Deductively Definable Logics of Induction

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Background Assumptions

The probability calculus is NOT the one universally applicable logic of induction. I am not a Bayesian.

There is NO universally applicable logic of induction. There are many logics, each specialized to the particular domains in which they are licensed (by facts).

The Project

Investigate a larger class of inductive logics suggested by rule:

Inductive strength $[A|B]$ for propositions A, B drawn from a Boolean algebra is defined fully by deductive structure of the Boolean algebra.

Find properties common to many logics of induction.

Plausibility of deductive definability

Induction as *partial* deduction.
Induction as *inverse* deduction.

Hypothetico-Deductive Confirmation

Evidence E confirms hypothesis H **if** H deductively entails E and further (deductively expressible?) ideas about explanation, simplicity, etc.

Instance confirmation

Instance I confirms generalization G **if** I is an instance of G.
Q(a) is an instance of (x)Q(x), etc.

Classical definition of probability

Probability measure induced on algebra of proposition **by** All atoms of algebra are equally probable.

Inductive notions — defined by — **Deductive** structure

Where we will end up

A No-Go theorem for a large class of inductive logics.

A viable class of inductive logics that are *deductively definable in preferred partitions and asymptotically stable* (includes probability calculus).

- Inductive independence is generic.
- There exist scale-free inductive logics.
- A limit theorem

Adapting an inductive logic to the deductive relations between propositions is actually responsible for many of the characteristics of inductive logics.

Deductive Notions

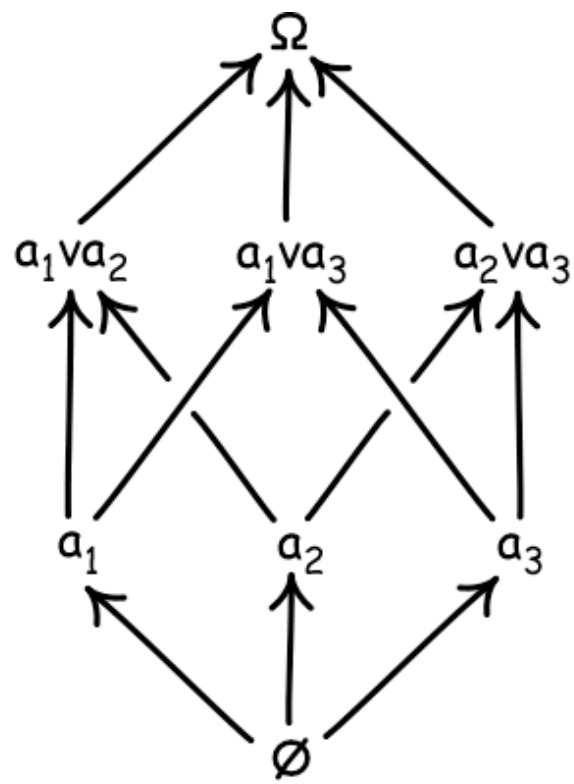
Finite Boolean Algebras

Goal: Lay bare the deductive structure of ordinary sentential logic, free of the distracting duplications: A = (AvA) = (A&A) = (Av (A&A)) = ….

All finite sets of sentences belong to one of a one, two, three, four, … atom Boolean algebra:

Three-atom algebra.

Atoms a_1, a_2, a_3 are the logically strongest propositions that are not ∅

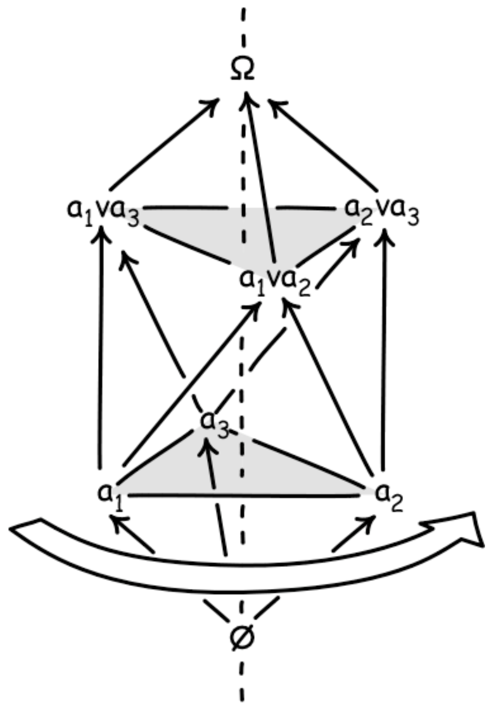


Finitely many propositions closed under ∨ (or), & (and), ~ (negation).

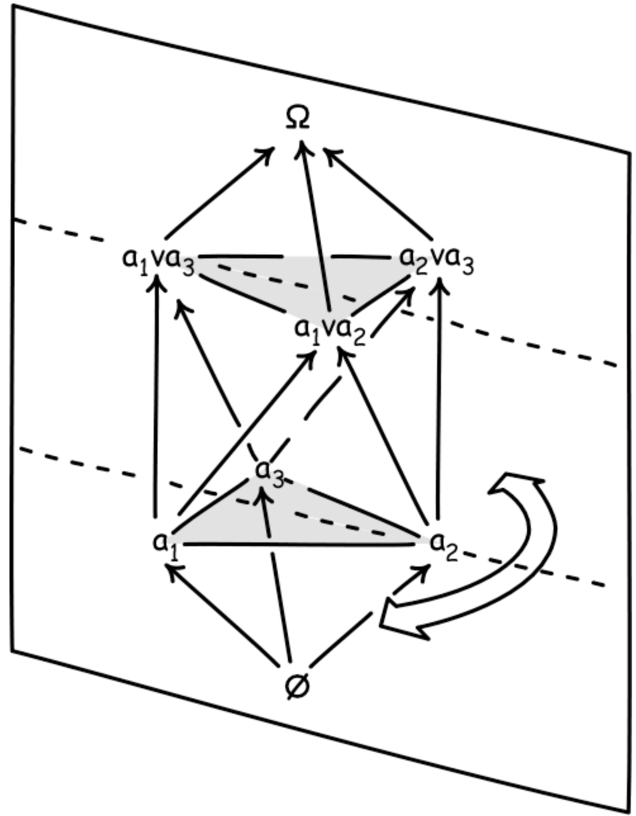
Ω universally true

∅ contradiction

Richness of Symmetries of a Boolean algebra



Cyclic permutation of atoms a_1, a_2, a_3



Exchange of atoms a_1 and a_3

Inductive Notions

Deductive Definability

Inductive
Strength of support

[A|B]

for propositions A and B

defined by

conditions that invoke
deductive structure
of the Boolean algebra.

A symmetry transformation of deductive structure does not change this.

Therefore a symmetry transformation of deductive structure does not change this.

So…

Symmetries of the deductive structure are also symmetries of the inductive logic.

Symmetry Theorem

Generalize…

$$[a_1 | a_1 \vee a_2] = [a_8 | a_8 \vee a_9]$$

They are the same under relabeling of atoms. Both are "one" conditioned on "two."

If the symmetries of the deductive structure of an N-atom Boolean algebra are also symmetries of the inductive structure,

then

$$[A|B] = f_N(\#A\&B, \#A\&{\sim}B, \#{\sim}A\&B)$$

"#" means "number of atoms."

Plausibility

All that matters in deductive structure is how many atoms are in each proposition and how many it shares with other propositions.

