Communication and Structured Correlation

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Philosophical/Scientific questions

1. Why is signaling stable?
2. How can reliable signaling come to emerge?
   - Under what conditions?
   - What learning/evolutionary processes?
   - Is there a difference in efficiency among different situations/processes?
3. What is the correct methodology?
   - Static equilibrium analysis and Nash refinements?
   - Dynamic systems?
4. What constitutes signaling?
5. Is there a continuity (or similarity) between human language and this type of signaling?

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   - Static equilibrium analysis and Nash refinements?
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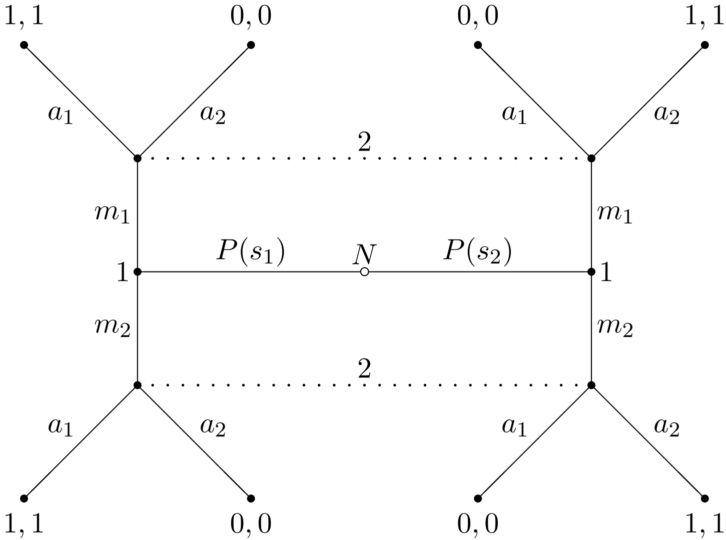
5.

Outline

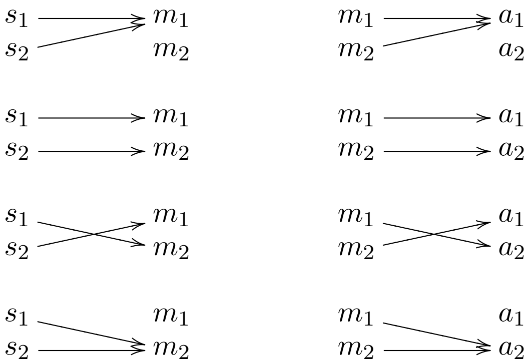
Sender-receiver games



Sender-receiver games

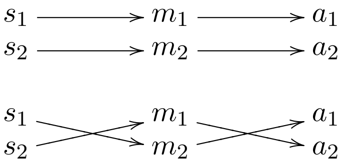
Sender-receiver games

Strategies . . .



Sender-receiver games

Signaling systems …



Sender-receiver games

With $P(s_1) = .5 \ldots$

	R_1	R_2	R_3	R_4
S_1	$\mathbf{.5, .5}$	$.5, .5$	$.5, .5$	$\mathbf{.5, .5}$
S_2	$.5, .5$	$\mathbf{1, 1}$	$0, 0$	$.5, .5$
S_3	$.5, .5$	$0, 0$	$\mathbf{1, 1}$	$.5, .5$
S_4	$\mathbf{.5, .5}$	$.5, .5$	$.5, .5$	$\mathbf{.5, .5}$

Sender-receiver games

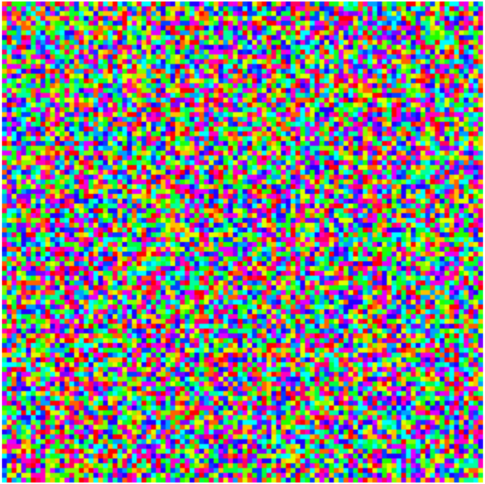
$n \times n \times n$:

- Signaling systems are the only ESS (Wärneryd 1993)

$2 \times 2 \times 2$, equiprobable states:

