Comment on
"Probable Probabilities"
by John L. Pollock

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Professor Pollock addresses the following question: suppose $p\{P \mid Q\}$ and $p\{P \mid R\}$ are known, is there a reasonable estimate of the value of $p\{P \mid Q \text{ and } R\}$. To address this question and many others, he reaches essentially to the principle of insufficient reason.

This principle, championed by Laplace and dominant in the 19^{th} century, says that, absent other information, one should derive probabilities by the proportion of favorable instances. This is the same idea that underlies Pollock's "assumption of finite proportions," and which he proposes to extend to infinite sets. As he points out, this principle has attracted attention from 20^{th} century philosophers such as Carnap and Reickenback. Kyburg and Pollock use it with respect to sets of probabilities.

However, the dominant trend, both in philosophy and in statistics, has been away from this principle. One reason, simply stated, is that the term "instance" is ambiguous. For example, consider two flips (not necessarily independent) of a (not necessarily fair) coin. By one method of accounting, the possible outcomes are HH, HT, TH and TT. If these are regarded as the "instances" to which the principle of insufficient reason is applied, they each have probability $\frac{1}{4}$, so in particular $p(HH) = \frac{1}{4}$.

However, there is another method of accounting, which says that the possible outcomes are no heads, one head, and two heads. If these are regarded as the "instances" to which the principle of insufficient reason is applied, they each have probability $\frac{1}{3}$, and in particular $p(\text{two heads}) = \frac{1}{3}$.

Why is one method of accounting correct, and the other one wrong? I think the most firmly based view of probability, given what we know know, is the personal or subjective view associated with Ramsey, deFinetti and Savage. In this interpretation, probability represents the personal uncertainty of a decision-maker. Avoidance of sure loss leads to the usual probability axioms (leaving aside the issue of finite versus countable additivity). From this perspective, there is nothing wrong with second-order probabilities – they are simply parameters in a hierarchical model. Since the axioms of probability allow $p(P \mid Q \text{ and } R)$ to be any number between 0 and 1, each such choice is coherent, and Pollock's preferred number,

$$Y(r, s \mid a) = \frac{rs(1-a)}{a(1-r-s)+rs}$$

where $a = p(P \mid Q) = r$ $p(P \mid R) = s$ and $p(P) = a$ is coherent, but has no special claim on our beliefs. Thus within this framework, the issue of not having a value for $p(P \mid QR)$ is an issue of incomplete elicitation.