Illustration

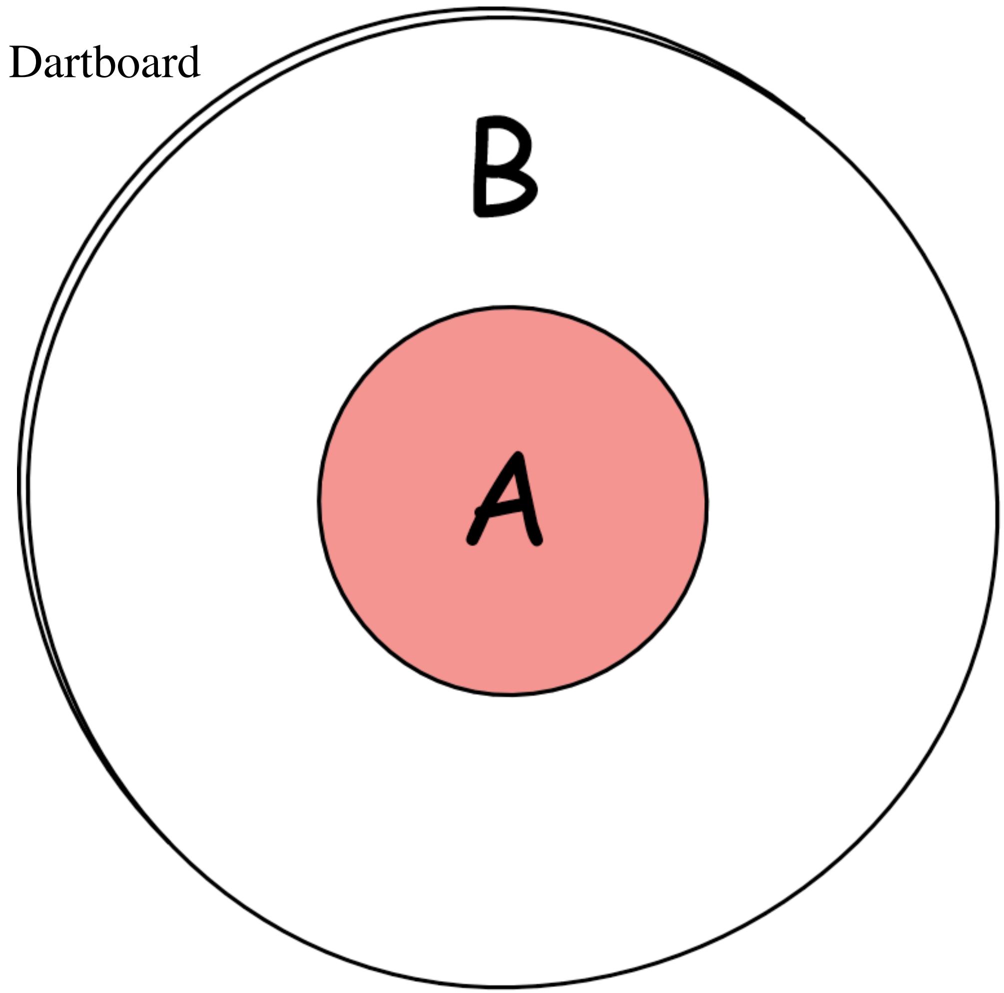
Classical probability in which the atoms are the equally likely cases

$$P(A|B) = \frac{\#A\&B}{\#B} = \frac{\#A\&B}{\#A\&B + \#{\sim}A\&B}$$

Deductively Definable, Asymptotically Stable

Inductive Logics

Deductive definability is not enough



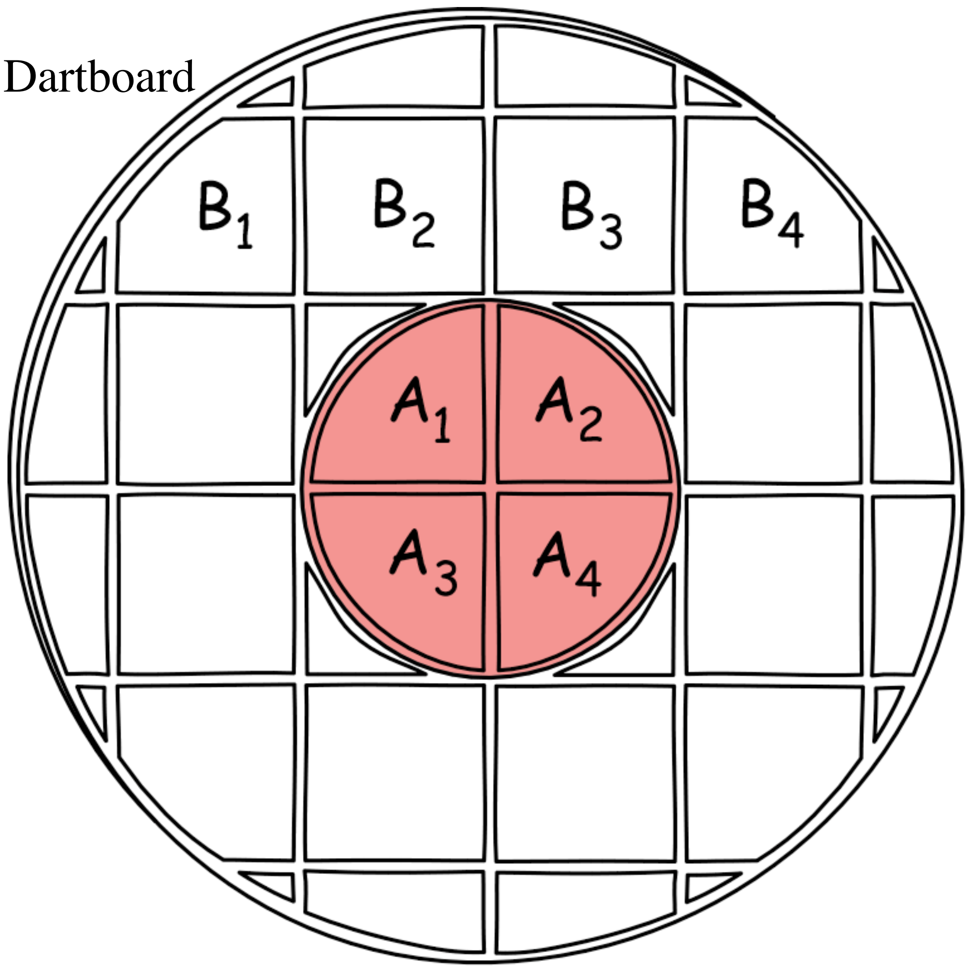
$[A \mid \Omega] = [B \mid \Omega]$

by symmetry theorem since each of A and B has just one atom.

Very small algebra.
Switch labels “A” as “B”.