- Discrete-time replicator dynamic simulations always converge to signaling systems (Skyrms 1996)
- Set of points that do not converge to a signaling system has Lebesgue measure zero in the replicator dynamic (Huttegger 2007)
- On a grid lattice with the imitate-the-best dynamic, signaling systems always evolve (Zollman 2005)

Regional meaning



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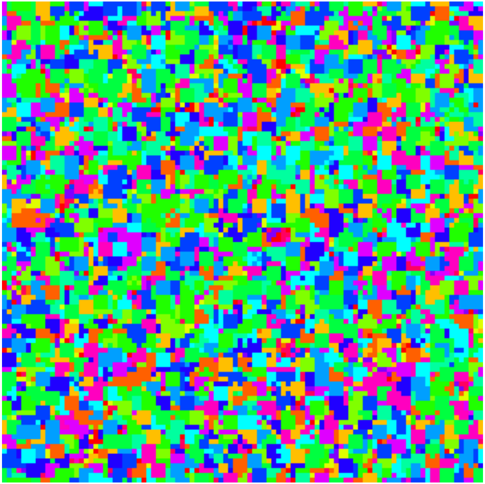

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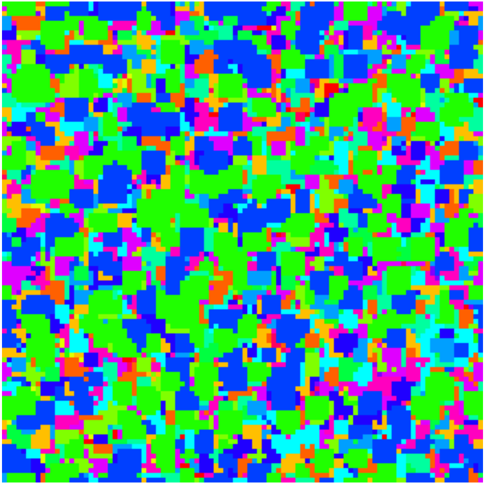

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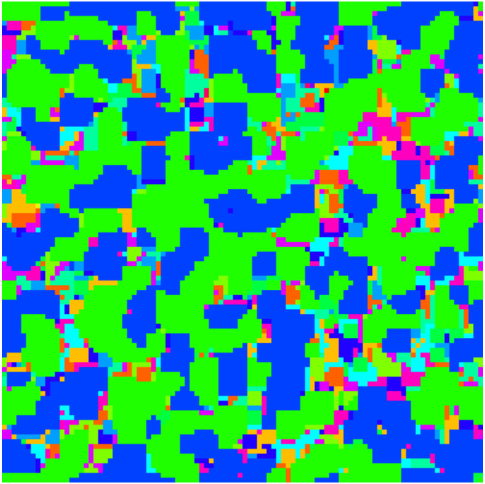

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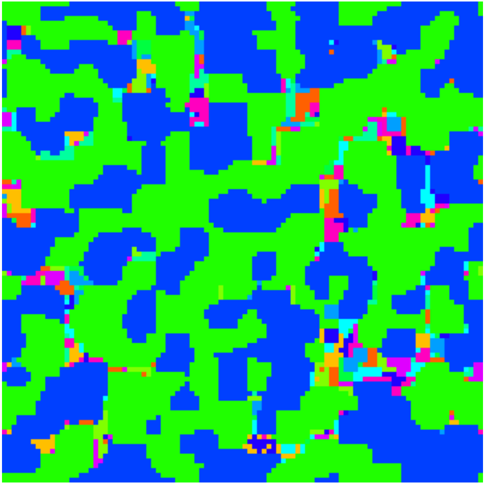

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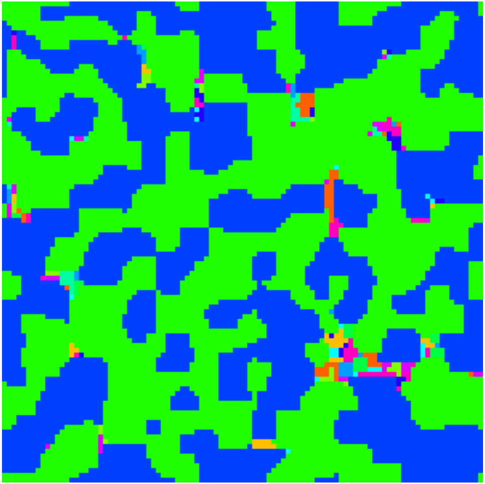

