The Rationalizability of Two-Step Choices

Ruth Poproski

Formal Epistemology Workshop
Carnegie Mellon University
rpoprosk@andrew.cmu.edu

June 21, 2009.

Two-Tier Choice Functions

$V_1(U)$			$V_2(U)$
a	b	ab	c
b	c	c	a
c	a		b

$$C_1(\{a,b,c\}) = \{a,b\}$$

$$\begin{aligned} C_2(C_1(\{a,b,c\})) &= C_2(\{a,b\}) \\ &= \{a\} \end{aligned}$$

Two-Tier Choice Functions

Definition

C is *a two-tier choice function* iff there is some $V_1(U)$ and $V_2(U)$ s.t. $C(S) = C_2(C_1(S))$ [for all $S \subseteq U$].

The Project

The Question

What properties characterize two-tier choice functions?

The Answer

C is two-tier rationalizable iff C satisfies properties TTα, TTIC(1) and TTIC(2).

Method of Proof

$\Leftarrow$: If C satisfies TTα, TTIC(1) & TTIC(2) then C can be represented as a two-tier choice function.

1. Let C be a well-defined choice function.
2. Construct $V_1(U)$ and $V_2(U)$ according to a set of construction rules (CR1-CR8).
3. Show that CR1-CR8 produce acyclic orderings in $V_1(U)$ and $V_2(U)$.
4. Show that if CR1-CR8 are used to construct $V_1(U)$ and $V_2(U)$ then $C(S) = C_2(C_1(S))$ for all $S \subseteq U$.

Relevant Properties

TTP1: If $\{x, y\} \subseteq C(T)$ then $x \sim_2 y$.

TTP2: If $x \in T$ & $\forall z \in C(T) x \in C(\{x, z\})$ then $x \succeq_2 C(T)$.

TTP3: If $T \subseteq S$ and $x \in C(S) \cap T \setminus C(T)$ then $C(T) \succ_2 x$.

TTP4: $C(S) \subseteq C_1(S)$

TTP5: If $x \in T \setminus C(T)$ & $x \succeq_2 C(T)$ then $x \notin C_1(T)$.

Second-Tier Properties

TTP1: If $\{x, y\} \subseteq C(T)$ then $x \sim_2 y$.

TTP2: If $x \in T$ & $\forall z \in C(T) x \in C(\{x, z\})$ then $x \succeq_2 C(T)$.

TTP3: If $T \subseteq S$ and $x \in C(S) \cap T \setminus C(T)$ then $C(T) \succ_2 x$.

Constructing the Second Tier

Construction Rules

CR1a: If $\{x, y\} \subseteq C(T)$ then set $x \sim_2 y$.

CR1b: If $x \in T$ & $\forall z \in C(T)\ x \in C(\{x, z\})$ then set $x \succeq_2 C(T)$.

CR2: If $T \subseteq S$ and $x \in C(S) \cap T \setminus C(T)$ then set $C(T) \succ_2 x$.

CR3: Ensure the partial order is transitive.

CR4: Complete the ordering for all x-$C(T)$ pairs where $x \in T \setminus C(T)$.

Constructing $V_1(U)$

TTP4: $C(S) \subseteq C_1(S)$

TTP5: If $x \in T \setminus C(T)$ & $x \succeq_2 C(T)$ then $x \notin C_1(T)$.

Construction Rules – for each $S \subseteq U$

CR5: Set $C(S) \succeq_s S$.

CR6: If $x \in T \setminus C(T)$ & $x \succeq_2 C(T)$ (by CR1-CR4) then pick some appropriate $y \in T \setminus \{x\}$ and set $y \succ_s x$.

CR7: Ensure the (partial) orderings are transitive.

CR8: Complete the orderings (while maintaining transitivity).

Tedious CR6

CR6: If $x \in T \setminus C(T)$ & $x \succeq_2 C(T)$ (by CR1-CR4) then pick some appropriate $y \in T \setminus \{x\}$ and set $y \succ_s x$.

Relieving Tedium

If $x \in T \setminus C(T)$ & $x \succeq_2 C(T)$ (by CR1-CR4) then $y \in T \setminus \{x\}$ is 'appropriate' iff ??????

Theorem

C is two-tier rationalizable iff C satisfies properties TTα, TTIC(1) and TTIC(2).