Disjunctive refinement adds essential inductive information

Dartboard



$A = A_1 \vee A_2 \vee A_3 \vee A_4$

4 atoms

$B = B_1 \vee B_2 \vee \ldots \vee B_{32}$

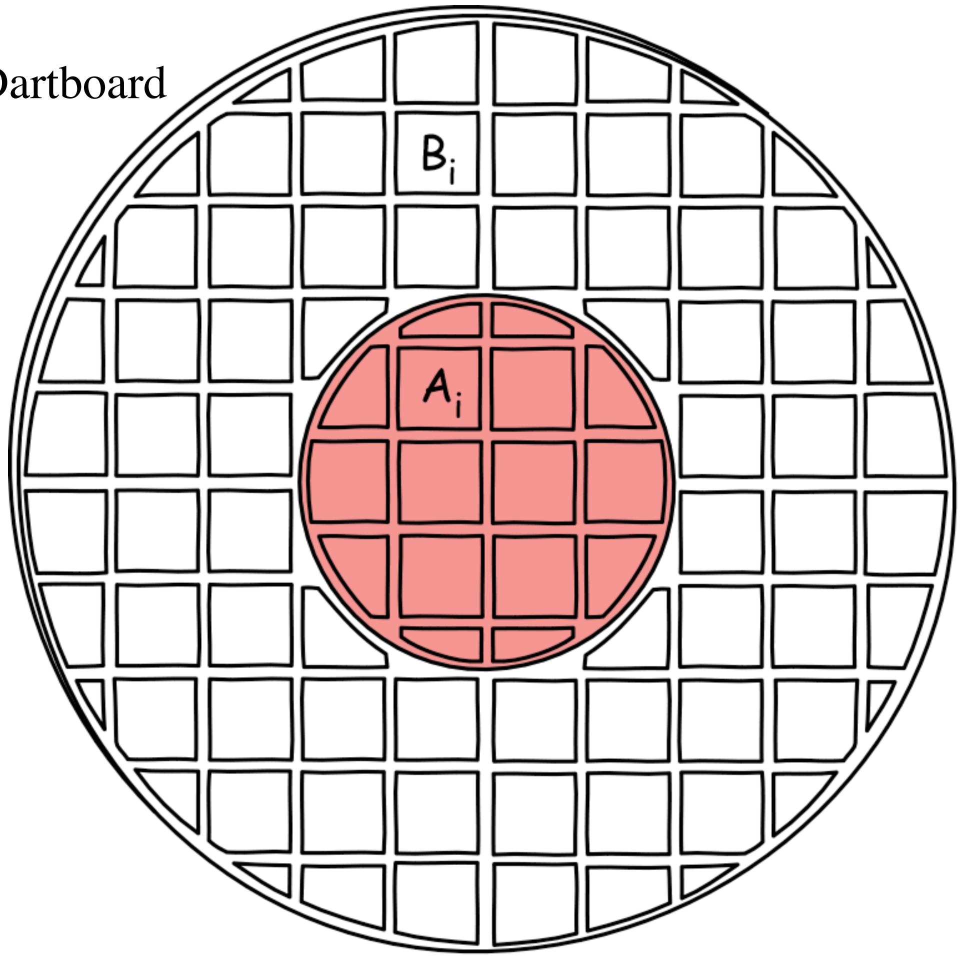
32 atoms

$[A \mid \Omega] < [B \mid \Omega]$

is now possible (depending on the form of the function f).

Further disjunctive refinement adds less and less inductive information

Dartboard



$A = A_1 \vee A_2 \vee \ldots$

16 atoms

$B = B_1 \vee B_2 \vee \ldots$

72 atoms

So consider inductive logics that are:

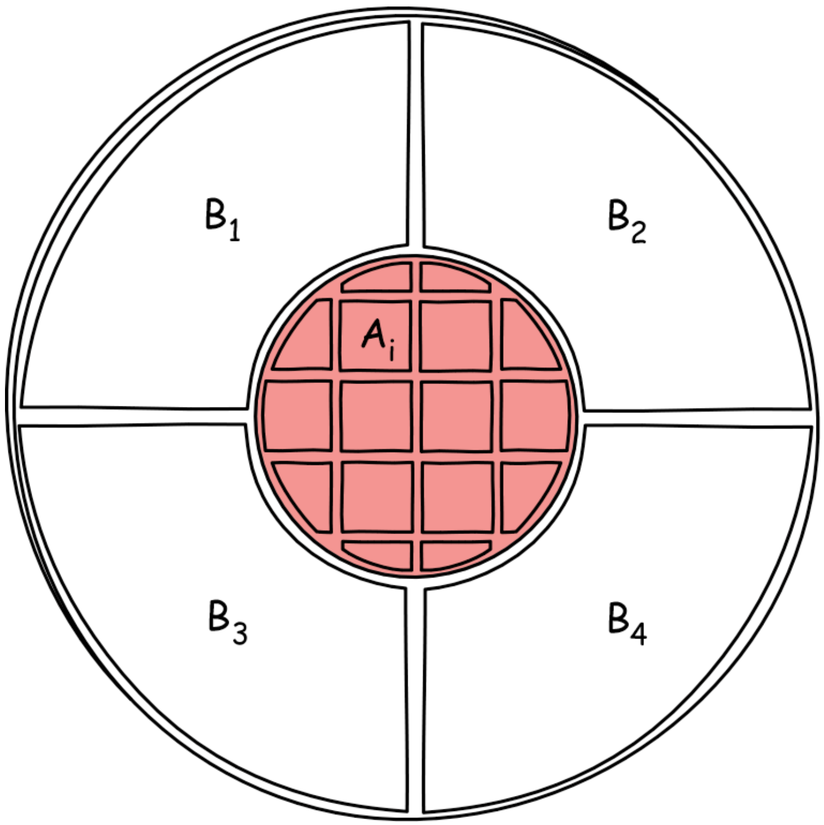
Deductively Definable.
The rules of the inductive logic are defined solely in terms of the deductive structure.

Asymptotically Stable under Disjunctive Refinement.
The logic stabilizes under repeated disjunctive refinement.

Learning that an outcome has disjunctive parts might initially affect our inductive assessments, but it will eventually become irrelevant--the splitting of logical hairs.

This class includes versions of:
hypothetico-deductivism,
instance confirmation,
classical approach to probability.

What if we refine poorly?



$A = A_1 \vee A_2 \vee \ldots$

16 atoms

$B = B_1 \vee B_2 \vee B_3 \vee B_4$

4 atoms

$[A \mid \Omega] > [B \mid \Omega]$

is now possible (depending on the form of the function f).

No-Go Theorem

The only *deductively definable, asymptotically stable* inductive logic assigns the same strength to all contingent propositions.

$[a_1 | $ [illegible] $] = [a_1 v\ a_2 | $ [illegible] $]$
$= [a_1 v\ a_2 v\ a_3 | $ [illegible] $] = \ldots$

Plausibility

$[a_1 | a_1 v\ a_2]$ vs
$[a_2 | a_1 v\ a_2]$

refine
a_1 to $b_1 v \ldots v b_{99}$

$[b_1 v \ldots v b_{99} | b_1 v \ldots v b_{99}\ v\ a_2]$ vs
$[a_2 | b_1 v \ldots v b_{99}\ v\ a_2]$

refine
a_2 to $c_1 v \ldots v c_{99}$

$[a_1 | a_1 v\ c_1 v \ldots v c_{99}]$ vs
$[c_1 v \ldots v c_{99} | a_1 v\ c_1 v \ldots v c_{99}]$

Escape: Add Essentially Inductive Content

Select special subset of partitions in which symmetries of deductive structure are also symmetries of the inductive logic.

Deductively Definable.
The rules of the inductive logic are defined solely in terms of the deductive structure IN INDUCTIVELY ADAPTED PARTITIONS.

Asymptotically Stable under Disjunctive Refinement.
The logic stabilizes under repeated disjunctive refinement.

Assume availability of inductively adapted partitions of arbitrarily large size.

Familiarity

Classical definition of probability.
Select preferred partitions in which atoms are equiprobable.

The Logics are…

In inductively adapted partitions

$$[A|B] = f_N(\#A\&B, \#A\&{\sim}B, \#{\sim}A\&B)$$

From symmetry theorem.

Each asymptotically stable function f defines a distinct logic.

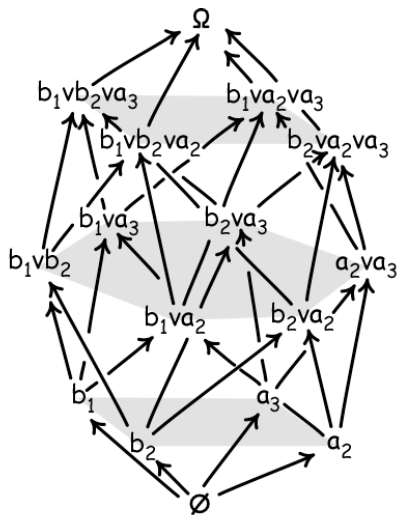
Illustration

Classical probability in which the atoms are the equally likely cases in preferred partitions.