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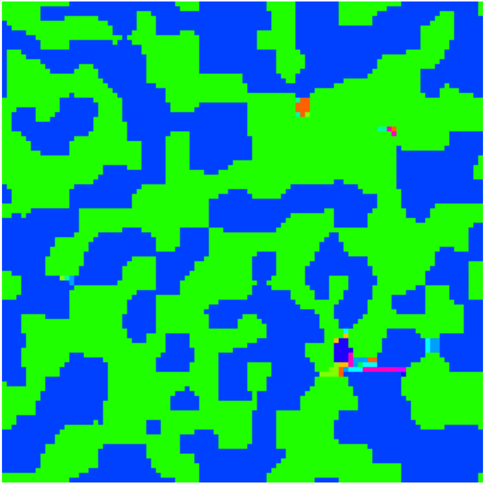
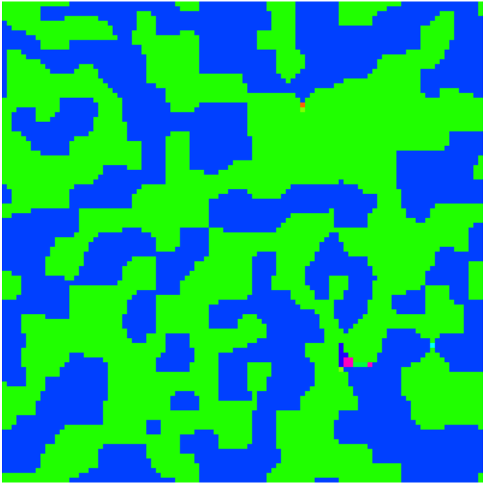


Regional meaning



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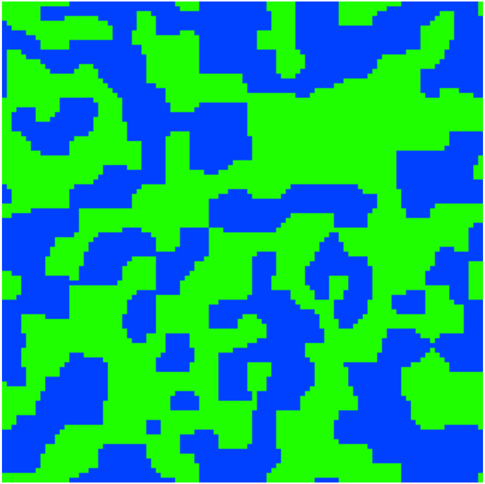


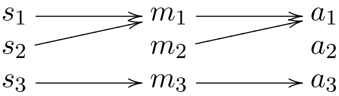
Sender-receiver games

Two complications . . .

- Partial pooling equilibria (Huttegger et al. 2009)
- Total pooling (babbling) equilibria (Huttegger 2007)

Partial pooling

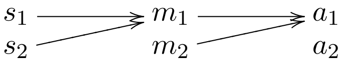
When $n > 2$, partial pooling equilibria attract a set of initial conditions with positive Lebesgue measure.



With $n = 3$, 4.7% of initial conditions converge to partial pooling under the discrete-time replicator dynamic.

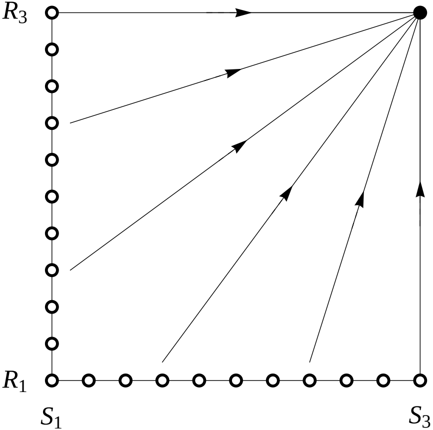
Babbling

When $P(s_1) \neq P(s_2)$, babbling equilibria attract a set of initial conditions with positive Lebesgue measure.

