Indicative Conditionals

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Outline

Introduction

- Many philosophers think that conditionals, especially indicative conditionals, do not express propositions and do not carry truth values.
  - Frank P. Ramsey, whose insights about conditionals have influenced much of the contemporary work in the field, was one of these philosophers.
  - Allan Gibbard offered an argument showing that (indicative) conditionals cannot carry truth values.

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- Despite this, there has been a fair amount of recent work on providing truth conditions for *epistemic modals*.
    - Obviously indicative conditionals are an example of a dyadic epistemic modality.
- A very recent (still unpublished, but forthcoming) paper by Niko Kolodny and John MacFarlane offers an example of an attempt to develop a truth-conditional account of indicatives (among many other things).

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- Our paper extends to iteration a probabilistic model presented by one of us in “Bayesian Epistemology and Epistemic Conditionals: On the Status of the Export-Import Laws,” (*Journal of Philosophy*, 2001) and by Rohit Parikh and one of us in “Conditional Probability and Defeasible Inference,” (*Journal of Philosophical Logic,* 2005).
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- The construction in terms of probability cores is crucially used to develop the probabilistic models and to make evident their semantic structure.

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Introduction: why iteration?

Adams gave examples of inferences that the classical theory regards as valid but that are utterly repugnant to English speakers. Here is one of them:

- The engine will start if Switch A and Switch B are both in the ON position.
- Therefore, either the engine will start if Switch A is in the ON position or the engine will start if Switch B is in the ON position.

Introduction: why iteration?

Nevertheless Adams' theory ended up focusing only on non-nested conditionals and he only allowed negation of conditionals in most of his writings.

As a result Adams did not have any way of explaining why the repugnant inferences of the type displayed above should be rejected. He was not able to explain either why certain nested principles like the export-import rule is indeed constitutive of sound reasoning with indicative conditionals. The theory presented here will be able to deal with arbitrary Boolean combinations of conditionals and some forms of iteration.

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- Our model bears an interesting relationship to two models presented in two papers:
  (1) Vann McGee's paper: "Conditional Probabilities and Compound of Conditionals."
  (2) A recent and still unpublished paper by Hannes Leitgeb, which offers a probabilistic model for counterfactuals.
- Our model shows how truth conditions can be defined probabilistically (as in Leitgeb's paper), and it is characterized by a syntax that includes most of the axioms proposed by McGee in this seminal paper on compounds of conditionals.

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Kolodny and MacFarlane on Indicative Conditionals

- Kolodny and MacFarlane characterize an *Information state* as a set of possible world-states.
- Intuitively, an information state is a set of descriptions which, given what is known, might depict the actual world.
- No other indication about the epistemological status of information states is provided.
- We do not have the time to present here the truth conditions for indicative conditionals (TCIC) that they propose, but we will make a few comments presently.

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Why (TCIC) seems Correct and Interesting

- Truth conditions are sensitive to the epistemic state of the agent, given that points have epistemic components.
- Truth conditions depend on a process of transformation of the current information state relative to the proposition expressed by the antecedent of the conditional.
  - This is usually understood as formalizing the corresponding notion of supposition involved in evaluating conditionals.
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Why (TCIC) seems Inadequate and Incorrect

- (TCIC) suggests that *i* is the only relevant epistemic component for evaluating conditionals. But we will see (via examples) that information sets do not suffice to capture well known conditional patters of inference.
- (TCIC) presupposes that the proposition expressed by the antecedent of the conditional is always compatible with *i*. But we wll show that this might be false in classical examples used in conditional logic (Oswald examples).
- (TCIC) is too weak to capture iterated patterns of inference that are usually considered as constitutive of the logic of indicatives (like Export-Import).

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Indicative Supposition

*...[T]he hallmark of matter-of-fact supposition is that it never forces the supposer to revise any of her views about the 'facts,' that is to say propositions that she is certain are true. Jim Joyce (*The foundations of causal decision theory*)*

Indicative Supposition and Certainty

- According to Joyce's analysis, an essential epistemic component of a model of indicatives is a representation of the certainties of the agent evaluating the conditional.
- It is unclear whether the state of information *i* used by Kolodny and MacFarlane represents certainties or another epistemic notion (plain belief?).
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Indicative Supposition and Context Sets

Stalnaker also has insights about indicative supposition:

> *The presuppositions will include whatever the speaker finds it convenient to take for granted, or to pretend to take for granted, to facilitate communication. What is essential is not that the propositions presupposed in this sense be believed by the speaker, but rather that the speaker believe that the presuppositions are common to himself and his audience. This is essential since they provide the context in which the speaker intends his statement to be received.*

Indicative Supposition: A Classical Example

A classical example (due to Adams) of an indicative conditional:

"If Oswald did not do it, then someone else did it."

