Probabilistic Notions of Deference

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Applications of Deference

Deference, as I will use the term, is a quasi-technical notion best introduced by citing its applications.

1. The connection between rational credence and credence about the chances is that rational agents defer to the chances (at least if they are sure they don't have spooky crystal ball evidence).
   - David Lewis's Principal Principle and Lewis and Ned Hall's New Principle are best seen as attempts to formalize this idea.
2. Bas van Fraassen's Reflection Principle is best seen as an attempt to formalize the (false) thesis that rational agents always defer to their future selves.
3. I am interested in deference because I want to develop a deference-based conception of rational belief updating, but there will be nothing about that in the talk.

Two probabilistic notions of deference

Deference is a relation between probability functions and probability function-valued random variables (functions from the set of worlds to the set of probability functions).

- ν locally defers to Q if $\nu(A \mid Q(A) = c) = c$ for all c such that $\nu(Q(A) = c) > 0$.
- ν globally defers to Q if $\nu(\cdot \mid Q = \mu) = \mu$ for all μ such that $\nu(Q = \mu) > 0$.

N.B. '$Q(A) = c$' $= \{w : Q_w(A) = c\}$. '$Q = \mu$' $=$ $\{w : Q_w = \mu\}$.

The indiscreteness problem

- Suppose $\nu(Q(A) = c) = 0$ for all A and c.
- Then ν locally and globally defers to Q.
- But if $\nu(Q(A) \geq 1/2) = 1$ and $\nu(A) < 1/2$ then, intuitively, ν doesn't defer to Q.
- Solution: ν tweakedly locally defers to Q if $\nu(Q(A) \in I) \in I$ for every closed interval I such that $\nu(Q(A) \in I) > 0$.[1]
- (Formulating a tweaked version of global deference is more subtle—I could use help.)
- Let's assume from here on in that there are only finitely many worlds, so the indiscreteness problem cannot arise. Footnotes mention some of the issues that arise in infinitary cases.

[1] The restriction to closed intervals is important only if ν can fail to be countably additive.

The imperfect introspection problem

- Q perfectly introspects if, writing μ for the value of Q in the actual world, $\mu(Q = \mu) = 1$.
- If ν is not certain that Q perfectly introspects then ν does not globally defer to Q.
- *Proof.* Let μ be such that $\nu(Q = \mu) > 0$ and $\mu(Q = \mu) < 1$. Then $\nu(Q = \mu \mid Q = \mu) = 1 \neq \mu(Q = \mu)$.
- Intuitively, though, ν might still defer to Q.
- The situation with local deference is more subtle, but we will see an example later where ν defers to Q without locally deferring to Q.
- If ν is certain that Q perfectly introspects then ν locally defers to Q iff ν globally defers to Q.
- In this case, both local and global deference are adequate notions of deference.

Undermining futures and strong undermining futures

- An undermining future at t is a proposition that has positive chance at t but cannot be true given history up to t and what the chance function at t is.
- Lewis argued that there will be undermining futures on any plausible reductionist account of chance.
- A strong undermining future at t is a proposition that has positive chance at t but cannot be true given what the chance function at t is.
- Since history up to t has chance 1 at t, for any undermining future F, the set X of worlds in which F is true and the chance function is as it is has chance zero, so $F \setminus X$ is a strong undermining future.
- So the existence of an undermining future is equivalent to the existence of a strong undermining future.

Strong undermining futures and imperfect introspection

- The existence of a strong undermining future is evidently equivalent to the chance function failing to perfectly introspect.
- So, if you have non-zero credence in there being an undermining future then local and global deference don't capture what it is to defer to the chances.
- Lots of ink has been spilled over this fact.
- But in the literature on the Reflection Principle, with few exceptions, no one ever notices that the principle doesn't work properly unless you are certain you will perfectly introspect.

NP-deference

- Lewis and Ned Hall introduced a principle called the New Principle to replace the Principal Principle.
- At its core is the following notion of deference.
- ν NP-defers to Q if $\nu(\cdot \mid Q = \mu) = \mu(\cdot \mid Q = \mu)$ for all μ such that $\nu(Q = \mu) > 0$.[2]
- There is no "local" version of NP-deference.
- Global deference counted ν as not deferring to Q just because, conditional on $Q = \mu$, ν was sure that $Q = \mu$ about but μ wasn't. That was unfair because ν might not have been sure that $Q = \mu$. NP-deference attempts to rectify the situation by giving μ the benefit of conditioning on $Q = \mu$.

