# Interactive Models of Closure and Introspection

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Formal Epistemology Workshop 2010



## The Many Faces of Closure and Introspection

### Initial Motivation

- K-axiom as *closure proper* and as *deductive omniscience*.
- 4-axiom as *access* and as *higher standard*.

### General Proposal

- Model individual agents as groups.
- Use different types of group-knowledge to distinguish between different *ways of knowing*.
- Focus on architecture-dependent aspects of closure and introspection.

### Additional Benefits

- Different kinds of explicit knowledge.
- Hintikka on positive introspection revisited.



## Overview





## The Multi-Component Characterisation

### General Features

- Using more expressive resources.
- What looks like communication from the inside, looks like reasoning from the outside.
- Components should be understood in a generic way, and component-knowledge is only knowledge by name.

### Component Knowledge and Interaction

- **S5**-knowledge.
- Can always be communicated.
- Communication always leads to knowledge as well as higher-order knowledge.

## Statics and Dynamics

- Component-knowledge is deductively closed and introspective in a static way.
- Differences in how knowledge is distributed lead to static differences in closure and introspection.
- Changes in how knowledge is distributed account for dynamic aspects of closure and introspection.

## Modelling Options: Why S5?

- Focus on how the distribution of component-knowledge and the interaction between components affect the logical properties of knowledge proper.
- To treat something as a single component is to ignore dynamic aspects of closure and introspection.



# Types of Group-Knowledge

## Distributed Knowledge

- $\phi$ is D-known in a group $\mathcal{G}$ iff:
  - Every non-$\phi$ world is excluded by some member of $\mathcal{G}$.
  - $\phi$ can be deduced by combining the knowledge of all members of $\mathcal{G}$.
- D-knowledge is **S5**-knowledge.
- What is merely D-known is only implicitly known.

## Individual Knowledge

- $\phi$ is S-known in $\mathcal{G}$ iff some member of $\mathcal{G}$ knows $\phi$.
- S-knowledge is not deductively closed, but is fully introspective because component-knowledge is.
- What is merely S-known is explicitly known because:
  - It is not deductively closed.
  - It can be communicated by some component.



## General Knowledge

- $\phi$ is E-known in $\mathcal{G}$ iff all members of $\mathcal{G}$ know $\phi$.
- E-knowledge is deductively closed, but is not fully introspective.
- What is E-known is explicitly known because:
  - It is also S-known.
  - It can be communicated by all components.

## Common Knowledge

- $\phi$ is E-known in $\mathcal{G}$ iff:
  - $\mathsf{E}^1\phi \leftrightarrow \mathsf{E}\phi$ and $\mathsf{E}^{k+1} \leftrightarrow \mathsf{E}\mathsf{E}^k\phi$.
  - $\phi$ is $\mathsf{E}^k$-known for all finite $k$
- C-knowledge is **S5**-knowledge.



# A Knowledge Hierarchy

## S and E-knowledge

- E-knowledge is explicit because it implies S-knowledge.
- They only differ in how knowledge is distributed among the components.

## Stalnaker's Objection

Knowledge can be explicit because it is explicitly stored, or because it is readily available, but a single implicit/explicit-distinction cannot play this double role because knowledge can be explicitly stored without being readily accessible, and can be readily accessible without being explicitly stored.

## Explicitly stored, but not readily available

- What is S-known is explicitly stored.
- When it is not E-known it is not readily accessible because the right component still needs to be queried.

## Explicitly stored and readily available

- What is E-known is still explicitly stored.
- It is also readily accessible because any component can be queried.

## Readily accessible but not explicitly stored

- What is merely D-known is not explicitly stored.
- If it can easily be made explicit (i.e. known by some designated component), it is still readily accessible because only one component needs to be queried.



# Dynamics of Closure and Introspection

## Basic Idea

- Focus on existence and outcome of communication-protocols.
- Knowledge can be made explicit if there is a way to upgrade D-knowledge to S-knowledge.
  This is a dynamic form of closure.
- Knowledge can be made readily available if there is a way to upgrade to E-knowledge.
  This is a dynamic form of positive introspection.
- Knowledge can be made transparent if there is a way to upgrade to C-knowledge.
  This is a dynamic form of full positive introspection.