- Suppose in addition that the person who accepts it is convinced of the truth of Warren's report.
- This person is convinced that someone assassinated Kennedy, one of many incontrovertible facts represented in the context set.
- This person might also plainly believe that Oswald did it alone (or she might attribute extremely high or maximal probability to that proposition).

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O: 'Oswald alone did shoot Kennedy.'

S: 'Someone else did it.'

J: 'Lyndon Johnson became president.'

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Then the following worlds are relevant:

$$w_1 = (O, \neg S, J, K)$$
$$w_2 = (\neg O, S, J, K),$$
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Now add the possible world:

$$w_4 = (\neg S, \neg O, \neg J, \neg K)$$

- This world is outside the context set.
- Yet when one supposes subjunctively, this world can be considered.
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Now observe that one can evaluate and accept the conditional:

‘If Oswald did not do it, someone else did.’

- Apparently the information set will be given by $i = \{w_1\}$.
- But one can suppose the antecedent of the conditional.
- This supposition is felicitous because the proposition expressed by the antecedent is compatible with the context set $\{w_1, w_2, w_3\}$.
- We therefore have a case in which we suppose indicatively a proposition that nevertheless contravenes the set of plain beliefs of the Warrenite (the set i of K&McF).

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Indicative Supposition: A Classical Example

- Indicative supposition can be counter-doxastic.
- The formal framework offered by Kolodny and MacFarlane cannot adequately represent counter-doxastic supposition of the sort common for indicatives.
- We contend that a theory of indicatives should include representations of both the plain beliefs of the agent and the set of certainties which are nonnegotiable when one engages in matter-of-fact supposition.

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Indicative Supposition: A Classical Example

Yet we need more than that.

- We also need a plausibility ordering ranking the options in the context set.
- Such an ordering would be given in the Oswald example by the following structure:

κ	Possible worlds
0	$O, \neg S, J, K$
1	$\neg O, S, J, K$
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- The lowest echelon in the ordering represents the plain beliefs of the agent (the information set i).
- The context set is given by the entire set of worlds and represents the certainties of the agent.
- Notice that we can perfectly well have a probabilistic representation for which all the measure is applied to the lowest echelon in the structure.
  - In this case, when one supposes $\neg O$, one has to consider a conditional probability where the conditioning event has measure zero.
  - This is the reason why Joyce proposes to model matter-of-fact supposition by so-called Popper functions or primitive conditional probability.

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$$\mathrm{P}(\{w_1\}) = .9,\ \mathrm{P}(\{w_2\}) = .009,\ \mathrm{P}(\{w_3\}) = .001.$$

Indicative Supposition: Minimal Epistemic Components

The previous analysis suggests that an adequate model of indicative supposition should contain at least the following epistemic components:

- A representation of basic facts presupposed (as certainties) by the speaker and the audience (context set).
- A weaker notion of credence.
    - Under a probabilistic point of view this can be given by the minimal proposition carrying measure zero (if it exists) or the strongest proposition with high probability (with respect to a given threshold).
- A representation of degrees of plausibility structurally similar to a system of spheres of Lewis (or Grove).

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Indicative Supposition

- We will suppose that all these epistemic elements are given, and we will propose truth conditions that depend on them.
  - Unlike the model of Kolodny and MacFarlane we will not add these elements as components of worlds.
  - We will relate them to worlds via a neighborhood function, which will afford a more elegant theory.
- In the final part of the talk, we will show that *all* the needed epistemic elements are derivable from primitive conditional probability, and we will propose a probabilistic model that is therefore epistemologically rich enough to represent indicative supposition.

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    - Unlike the model of Kolodny and MacFarlane we will not add these elements as components of worlds.
    - We will relate them to worlds via a neighborhood function, which will afford a more elegant theory.
- In the final part of the talk, we will show that *all* the needed epistemic elements are derivable from primitive conditional probability, and we will propose a probabilistic model that is therefore epistemologically rich enough to represent indicative supposition.