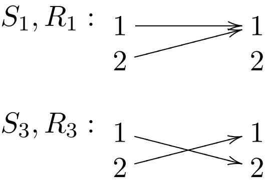


Babbling

$$P(s_1) = P(s_2) = .5$$

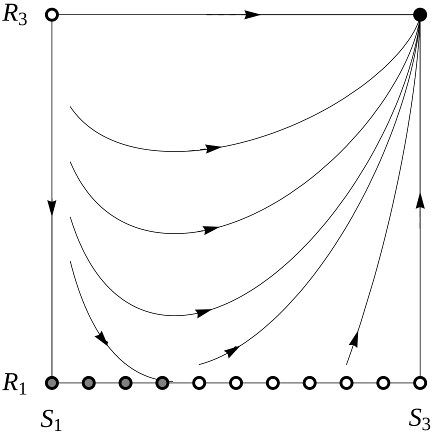


	R_1	R_3
S_1	**.5, .5**	.5, .5
S_3	.5, .5	**1, 1**

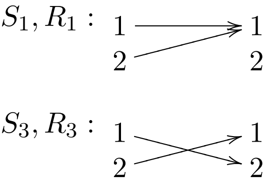


Babbling

$$P(s_1) = .6$$

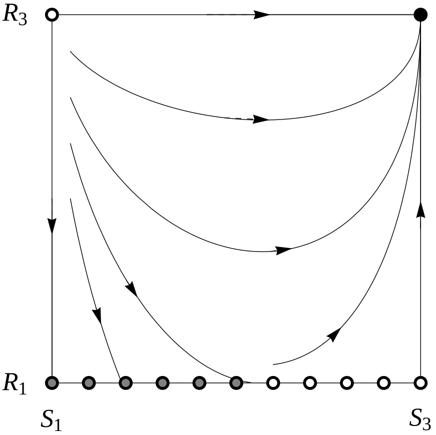


	R_1	R_3
S_1	**.6, .6**	.4, .4
S_3	.6, .6	**1, 1**



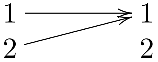
Babbling

$$P(s_1) = .7$$

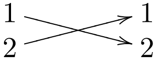


	R_1	R_3
S_1	**.7, .7**	.3, .3
S_3	.7, .7	**1, 1**

S_1, R_1 :



S_3, R_3 :



Local interaction

Local interaction

- allows cooperation to emerge and persist
- increases the likelihood of stag hunting
- aids the emergence of fair division in bargaining

Perhaps local interaction also assists the evolution of perfect communication in these unfavorable situations.

If so, we'd like to know why. In particular, what features of graph topology aid the evolution of communication?

$n > 2$ – simulations

$n = 3$

- 100×100 lattice: $\frac{10{,}000}{10{,}000}$ trials stabilized with regional signaling systems
- Discrete-time replicator: $\sim 95.3\%$

$n = 4$

- $1,000 \times 1,000$ lattice: $\frac{1{,}000}{1{,}000}$ trials stabilized with regional signaling systems
- Discrete-time replicator: $\frac{27}{38} \approx 71\%$

$n = 5$

- $10,000 \times 10,000$ lattice: $\frac{75}{75}$ trials stabilized with regional signaling systems
- Discrete-time replicator: ??

$n > 2$ – simulations

$n \times n \times n$ sender-receiver game:

- n^n sending strategies, n^n receiving strategies
- n^{2n} sending-receiving pairs
- But only $n!$ signaling systems

In the initial configuration, each agent uses a signaling system with probability $\frac{n!}{n^{2n}}$.

With $n = 10$, a $10^7 \times 10^7$ lattice is expected to contain less than 4 agents using signaling system strategies.

$n > 2$ – analysis

The key to a region's stability is interaction along the frontiers.

Case 1:

S	S	*S*
P	**P**	S
P	P	S

Case 2:

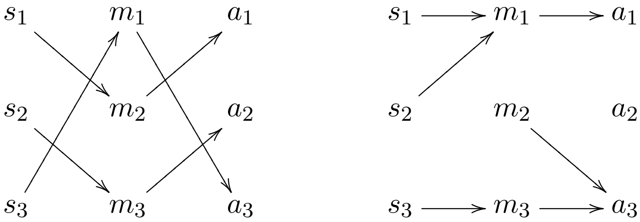
S	S	S
S	*S*	S
P	**P**	P

Case 3:

P	S	S
P	*S*	S
P	P	P

$n > 2$ – analysis

A signaling system and a partial pooling equilibrium that perfectly miscommunicate.



$n > 2$ – analysis

An interior partial pooler earns an average payoff of $\frac{2}{3}$.

A frontier signaler earns 1 when interacting with another signaler, and 0 when interacting with a partial pooler.

Frontiers are stable when interior partial poolers earn a higher average payoff than frontier signalers.

$n > 2$ – analysis

Case 1:	Case 2:	Case 3:
S S *S*	S S S	P S S
P **P** S	S *S* S	P *S* S
P P S	P **P** P	**P** P P
S's payoff $= \frac{14}{16}$	$\frac{10}{16}$	$\frac{6}{16}$
P imitates S	**P** imitates P	**P** imitates P

$n > 2$ – analysis

What if some information is exchanged? Supposed the signaler and the partial pooler get it right $\frac{1}{3}$ of the time . . .

