP}$ in some applications:

- Unlike $\mathcal{CP}$, $\mathcal{DB}$ requires an antecedently drawn distinction between premises and conclusions—or equivalently, requires assigning a privileged role to atomic sentences.

- For $\mathcal{DB}$ the collective outcome on p can depend on *any* proposition in the agenda, whereas for $\mathcal{CP}$ it only depends on the propositions that judges take to be relevant to p.

$\mathcal{DB}$ also has advantages over $\mathcal{CP}$. First, it requires a less complex input. Actual voting procedures only solicit opinions, rather than opinions with patterns of reasons along them.[22] Second, the requirement of strong cohesiveness makes $\mathcal{CP}$

[22] I take this point as a serious obstacle, though I do not think it is a damning objection against my framework: presumably we want a Judgment Aggregation framework that can do more than just give abstract representations of voting procedures.

very restrictive: some propositions may be rejected even when supported by a very large (though less than 100%) percentage of the judges because of a slight discrepancy in the reasons they produce. As I mentioned, this is a serious worry against $\mathcal{CP}$: it is time to see it addressed.

6 Amending Cohesiveness.

Strong cohesiveness is too strong. Two (or more) judges do not need to cite the exact same reasons for them to count as cohesive in their support of a proposition. We want to say that two (or more) judges can count as cohesive even if they adopt different reasons—provided that those reasons are not mutually undermining.

At the same time, Strong Cohesiveness is also too weak because it is only sensitive to the reasons for a particular proposition, but not to the reasons that support those reasons. Sam and Abe might support p for the same reason q but be wildly non-cohesive in their support of q.

In this section, I implement two changes to the definition of Cohesiveness to correct these problems.

6.1 Relaxing Strong Cohesiveness

I approach the first issue (*too strong*) by treating the notion of 'being mutual undermining' schematically. Consider an analogy: the Judgment Aggregation setup makes extensive use of the concepts of consistency and entailment. These concepts can be analyzed in various ways, but many of the central facts of Judgment Aggregation do not depend on one particular analysis of consistency.[23] The same modular architecture can be used for 'being mutually undermining': the details of this concept are difficult and no doubt there are multiple possible explications. However, insofar as we can sketch the broad structural features of these concepts and draw results out of them, we can leave the complexities for separate treatments.

Unlike the concepts of consistency and entailment, however, I am inclined to think that 'being mutually undermining' is in part dependent on features that are specific to the modeling application. Therefore, in every specific modeling application, I introduce an equivalence relation $\sim_q$ (for each q), holding between sets of reasons Σ and Σ' just in case they do not undermine each other q.

Definition 12 (General Cohesiveness). *$M = \{m_1, ..., m_k\}$ cohesively supports p iff there are sets $\Sigma_1, ..., \Sigma_k$ of propositions (drawn from $\mathcal{I}$) such that:*

[23]For a strong result to this effect, see Dietrich (2007).

(i) for all i, m_i accepts every member of Σ_i as well as p and endorses $\Sigma_i \hookrightarrow_i p$

(ii) for each i, $j \in \{1, ..., k\}$, $\Sigma_i \sim_p \Sigma_j$.

In the special case in which $\sim_q$ is the identity relation for all q, General Cohesiveness is obviously equivalent to Strong Cohesiveness. In defining $\mathcal{CP}$, we can replace 'strong cohesiveness' with 'general cohesiveness' and obtain a more liberal rule.

One cost of general cohesiveness is that it makes it more difficult to identify a collective reason. When I required Strong Cohesiveness for p, I could, in most cases, point to a unique collective reason—the reason of the largest cohesive group of p supporters (if there was one). Once we relax to General Cohesiveness even if there is a largest cohesive group of p supporters, we are not guaranteed to have a single reason on the basis of which they all support p.

I cannot offer have a definite formal solution to this problem. Lest this discourages you, I do note that the same problem would apply to the other accounts of collective reasons that I have argued against. Nonetheless, I think something more substantive (although not formal) about the problematic cases can be said. When a group of advisors S, Generally (but not Strongly) Cohesively support p, we can take the reason for p to be some proposition q such that:

- for all $i \in S$, $\{q\} \sim_p \Sigma_i$.
- for all $i \in S$, if i entertained q, and $\{q\} \hookrightarrow p$. they would accept both

Note that the condition does not assume that the proposition q be entertained or considered as relevant to p: it merely says that there must be such a proposition. Imagine a group of scientists each having their own datasets and data analyses pointing to the conclusion that p. Chances are that each scientist will produce a slightly different reason for p, so that they count as Generally but not Strongly Cohesive. The above proposal suggests that they will count as having a collective reason if there is a 'neutral' description of the experimental evidence that they are potentially disposed to accept and that they are disposed to take as supporting p. For obvious reasons, a condition like this is hard to write into the formal model, but I am skeptical that this problem can be solved purely within the formalism.