Outline

Introduction

- We will introduce models of greater and greater complexity.
- First we appeal to the idea of *conditional neighborhood*.
- The idea is to have a neighborhood function that maps points and propositions (expressed by the antecedent of the conditional) to sets of propositions.
- A conditional $A > B$ is true at a world w just in case the proposition expressed by B is in the neighborhood of w conditional on the proposition expressed by A.

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This model still does not have the required complexity.

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- It cannot represent degrees of plausibility and it is not clear how to differentiate plain belief from certainty by appealing to it.
- Nevertheless, we will see that some iterated properties of conditionals can be represented by this simple model.
- We will also see the syntax complete to this semantics and then we will move to a more complicated model that does include all the needed epistemological components.

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The basic language

We proceed cautiously by enlarging the conditional language in a non-problematic manner.

Definition

Let L_0 be a non-modal propositional language. We define LC recursively as follows: If $A, B \in L_0$, $\alpha, \beta \in LC$, $A > B$, $A > \alpha$, $\alpha \wedge \beta$, $\neg\alpha \in LC$.

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Propositions and maximal and consistent sets

- We will refer to a L_0-maximal and consistent set (world) by using the notation w_0.
- We will use the notation w to refer to a $L_0 \cup LC$-maximal and consistent set (world).
- By the same token, given a model $\mathcal{M}$ and $A \in L_0$, $(A)^{\mathcal{M}_0}$ stands for the proposition composed by all the L_0-worlds where A is true; while $(A)^{\mathcal{M}}$ represents the $L_0 \cup LC$-worlds where A is true.

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Models and Frames

Definition

(i) A *conditional neighborhood frame* is a pair $\mathfrak{F} = \langle W, N \rangle$, where W is a set of states, or worlds, and $N : W \times 2^{W^0} \to 2^{2^{W^0}}$ is a mapping.

(ii) Given a conditional neighborhood frame $\mathfrak{F} = \langle W, N \rangle$, a *conditional neighborhood model based on* $\mathfrak{F}$ is pair, where $V : \Phi_0 \to 2^{W^0}$ is a valuation function.

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Definition

Let $\mathfrak{F} = \langle W, N\rangle$ be a conditional neighborhood frame.

(i) Given $A \in 2^{W^0}$, we define $N_A : W \times 2^{W^0} \to 2^{2^{W^0}}$ by setting for every $C \in 2^{W^0}$ and $w \in W$,

$$N_A(w, C) := N(w, A \cap C).$$

(ii) We accordingly define $\mathfrak{F}_A := \langle W, N_A\rangle$.

Truth

The truth condition for non-nested formulas can therefore be expressed as follows:

$$\mathcal{M}, w \models A > B \quad \text{iff} \quad (B)^{\mathcal{M}_0} \in N_{(A)^{\mathcal{M}_0}}(w, (\top)^{\mathcal{M}_0}).$$

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More generally, when ψ is a sentence of *LC*:

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Armed with this notation we are now ready to introduce truth conditions for *LC*.

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Truth Conditions

$\mathcal{M}, w \models A > B$ iff $(B)^{\mathcal{M}_0} \in N(w, (A)^{\mathcal{M}_0})$.

$\mathcal{M}, w \models A > \alpha$ iff $\mathcal{M}_{(A)^{\mathcal{M}_0}}, w \models \alpha$

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Truth Conditions

The semantic turnstile defined above behaves classically with respect of Boolean combinations of sentences in *LC*:

$\mathcal{M}, w \models \alpha \wedge \beta$ iff $\mathcal{M}, w \models \alpha$ and $\mathcal{M}, w \models \beta$.

$\mathcal{M}, w \models \alpha$ and $\mathcal{M}, w \models \alpha \rightarrow \beta$, then $\mathcal{M}, w \models \beta$.

