Social Interaction and the Invasion of Adaptive Strategies

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The Dilemma of Cognition

Increased flexibility, but at a price...

Rupicapra rupicapra

Myotragus balearicus

Godfrey-Smith (2002): “Why has the expensive and delicate biological machinery underlying mental life evolved?”

Social Complexity

Machiavellian Intelligence

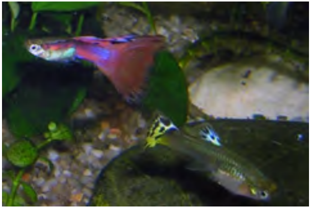
Byrne and Bates (2007). "Sociality, Evolution and Cognition." *Current Biology.*

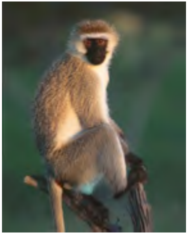
"Group living has many possible advantages...But living in close proximity to conspecifics also has clear disadvantages, in terms of direct resource competition."

"...other individuals present a 'moving target' of continually changing behaviour, able to respond to the self's strategies with their own."

Plasticity

Taking a step back...





Central Question

Under what conditions can social interaction (alone) provide the selective pressure necessary for the evolutionary invasion of plasticity?

Outline

1. Evolutionary Game Theory
2. Model I: Playing the Field
3. Model II: Playing Individuals
4. Concluding Remarks

Game Theory

A game G consists of the following:

- A set of players
- A set of strategies for each player $S = \{s_1, ..., s_n\}$
- A payoff function for each player π

We will focus on 2-player symmetric games.

	c	d
c	$2, 2$	$0, 3$
d	$3, 0$	$1, 1$

Other important concepts:

- Best Response
- Nash Equilibrium (NE)

Evolution and Population Games

Strategies in the game represent phenotypes in the population and we can think about the stability of populations.

The population space can be represented as:

$$P^n = \{(x_1, ..., x_n) \in \mathbb{R}^n_+ | \sum_{i=1}^{n} x_i = 1\}.$$

The corresponding expected utility is used to calculate the fitness of a type s against a population x:

$$F(s, x) = \sum_{i \in S} u(s, i) x_i$$

Stability and Invasion of Populations

A population plays a Nash iff it is a best response to itself.

Intuitively, a population x is *evolutionarily stable* when, after a small number of mutants are introduced, selection will return the population to state x.

Definition

- A strategy s is *weakly invasive* with respect to x *iff* $F(s,x) \geq F(x,x)$ and
- s is *strongly invasive* if $F(s,x) > F(x,x)$.

Evolutionary Dynamics

The Replicator Dynamics

$$\dot{x}_i = x_i[F(i, x) - F(x, x)]$$

Concepts of Dynamical Stability

- *Asymptotic Stability:* Selection will eliminate any invading strategies.
- *Lyapunov Stability:* Selection will not drive evolution (too far) away.

Strategic Plasticity

Plastic individuals will be represented as *adaptive strategies* $\mathcal{L}_0 ... \mathcal{L}_n$ which will be introduced as additional types in the population.

Each $\mathcal{L}_i$ will carry an associated cost $c_i > 0$.

Two modeling frameworks:

1. Learners adapting to the population.
2. Learners adapting to other individuals.

Model 1: Adapting to the Population

Individuals are paired to play several one shot games (with different opponents).

Learners adjust their behavior according to the average behavior of the population as a whole. Learners *do not* condition on their individual opponents.

Payoffs for are determined by how each strategy does against the long-run behavior of the population.

Example: Hawk-Dove

Hawk-Dove

	h	d
h	$0, 0$	$3, 1$
d	$1, 3$	$2, 2$

Population Types:

h, d, and $\mathcal{L}$

All $\mathcal{L}$ individuals adopt whatever mixed strategy brings the population behavior as close as possible to the ESS.

In this case:

- If there is $\geq$ 50% doves, all $\mathcal{L}$ adopt h.
- If there is $\geq$ 50% hawks, all $\mathcal{L}$ adopt d.
- Otherwise $\mathcal{L}$ mix to bring the behavior to 50% h, 50% d.

The Invasion of Learners

Theorem 1

If adaptive strategies respond to the population, population x has no adaptive strategies and is at a Nash equilibrium of G then no $\mathcal{L}$ is weakly invasive with respect to x.