6.2 Deep Cohesiveness.

The other worry required us to pin down a notion of Cohesiveness that is sensitive to more than just the immediate reasons for a given proposition. This is easily accomplished by defining cohesiveness with reference to the entire set of propositions that underlie an agent's belief in q.

Suppose that a believes that q. Say that p is a partial reason in favor of z (according to a) iff there is a set of propositions Σ, such that $p \in \Sigma$ and $\Sigma \hookrightarrow_a z$. Let PR^*_a be the transitive closure of a's partial reason relation. Say that the *cone above* q is the set: $\{p \mid PR^*_a(p,q)\}$. Informally, a's cone above q is the set of propositions that appear at some point in the chain of reasons that leads a to accept q.

Definition 13 (Deep Cohesiveness). $M = \{m_1, ..., m_k\}$ *cohesively supports p iff there are sets* $\Sigma_1, ..., \Sigma_k$ *of propositions (drawn from* $\mathcal{I}$*) such that:*

(i) for all i, m_i *accepts every member of* Σ_i *as well as p.*

(ii) Σ_i *is the cone above p (for i).*

(ii) for each $i, j \in \{1, ..., k\}$*,* $\Sigma_i \sim_p \Sigma_j$*.*

Notice that as defined Deep Cohesiveness already implements the ideas of §6.1. Moreover, all of the changes suggested in this section are easily implemented in the framework of §4, and do not affect the discussion of section §5.

7 Premises.

The aim of this section is to show that Premise-Based majority can be naturally defined in $\mathcal{R}$ without taking the premise-conclusion dichotomy as an external input to the framework (indeed, it can be defined as a small variation on $\mathcal{CM}$). Standardly, we think of the premise-conclusion roles as fixed as independent parameters. In $\mathcal{R}$, however, we can construct those roles, in an elementary way, from the individual basing relations. Suppose that p_1, ..., p_k are pairwise independent propositions in the agenda. For each individual i, let $\|p_1, ..., p_k\|^i$ denote the maximally consistent subset of $\{p_1, ..., p_k, negations\}$ that i accepts (such a subset must of course exist because i accepts some maximally consistent subset of the agenda). Similarly let $\|c\|^i$ be whichever of c and $\sim c$ i accepts.

Definition 14. *A sequence of individual basing relations* $\langle \hookrightarrow_1, ..., \hookrightarrow_n \rangle$ *over an agenda* $\mathcal{I}$ *admits* $\{p_1, ..., p_k, negations\}$ *as premises for conclusion c iff*

(i) Every maximally consistent subset of $\{p_1, ..., p_k, negations\}$ *settles c (i.e. every such combination entails c or* $\sim c$ *relative to background knowledge).*

(ii) For all $i \in \mathcal{G}$*,* $\|p_1, ..., p_k\|^i \hookrightarrow_i \|c\|^i$*.*

With the help of Definition 7, Premise-based majority can be defined roughly as follows:

Definition 15. *Provided that the basing relations in $\vec{j}$ admit $\{p_1, ..., p_k\}$ as premises for c:*

$$q \in PB(\vec{j}) \text{ iff there is a set of judges } \mathcal{S} \subseteq \mathcal{G}$$

(i) members of S strongly cohesively support p.

(ii) the reasons that members of S produce for p are also in $PB(\vec{j})$.

(iii)

$$\frac{|S|}{|S| + |\mathcal{G}[\sim p]|} > 1/2$$

The sense in which the definition is 'rough' is that $PB(\vec{j})$ appears on both sides of the biconditional. This is innocuous, however, because when a basing relation admits some propositions as premises for a conclusion, Definition 9 can be converted into a two-step definition (step 1, handle the premises; step 2, handle the conclusion). The presentation in Definition 9 is nonetheless useful, however, because it shows that, in essence, Premise-Based Majority is a variant of $\mathcal{CM}$ which (a) is defined only for some particular basing relations and (b) requires that the reasons be accepted (see again Example 3 above for confirmation that this is not the case with $\mathcal{CM}$).

8 Conclusion

In the interest of modular design, I have left open several questions surrounding this framework. Among them, the central issue seems to me to be an analysis of the epistemic value of cohesiveness, as well as an analysis of the types of new rules that are available in this framework and how they affect the interpretation of results that were established for the standard framework.

Let us focus then on what I think this paper accomplishes: I contributed a new proposal for the analysis of collective reasons in judgment aggregation. A natural account of collective reasons emerges if we represent individuals' epistemic states as featuring also a basing relation. In particular, this involves specifying a notion of cohesive support of a proposition, as well as defining rules according to which acceptance of *p* requires cohesive support. This framework helps clarify the sense in which standard premise-based approaches have limited applicability (by showing premise-based majority to be definable as a special case of more general rules). Even in the framework I have proposed, collective reasons need not always be available—sometimes a group's opinion is just too fragmented (as we saw in Example 3). My framework allows a natural weakening of Independence assumptions that 'breaks' some existing Impossibility results, while at the same type limiting the deviation from Independence to a naturally circumscribed range.

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