Case 1:

S	S	*S*
P	**P**	S
P	P	S

S's payoff $= \frac{43}{48} \approx .9$

P imitates S

Case 2:

S	S	S
S	*S*	S
P	**P**	P

$\frac{11}{16} \approx .69$

P imitates S

Case 3:

P	S	S
P	*S*	S
P	P	P

$\frac{23}{46} \approx .48$

P imitates P

$n > 2$ – analysis

Regions of partial pooling can be stable on the Moore-8 toroidal lattice provide that:

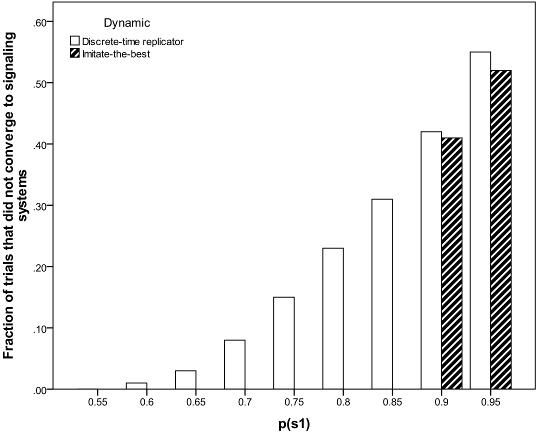
1. The partial poolers exchange no information with their neighboring signalers

   –and–

2. The region either occupies an entire slice of the toroidal lattice (case 2) or surrounds a rectangle of signalers (case 2+3)

Simulation results indicate that it is highly unlikely that the two conditions are ever simultaneously fulfilled.

$P(s_1) \neq P(s_2)$ – simulations



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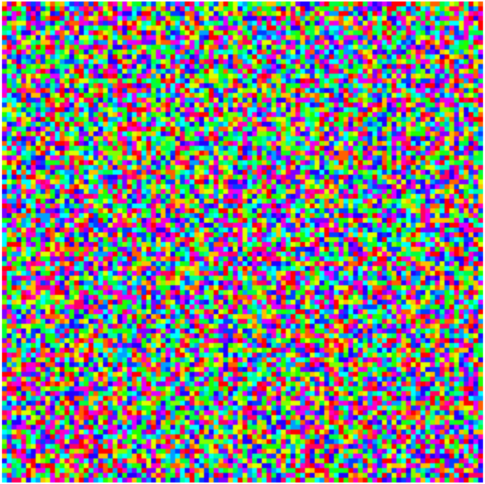

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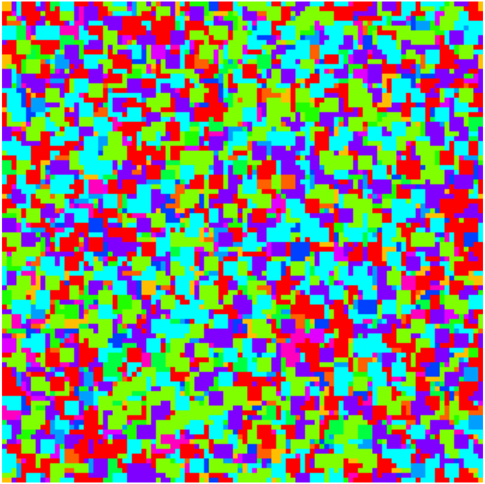

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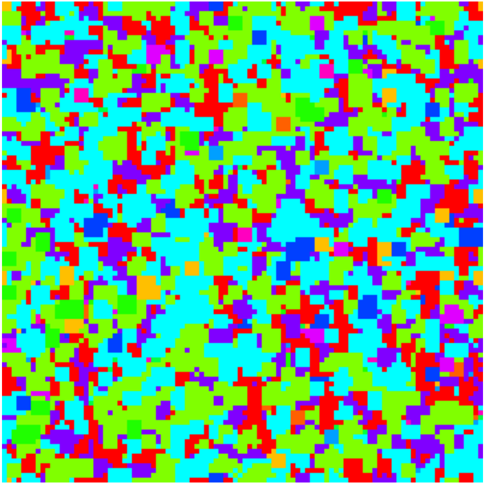

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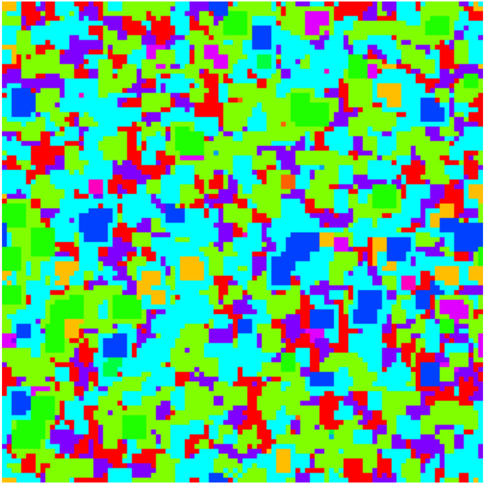