[2] Formulating a tweaked version of NP-deference is easy once one knows how to formulate a tweaked version of global deference.

Two plausible necessary conditions for deference

- For the first condition, I'll start with a special case, then give a more general special case, and finally give the general case.
  1. If $\nu(Q = \mu) = 1$ then ν doesn't defer to Q unless $\nu = \mu$.
  2. If $\nu(Q(A) = c) = 1$ then ν doesn't defer to Q unless $\nu(A) = c$.
  3. If $\nu(Q \in S) = 1$, where S is a convex set of probability functions, then ν doesn't defer to Q unless $\nu \in S$.[3]
- The second condition is that if ν defers to Q and $\nu(A) > 0$ then $\nu(\cdot \mid A)$ defers to $Q(\cdot \mid A)$.
- ν can NP-defer to Q without satisfying either of these plausible necessary conditions for deference, so NP-deference is insufficient (though necessary) for deference.

[3] In infinitary cases we must put further restrictions on S, such as closure under countable mixtures.

Is the New Principle too weak to express the deference we owe the chances?

- I'm not sure how to think about this.
- I don't have any example where you satisfy the New Principle but don't do the chances their due.
- But I haven't thought very hard about it.

S-deference

- S-deference is the largest relation satisfying the two plausible necessary conditions.
- More explicitly, ν S-defers to Q if $\nu(\cdot \mid A) \in S$ for every proposition A such that $\nu(A) > 0$ and convex set S such that $\nu(Q(\cdot \mid A) \in S \mid A) = 1$.
- S-deference entails NP-deference.
- I haven't found any ways in which it is an inadequate formalization of deference.

A sufficient condition for deference and an example

- A plausible sufficient condition for deference is that if, for every proposition A, ν is certain that $Q(A)$ is at least as close to the truth value of A as $\nu(A)$ (where we identify the True with 1 and the False with 0) then ν defers to Q.
- Suppose there are just two worlds w_1 and w_2.
- Suppose ν assigns each world probability $1/2$, but Q gives $2/3$ to w_1 in w_1 and $2/3$ to w_2 in w_2.
- Then ν satisfies the plausible sufficient condition, and Q doesn't perfectly introspect.
- ν doesn't locally defer to Q.

Two vague ideas about how to find the true formalization of deference and demonstrate that it is the one

1. Find a weaker plausible sufficient condition for deference and show it's equivalent to S-deference. Maybe it would involve accuracy measures.
2. Say that (ν, Q) is diachronically Dutch bookable if there are packages of bets Γ and Δ such that ν judges Γ fair, and, in every world, Q judges Δ fair, but taken together, Γ and Δ result in a sure loss.
   - Maybe there is a connection between diachronic Dutch bookability and deference.

The At Most One and At Least One properties

- A notion of deference has the At Most One property if, for all ν and Q, there is at most one ν' that has the same distribution as ν on what Q is and defers to Q.
- A notion of deference has the At Least One property if, for all ν and Q, there is at least one ν' that has the same distribution as ν on what Q is and defers to Q.
- Local and global deference have these properties restricted to ν and Q such that ν is certain Q perfectly introspects.
- In fact, one way to prove the equivalence of local and global deference is to directly show that global deference implies local deference and to show the opposite implication by proving that local deference has the (restricted) At Most One property and global deference has the (restricted) At Least One property.

The special status of NP-deference

- NP-deference has the At Most One and At Least One properties restricted to those ν and Q such that there is no μ such that $\nu(Q = \mu) > 0$ and $\mu(Q = \mu) = 0$, which is about as much as one could hope for.
- Any strictly stronger notion, such as S-deference, must fail to have the At Least One property, even for ν and Q that it is supposed to apply to.
- In other words, sometimes, you can't come to defer to someone without changing your opinion about what they think.
- This happens in cases where you are highly opinionated about what someone thinks, and you think they are unsure what they think.
- Does this phenomenon have implications for the debate on peer disagreement?

A couple last things

- There is a probability function-valued random variable Q whose value in every world assigns positive probability to every world but that no probability function S-defers to.
  - See Appendix 2 of the paper for the example. It only involves three worlds.
- Is there any interest to the notion of practical deference?
  - See Appendix 1 of the paper for a brief discussion.