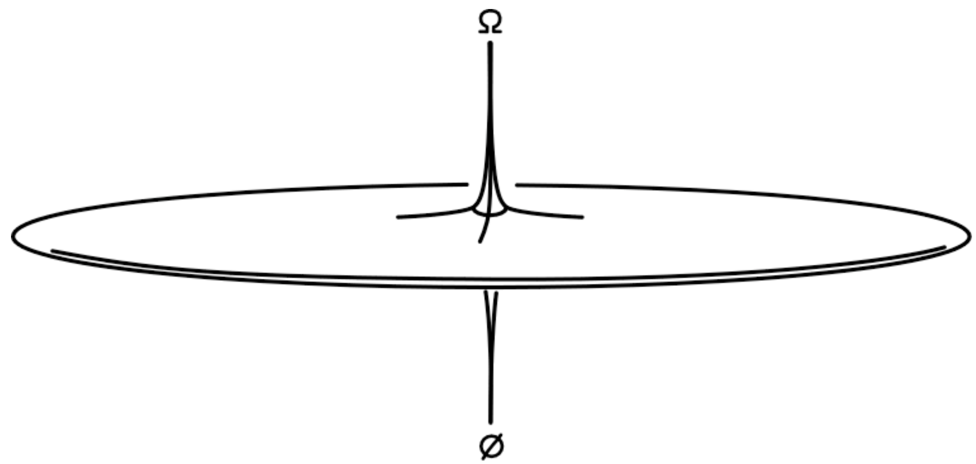
$$P(A|B) = \frac{\#A\&B}{\#B} = \frac{\#A\&B}{\#A\&B + \#{\sim}A\&B}$$

Inductive Independence is Generic

Fact about Deductive Structure of Boolean Algebra



Very small Boolean algebra



Very big Boolean algebra

Number of propositions in W_N with n atoms $= \dfrac{N!}{n!(N-n)!}$

Nearly all propositions have N/2 atoms…

…and nearly all pairs of propositions agree on N/4 atoms.

For most common pairs of propositions A and B

In inductively adapted partitions
$[A|B] = f_N(\#A\&B, \#A\&{\sim}B, \#{\sim}A\&B)$

$$[A|B] = f_N(N/4, N/4, N/4)$$
$$[A|{\sim}B] = f_N(N/4, N/4, N/4)$$

True for most common pairs. Very nearly true for most of the rest.

Therefore $[A|B] = [A|{\sim}B]$

Independence

Discharging the "very nearly" in a precise proof requires a lot of accountancy.

Scale-Free Inductive Logics

A Scale-Free Inductive Logic is…

…invariant under changes of scale.

Implement as:

Same value [A|B] at all scales.

Uniform refinements between inductively adapted partitions.

$\square_3 = (a, b, c)$

$\square_6 = (a_1, a_2, b_1, b_2, c_1, c_2,)$

Asymptotically stable inductive logics eventually stabilize under repeated disjunctive refinements.

Scale-free inductive logics are already at a limiting stable logic.

Theorem about Scale-Free Inductive Logics

In inductively adapted partitions

$$[A|B] = f_N(\#A\&B, \#A\&{\sim}B, \#{\sim}A\&B)$$

Family of functions f_N replaced by a single function g.

$$[A|B] = g(\#A\&B/N, \#A\&{\sim}B/N, \#{\sim}A\&B/N)$$

Sample Scale-Free Logics

$$[A|B] = \frac{\#A\&B}{\#B}$$ Probability

$$[A|B] = \frac{(\#A\&B)^2}{\#A.\#B}$$ "Specific conditioning" logic

$$[A|B] = \left(\frac{\#A\&B}{\#B}\right)^{m(B)}$$ where m(B) = (1+[illegible]) / (1+[illegible]-#B/N)   Partial ignorance logic

A Limit Theorem

for Narrow Inductive Logics

A Limit Theorem in the Probability Calculus

If H entails $E_1, E_2, \ldots, E_n$, and $P(H) > 0$

then $\text{Lim}_{n \to \infty} P(E_n \mid E_1 \& E_2 \& \ldots \& E_{n-1}) = 1$

Finitely many successful predictions E_{n-1} are enough to make us arbitrarily sure of the n-th consequence of hypothesis H.

A Limit Theorem

If an inductive logic is narrow,
H entails $E_1, E_2, \ldots, E_n$,
and [H|□] is not [null|□],

then $\mathrm{Lim}_{n□□}$ [E_n | E_1& E_2& …& E_{n-1}] = certainty

Narrowness.
[A|B] = [A&B|B]

It is an inductive expression of the deductive fact:
F_1=E_1,
F_2=E_1&E_2,
F_3=E_1&E_2 &E_3,
…
has a decreasing number of atoms that must always be greater than or equal to those of H.

Technical complication. Taking the limit is messy since all the algebras considered are finite. To achieve arbitrarily large n, we must repeatedly expand algebra as needed.

Conclusion

Conclusions

Deductively defined inductive logics, without inductive supplement, fail. (No-go theorem)

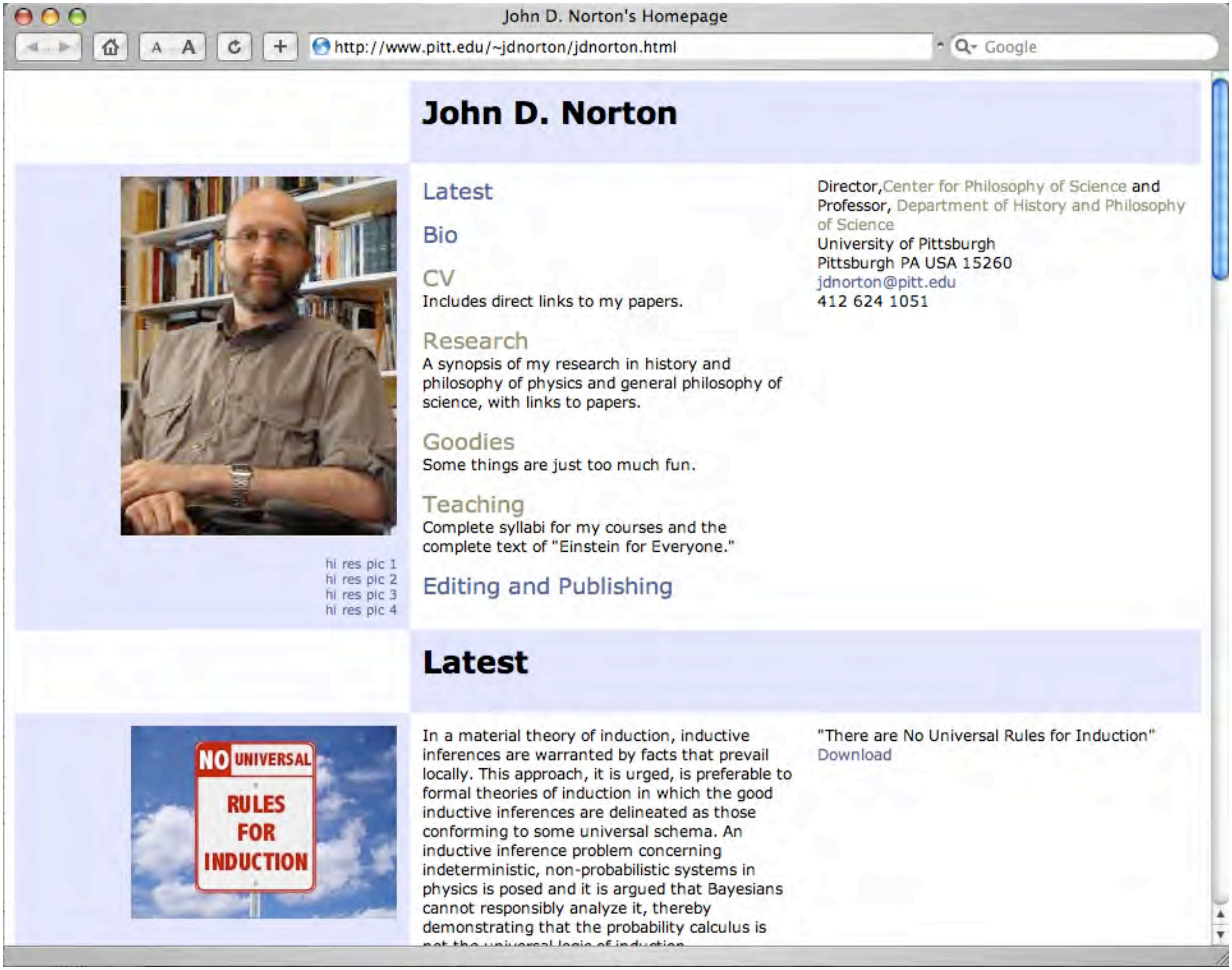
A viable class of inductive logics are *deductively defined in preferred partitions and asymptotically stable*.