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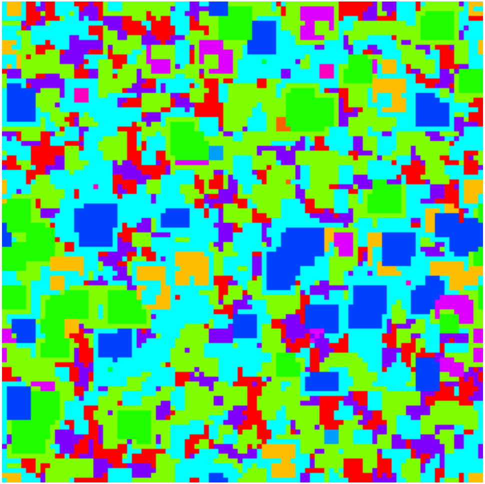

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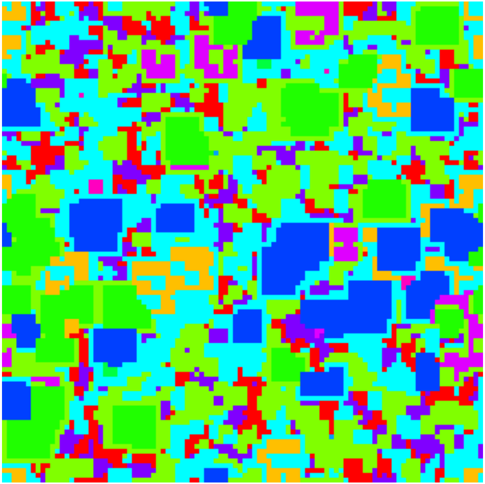

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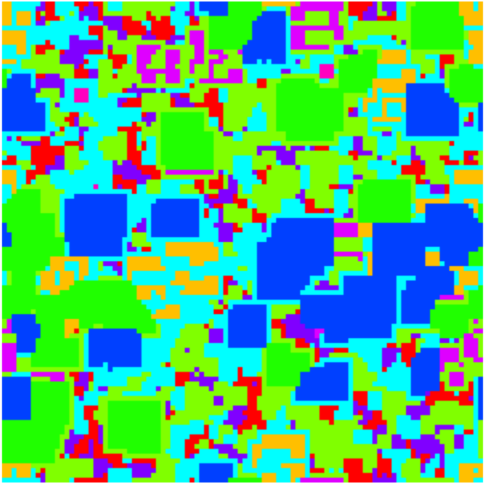

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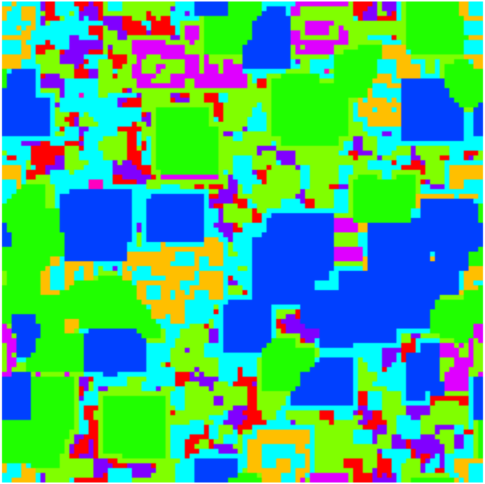

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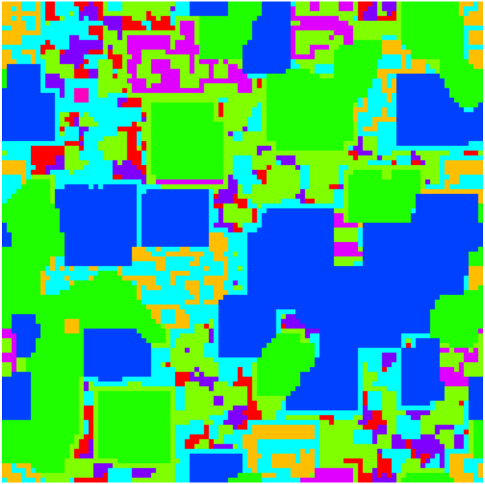