## Problems for closure

- Making distributed knowledge implicit required *full-communication* models (we leave this issue aside).
- Fitch-like phenomena.

## Fitch's Paradox

- From $\forall p(p \to \Diamond \mathsf{K}p)$, and $\exists(p \wedge \neg \mathsf{K}p)$ we can derive a contradiction.
- Hence, unrestricted knowability implies that all truths are known.

**Question**

Can all D-knowledge be upgraded to S-knowledge?



## A Fitch-style Result

- From

$$\forall p(\mathsf{D}p \to \Diamond \mathsf{S}p)$$

and

$$\exists p(\mathsf{D}(p \wedge \neg \mathsf{S}p))$$

we can derive a contradiction.

- Hence, we can prove that:

$$\forall p\Big((\mathsf{D}p \to \Diamond \mathsf{S}p) \to (\neg \mathsf{D}(p \wedge \neg \mathsf{S}p))\Big)$$

Thus, either some implicit knowledge cannot be made explicit, or some truths (e.g. $p$ is true but not explicitly known) cannot be implicitly known.

## Upgrading and Protocols

- Knowledge of one type can be upgraded to a stronger type iff there exists a communication-protocol that leads to the intended result.
- Fitch-like phenomena show that upgrading is sometimes blocked because some announcements are not successful.
- Only upgrading to C-knowledge requires a special kind of announcement.

## Two Definitions of Common Knowledge

$$\mathsf{E}p \wedge \mathsf{EE}p \wedge \ldots \wedge \mathsf{E}^k p \wedge \ldots$$

$$\mathsf{E}(p \wedge \mathsf{C}p)$$



## Two Types of Announcements

- Private and semi-private announcements cannot ensure $\mathsf{E}^k p$ for all finite $k$.
- Only public announcements can at once ensure that everybody knows $p$ *and* that this is transparent to all parties involved.

## Conclusion

- Knowledge can be made explicit *and* readily available by passing the relevant information around.
- Knowledge can only be made transparent when all the components share the same informational context (i.e. where all information can be made public).



# Hintikka Revisited

## Background

- Based on *defensibility* as possibility to coherently know.
- Auto-epistemology and strong rationality postulates.

## The KK-Argument

1. If $\{\mathsf{K}_a\phi, \neg\mathsf{K}_a\neg\psi\}$ is consistent, then $\{\mathsf{K}_a\phi, \psi\}$ is also consistent.
2. If $\{\mathsf{K}_a\phi, \neg\mathsf{K}_a\neg\neg\mathsf{K}_a\phi\}$ is consistent, then $\{\mathsf{K}_a\phi, \neg\mathsf{K}_a\phi\}$ is also consistent.
3. Since $\{\mathsf{K}_a\phi, \neg\mathsf{K}_a\phi\}$ is inconsistent, $\{\mathsf{K}_a\phi, \neg\mathsf{K}_a\mathsf{K}_a\phi\}$ is also inconsistent.



## Three Versions

- E-version: $\mathsf{E}p$, but $\neg\mathsf{EE}p$
- EC-version: $\mathsf{E}p$, but $\neg\mathsf{C}p$
- C-version: $\mathsf{C}p$, but $\neg\mathsf{CC}p$

## Two Knowability-Principles

- $\Diamond$EC-version: $(\mathsf{E}p \wedge \neg\mathsf{C}p) \rightarrow \Diamond\mathsf{E}(\mathsf{E}p \wedge \neg\mathsf{C}p)$
- $\Diamond$CC-version: $(\mathsf{E}p \wedge \neg\mathsf{C}p) \rightarrow \Diamond\mathsf{C}(\mathsf{E}p \wedge \neg\mathsf{C}p)$

If knowability is understood as “there is a way to make this announcement in a successful manner,” then ($\Diamond$EC) is true in virtue of the possibility to announce $\mathsf{E}p \wedge \neg\mathsf{C}p$ privately to all components.