Many characteristics of inductive logics are merely inductive reflections of facts in the deductive structures to which the logics are adapted.

Most propositions have N/2 atoms. → Inductive independence is generic.

Conjunctions of consequences of H are still consequences of H. → The limit theorem.

Read



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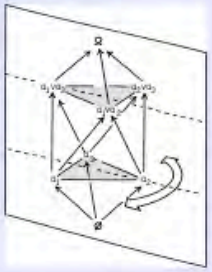
John D. Norton's Homepage

http://www.pitt.edu/~jdnorton/jdnorton.html

Latest

In a material theory of induction, inductive inferences are warranted by facts that prevail locally. This approach, it is urged, is preferable to formal theories of induction in which the good inductive inferences are delineated as those conforming to some universal schema. An inductive inference problem concerning indeterministic, non-probabilistic systems in physics is posed and it is argued that Bayesians cannot responsibly analyze it, thereby demonstrating that the probability calculus is not the universal logic of induction.

"There are No Universal Rules for Induction" Download



What if, like me, you don't think that the probability calculus is the One, True Logic of Induction? Then you want to know what other logics are possible. Here I map out a large class of inductive logics that originate in the idea that the inductive support B affords A, that is "[A|B]," is defined in terms of the deductive relations among propositions. I demonstrate some very general properties for these logics. In large algebras of propositions, for example, inductive independence is generic in all of them. A no-go result forces all the logics to supplement the deductive relations among propositions with intrinsically inductive structures.

"Deductively Definable Logics of Induction" Download.

For a less formal development, see "What Logics of Induction are There?" in Goodies.

While Bayesian analysis has enjoyed notable success with many particular problems of inductive inference, it is not the one true and universal logic of induction. I review why the Bayesian approach fails to provide this universal logic of induction. Some of the reasons arise at the global level through the existence of competing systems of inductive logic. Others emerge through an examination of the individual assumptions that, when combined,

"Challenges to Bayesian Confirmation Theory," Prepared for Prasanta S. Bandyopadhyay and Malcolm Forster (eds.), *Philosophy of Statistics: Vol. 7 Handbook of the Philosophy of Science*. Elsevier. Download draft.

0. Logics of Induction

http://www.pitt.edu/~jdnorton/Goodies/logics_induction/index.html

What Logics of Induction are There?

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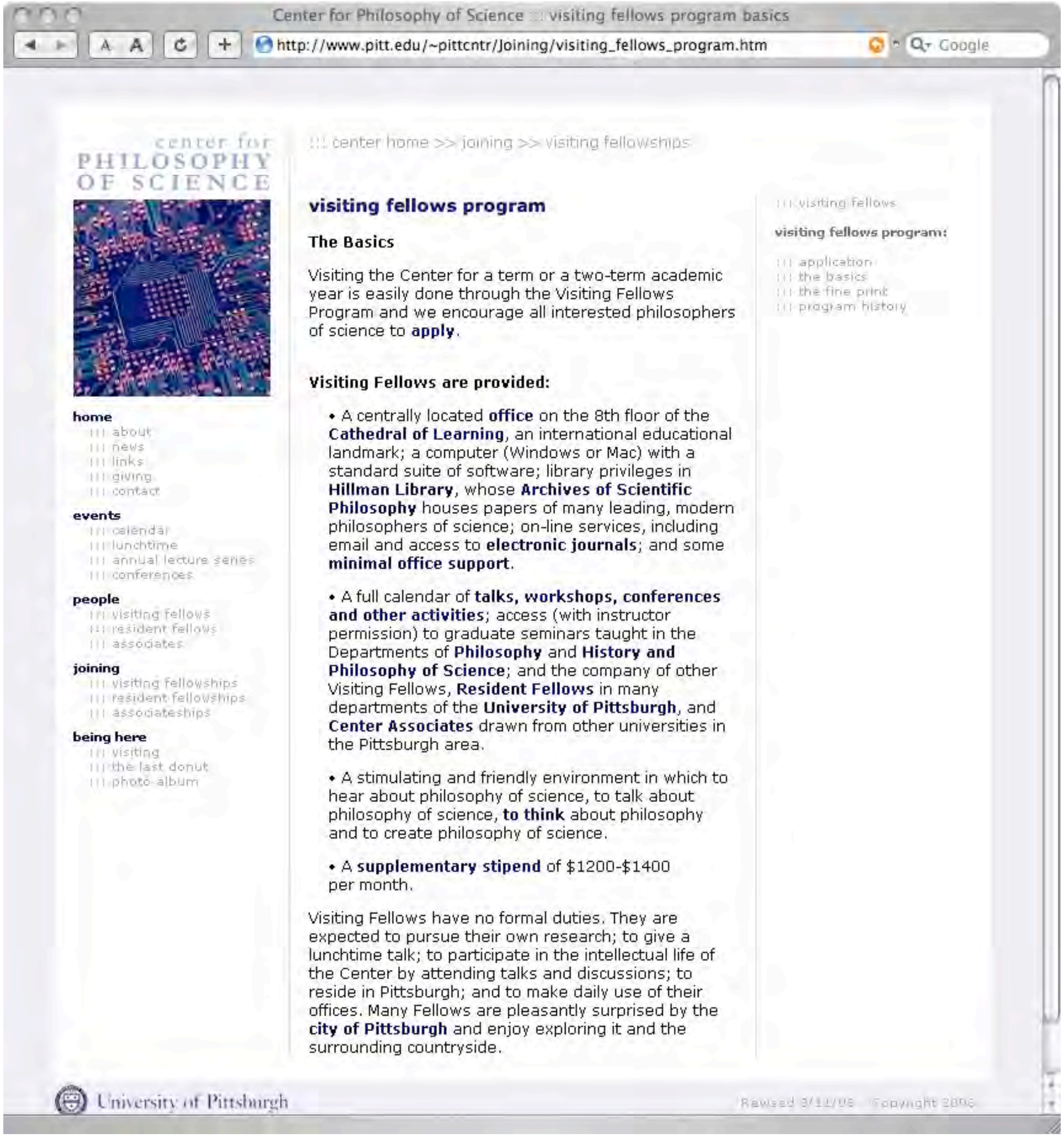
This page at www.pitt.edu/~jdnorton/Goodies

Most work on inductive logic is restricted to considering just one logic of induction that happens to be favored by the researcher. Since I don't think any one of these is the One True Logic, my goal is to broaden the perspective by considering a large class of inductive logics. A very common way of specifying inductive import in the literature is through an examination of the deductive relations among propositions. This suggests that we delineate a class of inductive logics that are "deductively definable." It will be shown here that no purely deductively definable logic is admissible, so that many favored approaches to induction in the literature fail. They all need some sort of inductive supplement. I show a simple way to provide this inductive supplement and display the general properties of the resulting class of inductive logics.

The ideas presented here are informal versions of material developed more precisely in my "Deductively Definable Logics of



Center for Philosophy of Science ::: visiting fellows program basics

http://www.pitt.edu/~pittcntr/Joining/visiting_fellows_program.htm

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visiting fellows program

The Basics

Visiting the Center for a term or a two-term academic year is easily done through the Visiting Fellows Program and we encourage all interested philosophers of science to **apply**.

Visiting Fellows are provided:

- A centrally located **office** on the 8th floor of the **Cathedral of Learning**, an international educational landmark; a computer (Windows or Mac) with a standard suite of software; library privileges in **Hillman Library**, whose **Archives of Scientific Philosophy** houses papers of many leading, modern philosophers of science; on-line services, including email and access to **electronic journals**; and some **minimal office support**.
- A full calendar of **talks, workshops, conferences and other activities**; access (with instructor permission) to graduate seminars taught in the Departments of **Philosophy** and **History and Philosophy of Science**; and the company of other Visiting Fellows, **Resident Fellows** in many departments of the **University of Pittsburgh**, and **Center Associates** drawn from other universities in the Pittsburgh area.
- A stimulating and friendly environment in which to hear about philosophy of science, to talk about philosophy of science, **to think** about philosophy and to create philosophy of science.
- A **supplementary stipend** of $1200-$1400 per month.