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Syntax: The System **M**

A1 $(A > (\phi \wedge \psi)) \leftrightarrow ((A > \phi) \wedge (A > \psi))$

A2 $(A > (\phi \vee \psi)) \leftrightarrow ((A > \phi) \vee (A > \psi))$

A3 $(A > \psi) \vee ((A > \neg\phi) \leftrightarrow \neg(A > \phi))$

A4 $(A > (B > \phi) \leftrightarrow ((A \wedge B) > \phi)$

A5 $(\top > \phi) \leftrightarrow \phi$

$$LLE > \quad \frac{\models (A \leftrightarrow B), A > \phi}{B > \phi}$$

$$RW > \quad \frac{\models (\phi \rightarrow \psi), A > \phi}{A > \psi}$$

Completeness for **M**

Theorem

The system **M** *is sound and complete with respect to the class of conditional neighborhood frames.*

Outline

Core Systems

Definition

Let W_0 the set of L_0-maximal and consistent sets (worlds), and let W denote the set of $L_0 \cup LC$-maximal and consistent sets (worlds). A *core system centered on B* is a nested set of subsets of W_0 with innermost core B and outermost core C.

Update of Cores

We define an operation on core neighborhoods that will help with the definition of a conditional core neighborhood:

Definition

Let $\mathcal{C}$ be a core system centered on B. For any proposition X compatible with the outermost core C of $\mathcal{C}$ let $\mathcal{C}_X = \{Y \cap X \mid Y \in \mathcal{C}\}$. We require in addition the existence of an innermost core of $\mathcal{C}_X$, which is called C_X.[1]

[1] This condition is tantamount to imposing the so-called Limit Assumption of Lewis.

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Core Neighborhoods

Definition

A *conditional core neighborhood* is a pair $\langle N^{\mathbf{c}}, \mathcal{C} \rangle$, where $N^{\mathbf{c}}$ is a conditional core neighborhood and $\mathcal{C}$ is an associated core system, such that for every $w \in W$ and every proposition $X \in 2^{W_0}$ such that X compatible with the outermost core C of $\mathcal{C}$,

$$N^{\mathbf{c}}(w, X) = \{Y \in 2^{W} : C_X \subseteq Y\}.$$

When X is incompatible with the outermost core C, C_X is identical with the empty set.

Core Neighborhoods

What is the epistemological role played by the core system in the core neighborhood?

- The intersection of the neighborhood coinciding with the innermost core represents the plain beliefs of the agent.
- The core system can in addition be used to represent a *plausibility ordering* of worlds outside of B.
- This plausibility ordering plays a crucial role in representing counter-doxastic suppositions which are nevertheless compatible with the certainties of the agent (represented by the outermost core of the core system).

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Truth

The main modification occurs in the basic case for the truth conditions. Let $\mathcal{M}$ be a conditional core neighborhood model whose associated frame contains a *conditional core neighborhood* $\langle N^{\mathbf{c}}, \mathcal{C} \rangle$. Define:

$$\mathcal{M}, w \models A > B \text{ iff } (B)^{\mathcal{M}_0} \in N^{\mathbf{c}}(w, (A)^{\mathcal{M}_0}) \text{ iff } C_{(A)^{\mathcal{M}_0}} \subseteq (B)^{\mathcal{M}_0}.$$

$$\mathcal{M}, w \models \Box(A) \text{ iff } (A)^{\mathcal{M}_0} \in N^{\mathbf{c}}(w, (\top)^{\mathcal{M}_0}) \text{ iff } C_{(\top)^{\mathcal{M}_0}} \subseteq (A)^{\mathcal{M}_0}$$

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Syntax: Axioms for System $\mathcal{IC}$

R1 $A > A$

R2 $((A > B) \wedge (A > C)) \rightarrow (A > (B \wedge C))$

R3 $((A > C) \wedge (B > C)) \rightarrow ((A \vee B) > C)$

R4 $((A > B) \wedge (A > C)) \rightarrow ((A \wedge B) > C)$

R5 $((A > B) \wedge ((A \wedge B) > C)) \rightarrow (A > C)$

R6 $(\neg(A > \neg B) \wedge (A > C)) \rightarrow ((A \wedge B) > C)$

*A*1 $(A > (\phi \wedge \psi)) \leftrightarrow ((A > \phi) \wedge (A > \psi))$

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*A*4 $(A > (B > \phi) \leftrightarrow ((A \wedge B) > \phi)$

Syntax: Rules of Inference for System $\mathcal{IC}$

A5 $\Box A \wedge \Box B \rightarrow A > B$

A6 $A > B \rightarrow \Box(A \rightarrow B)$

B1 $\Box A \wedge \Box B \leftrightarrow \Box(A \wedge B)$

B2 $\Box \top$

Syntax: Rules of Inference for System $\mathcal{IC}$

$$LLE > \quad \frac{\models (A \leftrightarrow B), A > \alpha}{B > \alpha}$$

$$RW > \quad \frac{\models (\alpha \rightarrow \beta), A > \alpha}{A > \beta}$$

As above we will use the letters A, B to denote formulas of L_0. We will use ϕ,ψ to denote formulas of LC and letters α,β to denote formulas of $LC \cup L_0$.