Theorem 2

If adaptive strategies respond to the population, c is sufficiently small, x has no adaptive strategies and x is not at a NE of G, then there is some $\mathcal{L}$ that is strongly invasive with respect to x.

Moral: We should only expect adaptive strategies to invade changing populations.

Character of the Adaptive Strategies?

Maynard-Smith (1982). *Evolution and the Theory of Games*. Cambridge University Press.

Harley (1981). "Learning the ESS." *J. Theor. Bio.*.

Harley's "Theorem"

If $\mathcal{L}$ is an evolutionarily stable learning rule, then it is a rule that will bring the population to the ESS of G.

Character of the Adaptive Strategies?

Rock-Scissors-Paper with outside option

	r	s	p	o
r	$2,2$	$4,0$	$0,4$	$0,0$
s	$0,4$	$2,2$	$4,0$	$0,0$
p	$4,0$	$0,4$	$2,2$	$0,0$
o	$0,0$	$0,0$	$0,0$	$1,1$

Consider a population of $\mathcal{L}_i$ that plays the $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0)$ NE. This population cannot be invaded by any other $\mathcal{L}_j$ unless $c_j < c_i$.

Moral: At this point, no conclusions can be drawn about the character of the evolving adaptive strategies.

Summary

Model 2: Adapting to Individuals

Interactions are repeated and learners adjust behavior based on the individual they are interacting with.

Results of the interactions:

- Fixed Strategy vs. Fixed Strategy
- Learner vs. Fixed Strategy
- Learner vs. Learner

The payoffs are the expected utility in the long run of the interaction.

Example: Hawk-Dove

Assume a population with types h, d, and $\mathcal{L}_{br}$

$\mathcal{L}_{br}$ will adopt a best response against pure strategies and play a NE against itself.

Hawk-Dove

	h	d
h	0	3
d	1	2

G

Example: Hawk-Dove

Assume a population with types h, d, and $\mathcal{L}_{br}$

$\mathcal{L}_{br}$ will adopt a best response against pure strategies and play a NE against itself.

Hawk-Dove

	h	d
h	0	3
d	1	2

G

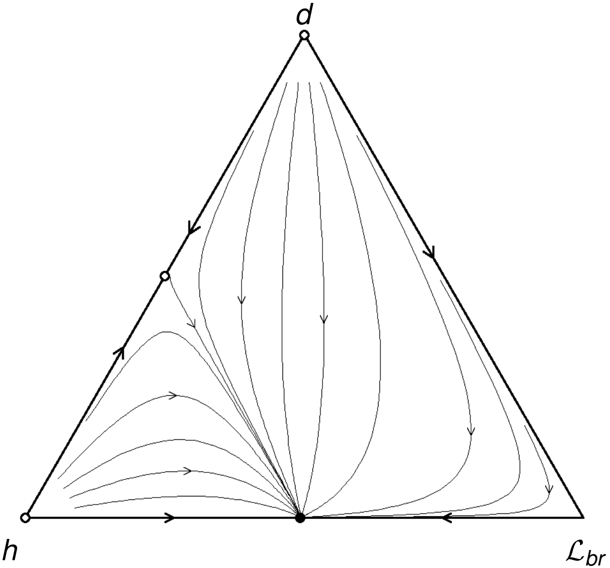
$\longrightarrow$

Hawk-Dove with Best Response

	h	d	$\mathcal{L}$
h	0	3	3
d	1	2	1
$\mathcal{L}$	$1 - c_{\mathcal{L}}$	$3 - c_{\mathcal{L}}$	$2 - c_{\mathcal{L}}$

G^L

Hawk-Dove with Best Response



When Learners *Can't* Invade

Theorem 3

If adaptive strategies respond to individuals and a population x without adaptive strategies is playing a pure strategy NE of G, then no $\mathcal{L}$ is weakly invasive with respect to x.

Moral: A mixed population will be needed for the invasion of adaptive strategies.