Visiting Fellows have no formal duties. They are expected to pursue their own research; to give a lunchtime talk; to participate in the intellectual life of the Center by attending talks and discussions; to reside in Pittsburgh; and to make daily use of their offices. Many Fellows are pleasantly surprised by the **city of Pittsburgh** and enjoy exploring it and the surrounding countryside.

::: visiting fellows

visiting fellows program:
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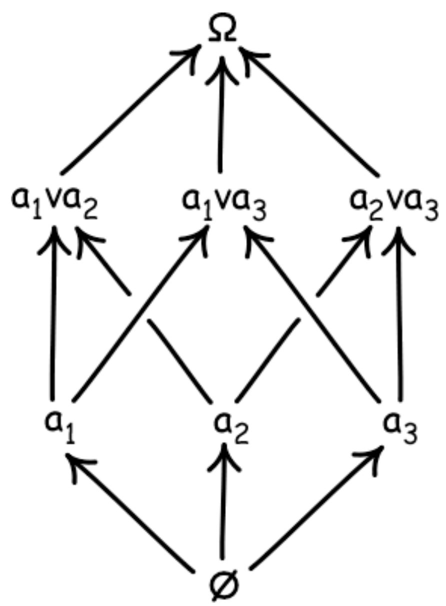
Appendices

How to make a Boolean Algebra Bigger

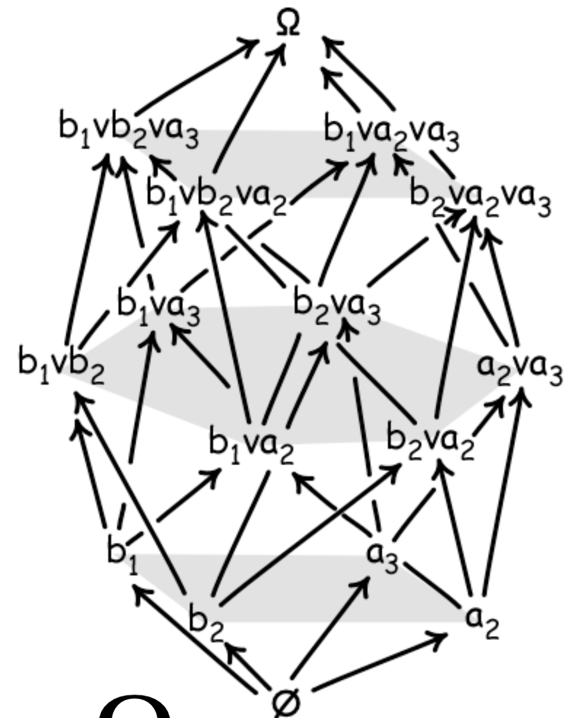


Ω_3

Atoms a_1, a_2, a_3

disjunctive refinement
a_1 expanded to b_1v b_2

disjunctive coarsening
b_1v b_2 collapsed to a_1



Ω_4

Atoms b_1, b_2, a_2, a_3

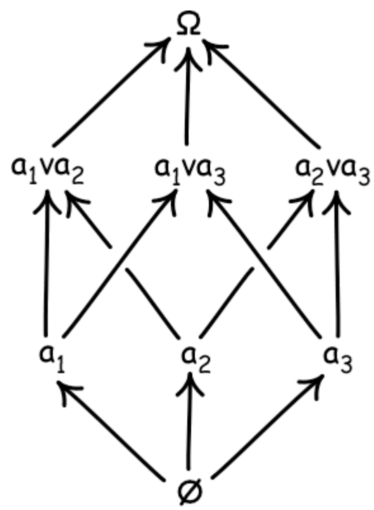
Why bother?

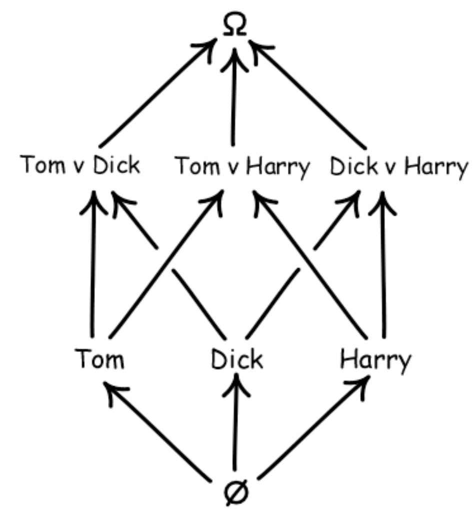
We can now proceed as far as we like along an unbounded sequence of propositions A_1, A_2, A_3, A_4, … without ever needing a single algebra with infinitely many propositions.

Symmetries of a Boolean algebra

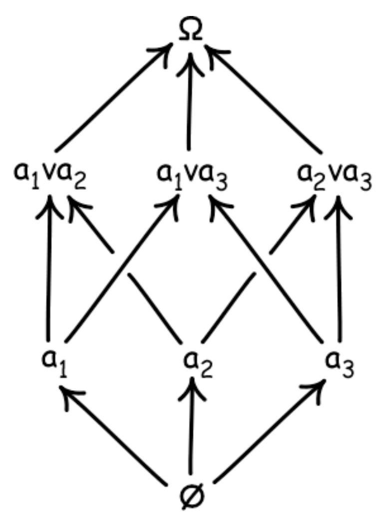
Same deductive structure if we…

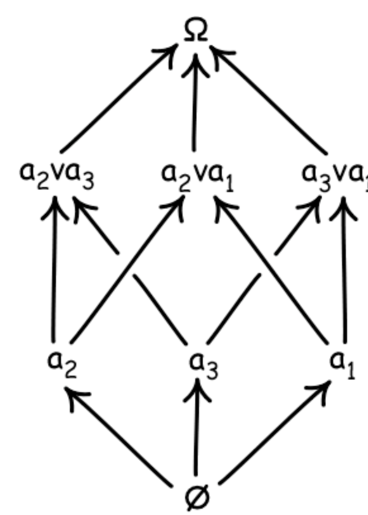
… relabel the atoms arbitrarily.