Completeness for $\mathcal{IC}$

Theorem

The system $\mathcal{IC}$ is sound and complete with respect to the class of conditional core neighborhood frames.

Outline

Probabilistic Models of the Conditional

- The gist of the idea is to use primitive conditional probability in the determination of acceptability conditions for indicative conditionals.
- The probabilistic test in question derives from suggestions by various authors, beginning with the seminal work of Frank Ramsey and continuing with the more recent work of Ernest Adams and (centrally) Vann McGee in a series of more recent papers (especially "Learning the Impossible").

Definition

(Ramsey-Adams-McGee) $a > b$ is accepted with respect to $P(\cdot|\cdot)$ if and only if $\mathrm{P}(B|A) = 1$, where A and B are the propositions expressed by a and b, respectively.

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Conditional Probability

A *space* is a couple S = $\langle W, \mathscr{F} \rangle$, where W is a non-empty set and $\mathscr{F}$ is a σ-field on W. A two-place probability measure $P(\cdot|\cdot)$ on space S is a mapping $\mathscr{F} \times \mathscr{F} \rightarrow \mathbb{R}$ such that:

(i) For every $A \in \mathscr{F}$, the function $P(\cdot|A)$ is either a (countably additive) probability measure or has constant value 1.

(ii) For every $A \in \mathscr{F}$, $P(A|A) = 1$.

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- The probability (*simpliciter*) of $A \in \mathscr{F}$, $pr(A)$, is $P(A|W)$.
- We will follow established terminology by referring to (II) as the *Multiplication Axiom*.
  - This axiom appears under the name 'W. E. Johnson's product rule' in H. Jeffreys's book on probability.
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Normality

- Given $A \in \mathscr{F}$, we say that A is *normal* if $P(\cdot|A)$ is a probability measure and *abnormal* otherwise (i.e., when $P(\cdot|A)$ has constant value 1 and, in particular, $P(\emptyset|A) = 1$).
- A normal set *may* have measure 0, in which case we do not *expect* to believe it, but we still *might*.
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Normality

van Fraassen proves in a JPL article where he introduce the notion of normality that supersets of normal sets are normal and that subsets of abnormal sets are abnormal. Assuming that the whole space is normal, abnormal sets must have measure 0. However, of course the converse need not hold, a normal set may also have measure 0.

Probability Cores

Slightly modifying van Fraassen's presentation, we define a *core* as a set K which is normal and satisfies the *strong superiority condition* (SSC) i.e., if A is a nonempty subset of K and B is disjoint from K, then $P(B|A \cup B) = 0$ (and so $P(A|A \cup B) = 1$).

Probability Cores: nesting

Lemma (Finesse)

All non-empty subsets of K are normal.

Lemma

The family of cores induced by a two place probability function P is nested, i.e. given any two cores K_1, K_2 either $K_1 \subseteq K_2$ or $K_2 \subseteq K_1$ (van Fraassen, 1995).

Probability Cores

Theorem (Descending Chains)

The chain of belief cores induced by a countably additive 2-place function P cannot contain an infinitely descending chain of cores. (Arló-Costa, 1999).

Probability Cores: Properties

Lemma

There is a smallest as well as a largest core. Moreover, the smallest core (and hence every core) has measure 1.

Core and Probability Dynamics

- We can confirm that the structure arising from probability has the right semantic properties by considering the dynamics of core systems and probability functions.

Update $P[Z](X|Y) = P(X|Y \cap Z)$

- This Bayesian definition of update induces a familiar dynamics of cores: In fact (Arló-Costa, 2001) shows that if $\mathcal{C}$ is the core system of $P(\cdot|\cdot)$, then the core system of $P[Z](X|Y)$ is $\mathcal{C}_Z := \{C \cap Z : C \in \mathcal{C}\}$.

Core and Probability Dynamics

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Probabilistic Models

Definition

A probabilistic model is a triple $\mathcal{M}_P = \langle W, (P_w)_{w \in W}, V \rangle$, where P_w is function assigning a probability function to each world. The probability function is constructed over a σ-field built over W_0 and a valuation V.