When Learners *Can* Invade

Theorem 4

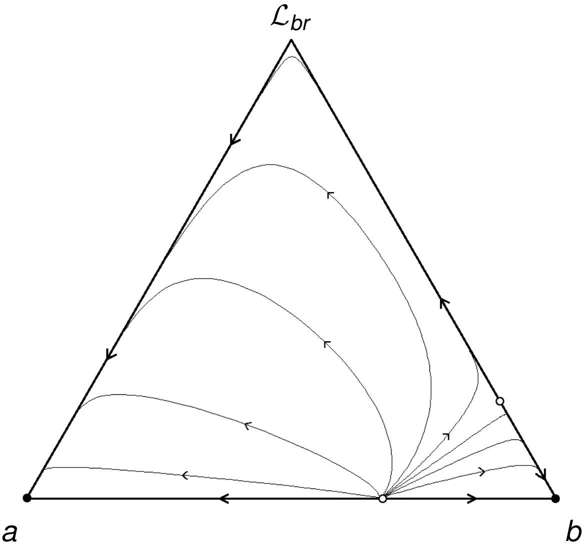
For any game G and any polymorphic population x without adaptive strategies such that for some s represented in x and some $t \in S$, $\pi(s, s) < \pi(t, s)$, if adaptive strategies respond to individuals and c is sufficiently small, then there exists a $\mathcal{L}$ that is strongly invasive with respect to x.

Moral: A wide range of mixed populations will open the possibility of a learner invasion.

Coordination Game with Best Response

Coordination Game

	a	b
a	3, 3	0, 0
b	0, 0	1, 1



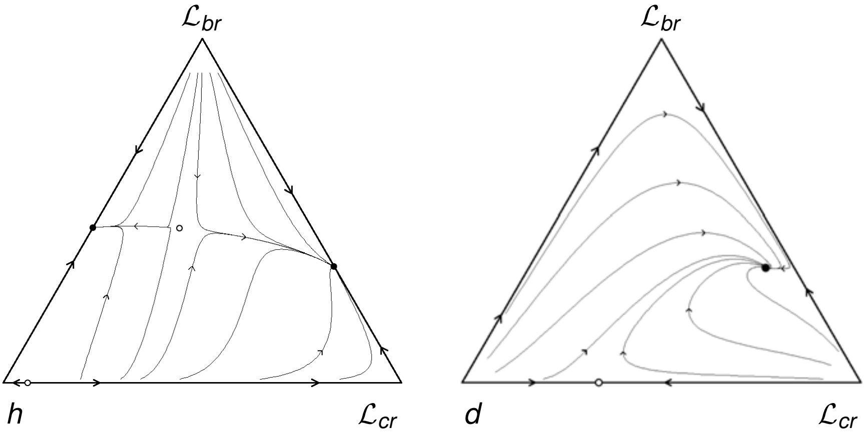
Best Response?

There are reasons to think learners that "best respond" will *not* dominate the population.

- Best Response is often susceptible to kinds of "exploitation."
- Including other plausible but non-traditional learning rules often results in stable polymorphisms.

Consider an alternative $\mathcal{L}$: *Competitive Response*.

Stable Polymorphism of Adaptive Strategies

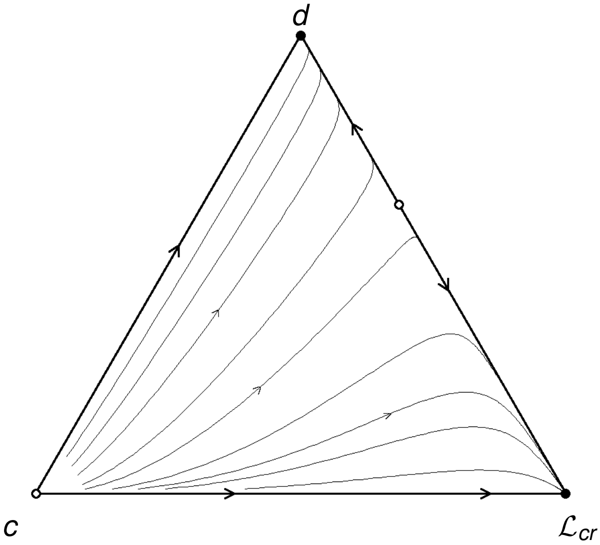


Hawk-Dove with learners in the replicator dynamics.

Prisoner's Dilemma with Competitive Response

Prisoner's Dilemma

	c	d
c	$2, 2$	$0, 3$
d	$3, 0$	$1, 1$



Summary

Concluding Remarks

1. Social interaction and the invasion of adaptive strategies
   - Unstable or polymorphic populations
   - Responsiveness to individuals
2. Evolution and best response learning rules
3. Future research