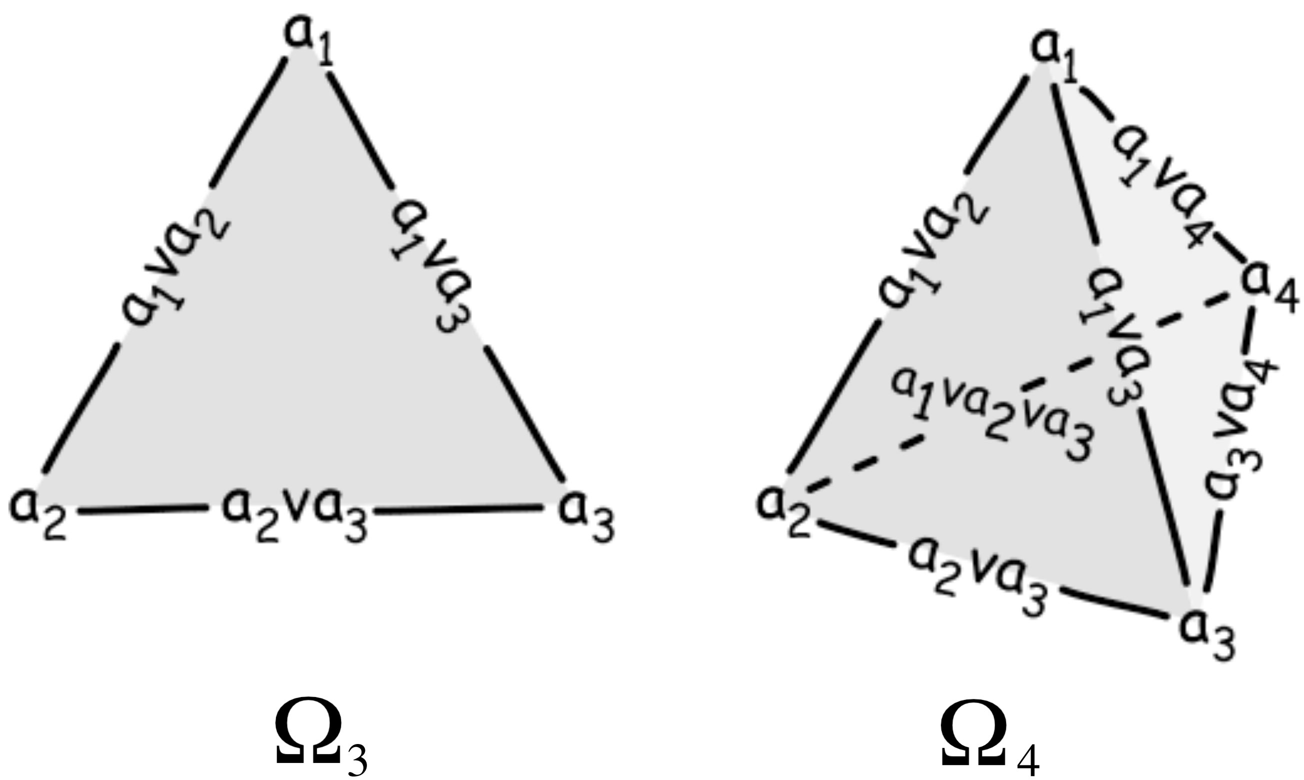
… permute the atomic labels arbitrarily.





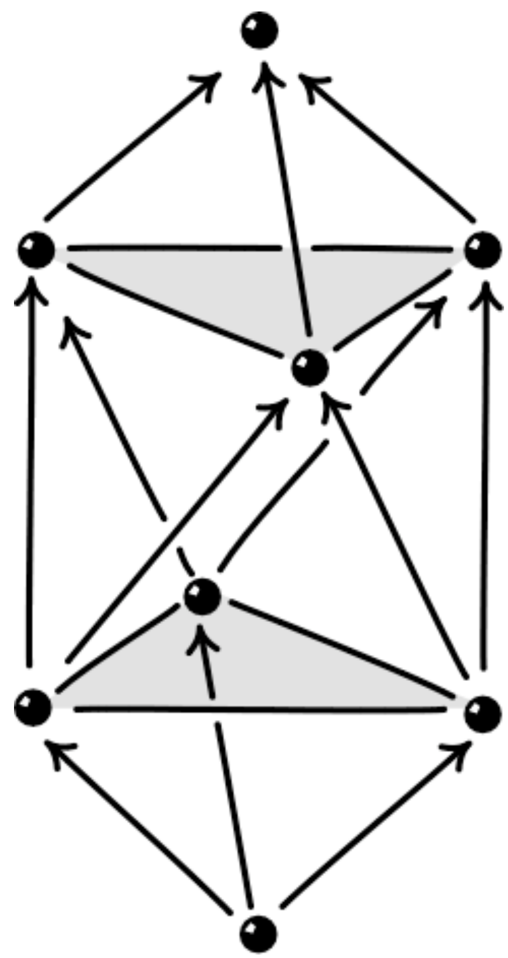
Richness of Symmetries of a Boolean algebra

Represented through simplices.

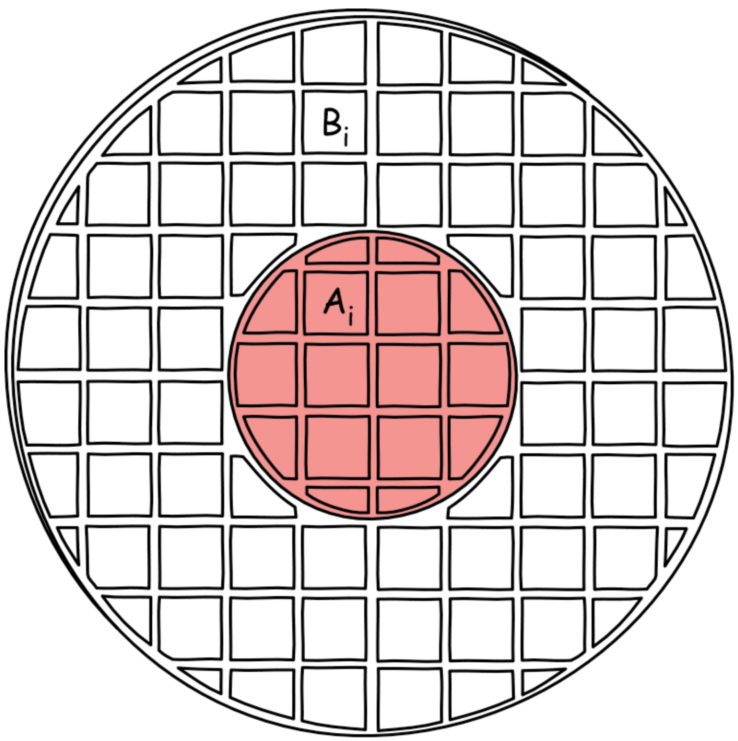


Bare Picture of the Deductive Structure of a Boolean Algebra

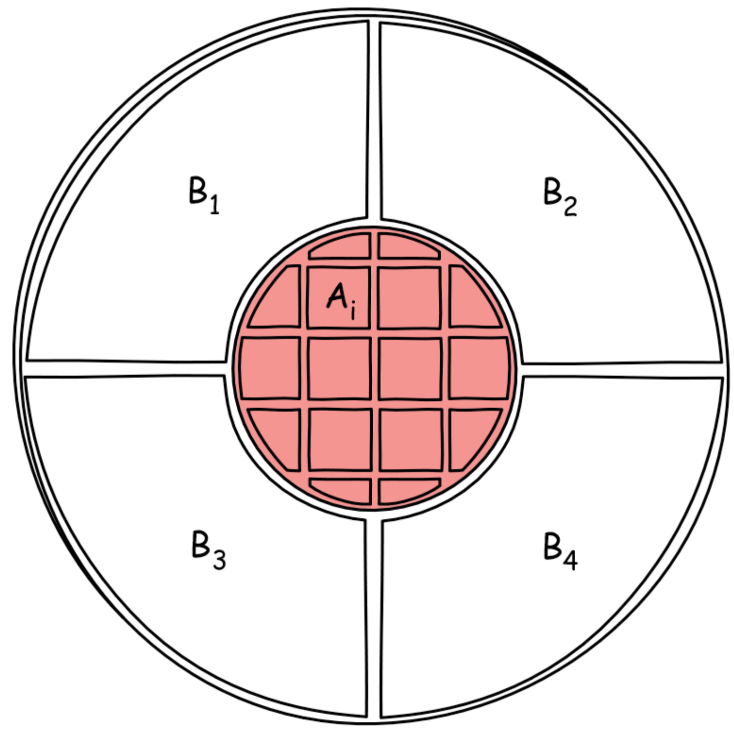
After symmetries have washed away surplus structure.