Truth defined Probabilistically

$\mathcal{M}^P, w \models A > B$ iff the smallest core of $P_w[(A)^{\mathcal{M}_0^P}]$ entails $(B)^{\mathcal{M}_0^P}$ iff $P_w((B)^{\mathcal{M}_0^P} | (A)^{\mathcal{M}_0^P}) = 1$

$\mathcal{M}^P, w \models A > \alpha$ iff $\mathcal{M}^P_{(A)^{\mathcal{M}_0^P}}, w \models \alpha$.

Where $\alpha \in LC$, $A, B \in L_0$; and $\mathcal{M}^P_{(A)^{\mathcal{M}_0^P}}$ evaluates formulas with respect to $(P_w)[(A)^{\mathcal{M}_0^P}]$.

Truth defined probabilistically

As before, the case that is particularly important is $\alpha := B > \theta$. In this case we have:

$$\mathcal{M}^P, w \models A > \alpha \text{ iff } \mathcal{M}^P_{(A)^{\mathcal{M}_0^P} \cap (B)^{\mathcal{M}_0^P}}, w \models \theta.$$

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And the case where $\theta := C$ is resolved as follows:

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$$w \in (A > B)^{\mathcal{M}^P} \text{ iff } P_w((B)^{\mathcal{M}_0^P} | (A)^{\mathcal{M}_0^P}) = 1$$

$$w \in (A > \alpha)^{\mathcal{M}^P} \text{ iff } w \in (\alpha)^{\mathcal{M}^P_{(A)^{\mathcal{M}_0^P}}}$$

$$w \in (A > (B > \beta))^{\mathcal{M}^P} \text{ iff } w \in (\beta)^{\mathcal{M}^P_{(A)^{\mathcal{M}_0^P} \cap (B)^{\mathcal{M}_0^P}}}$$

Probabilistic and Neighborhood Models

The probabilistic models and the neighborhood models are elementary equivalent in the sense that for $\phi \in LC$:

$$\mathcal{M}, w \models \phi \text{ iff } \mathcal{M}^P, w \models \phi$$

- The proof is rather direct.
  (⇒) Take an arbitrary world and its core neighborhood $\mathcal{C}$. Then select a probability function with this core system P_w. It is clear that we have then the direction from left to right.
  (⇐) Take an arbitrary world w and the corresponding P_w. Then take the core system of this probability function and construct a core neighborhood with this associated core. Then we verify that we have the converse.

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Probabilistic and Neighborhood Models

- Clearly the semantic structure that matters in the probabilistic model is given by its core system.
- We can think about this in a different manner:
  - For each world $w \in W$, define the core neighborhood for w by appealing to a probability function P_w.
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Leitgeb's Models

Hannes Leitgeb proposed a semantics for counterfactuals that in our notation can be written as follows:

$$w \in (\phi > \psi)^{\mathcal{M}_P} \quad \text{iff} \quad P_w((\psi)^{\mathcal{M}_P} | (\phi)^{\mathcal{M}_P}) = 1$$
$$\text{iff} \quad \text{the smallest core for } P[(\phi)^{\mathcal{M}_P}] \text{ entails } (\psi)^{\mathcal{M}_P}.$$

- Obviously this semantics does not validate Export-Import.
- Moreover, we cannot impose Export-Import to this model on pain on triviality.
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One can see our models as retaining Leitgeb's idea for the case of non-nested conditionals and then circumventing triviality by avoiding the following identity:

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Open Problems: Iteration to the Left

Is it possible to extend the model adding iteration to the left?

- In this case one needs to condition on conditional propositions and so it seems that one has to work with core systems constructed over an algebra allowing for conditional propositions.
- This could be problematic even for the neighborhood models.
- The completeness proof for conditional core frames indicates that core dynamics induces an AGM revision operator and at the same time as long as the model is weakly centered one half of the so-called Ramsey test is validated.

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Open Problems: High probability

The core construction can be generalized as follows:

Definition

Let P be a two-place probability function on an algebra $\mathscr{A}$, let $K \in \mathscr{A}$ be normal, and let $r \in [.5, 1)$. We say that K is a *generalized high probability core*^{r} if for every $A, B \in \mathscr{A}$ such that $\emptyset \neq A \subseteq K$ and $K \cap B = \emptyset$, $P(A \mid A \cup B) > r$.

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