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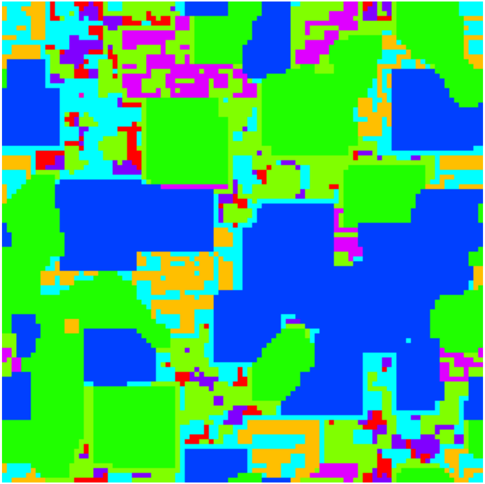

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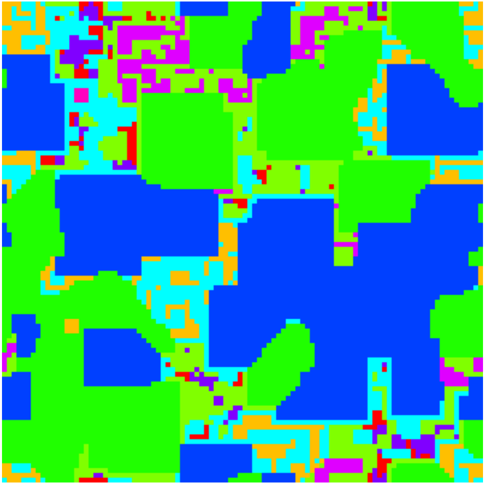

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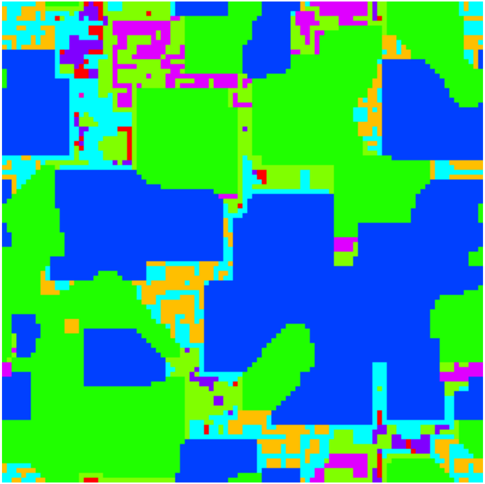

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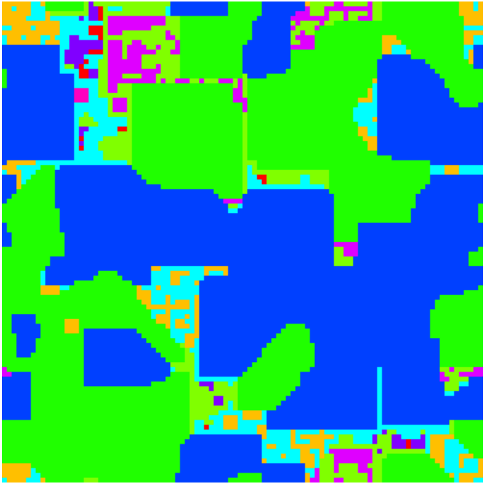

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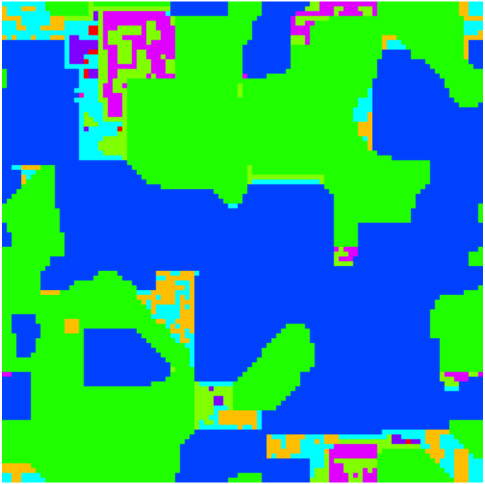

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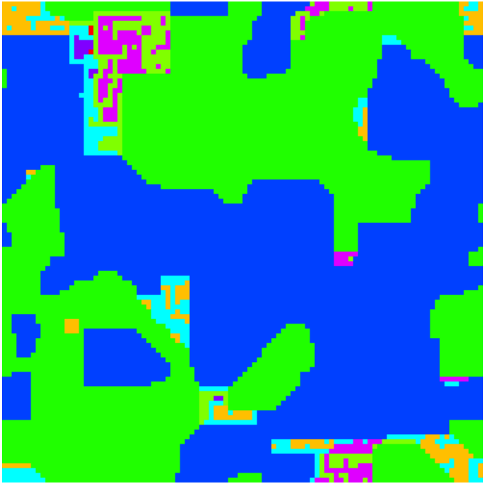