Adapted Partition

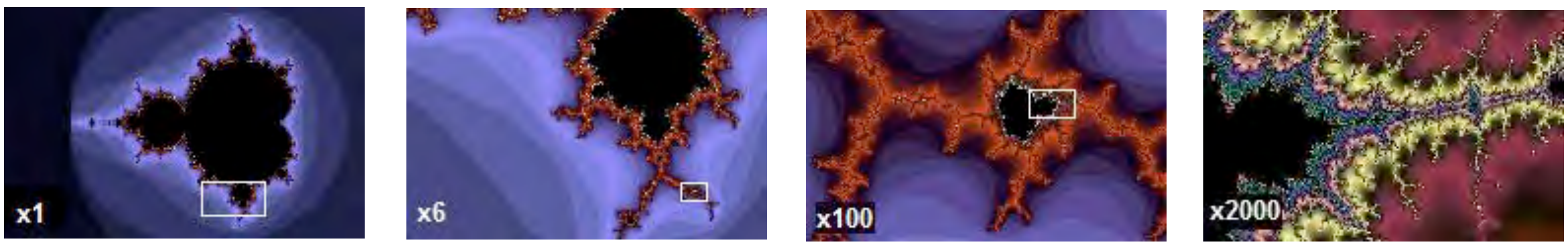


Not Adapted Partition



Familiar Examples

The Mandelbrot set is self-similar under magnification.



Images from “Fractal,” http://en.wikipedia.org/wiki/Fractal

Scale-free network

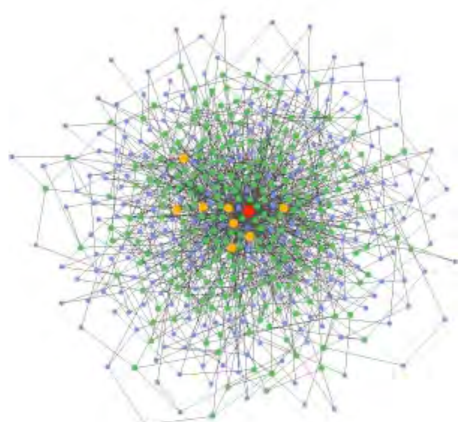
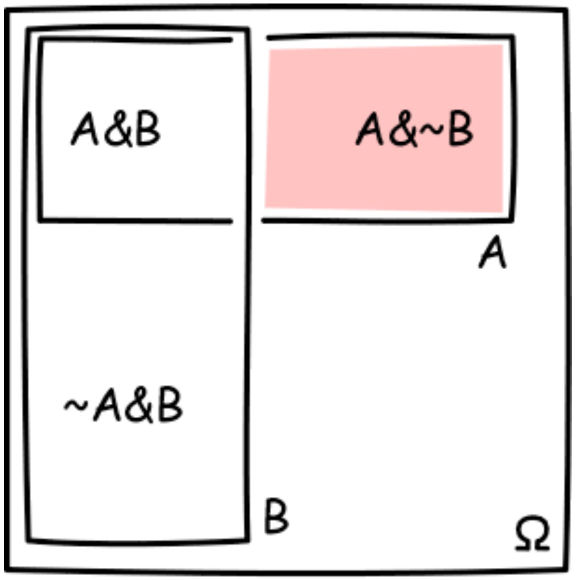


Image from http://home.uchicago.edu/~poikonom/research.html

Narrowness of Conditional Probability

$$P(A|B) \;=\; P(A\&B \vee A\&{\sim}B \mid B) \;=\; P(A\&B \mid B)$$

no contribution
to conditional
probability



For some unknown animal:

P(canary v whale | bird) = P(canary | bird)

P(canary v whale | canary) = P(canary | canary)

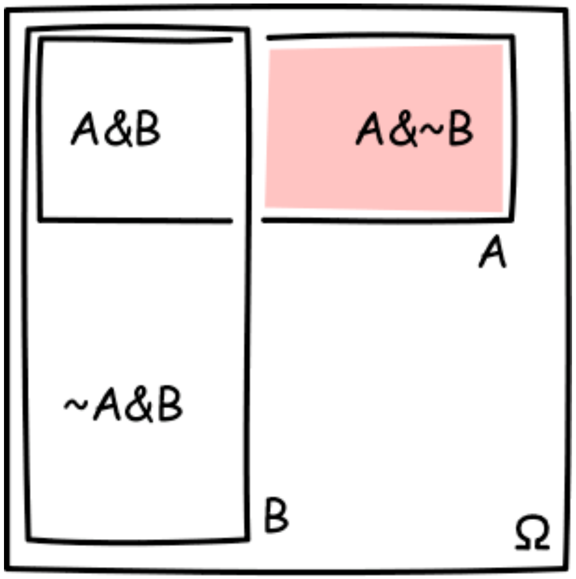
Isn't there more to say? In both cases the total evidence points more *specifically* to "canary." We discount "whale" and think "canary."

Specific Conditioning Logic

$$[A|B] = \frac{(\#A\&B)^2}{\#A.\#B} = \frac{\#A\&B}{\#B} \cdot \frac{\#A\&B}{\#A}$$

alone yields ordinary conditional probability

penalizes A for extending beyond total evidence B



For some unknown animal:

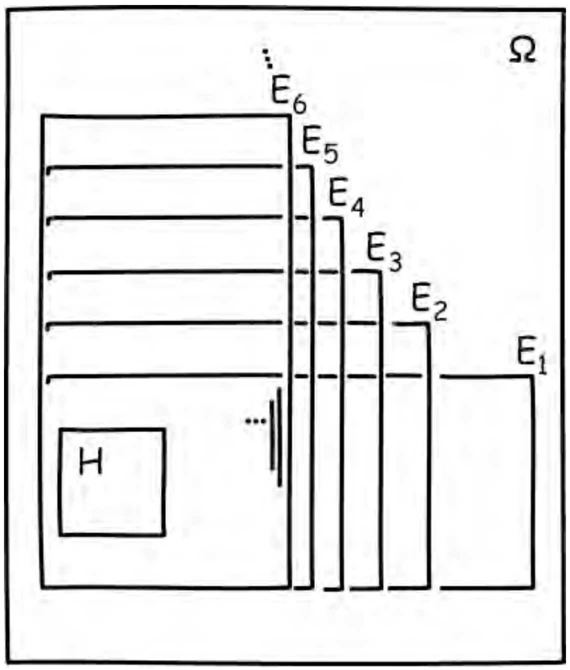
[canary v whale | bird] < [canary | bird]

[canary v whale | canary] < [canary | canary]

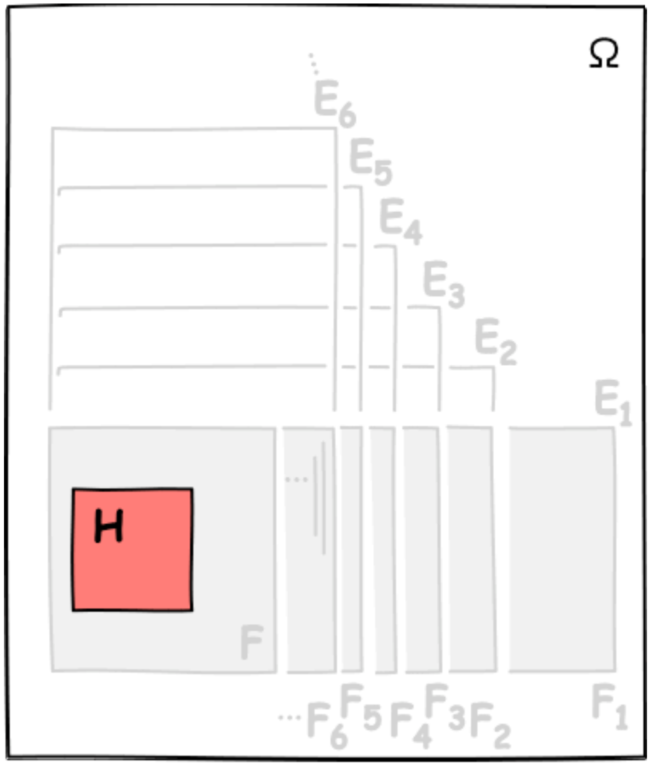
Symmetry [A|B] = [B|A] A penalized equally for extending beyond B and failing to exhaust B.

Theorem Depends on a Fact of Deductive Structure:

$E_1, E_2, E_3, \ldots$



Intersections
$F_1=E_1$, $F_2=E_1\&E_2$, $F_3=E_1\&E_2\&E_3$, …
approach a limit F that is a deductive consequence of H.



Re-express this fact in a more general inductive logic…