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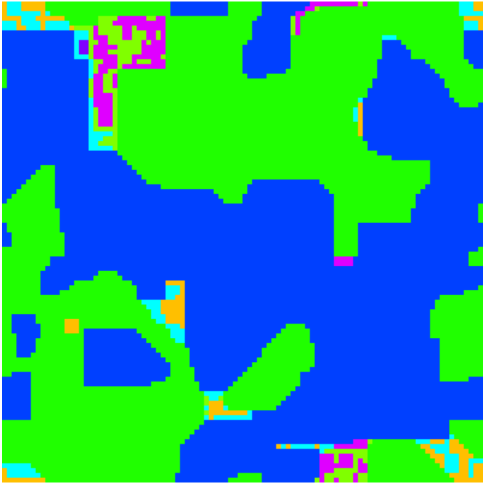

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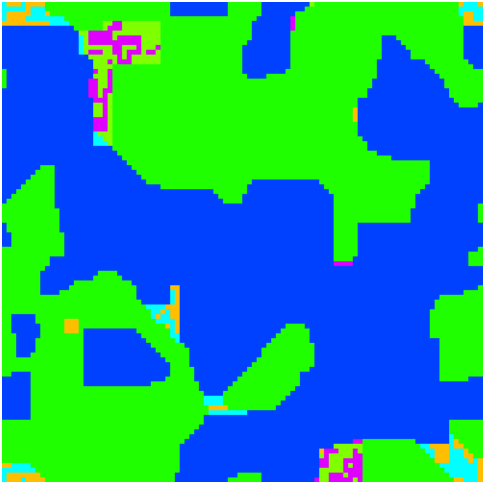

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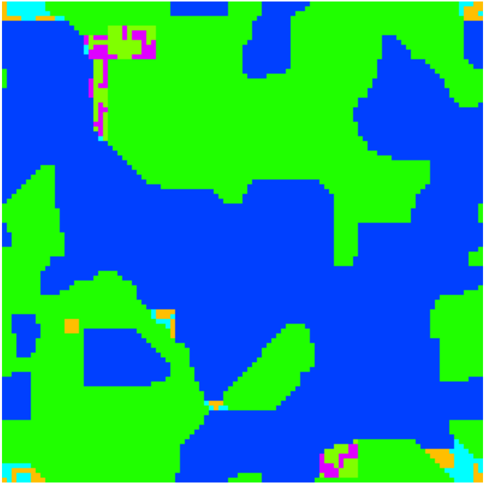

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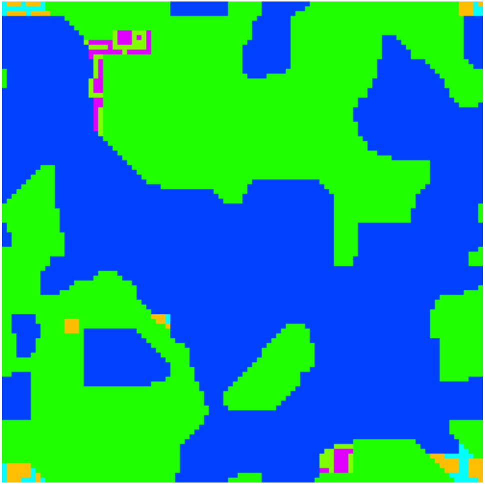

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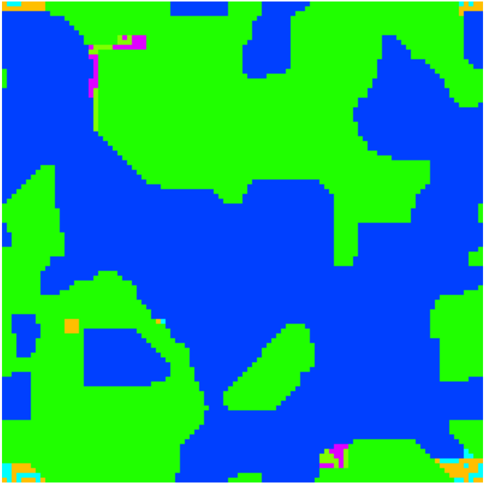

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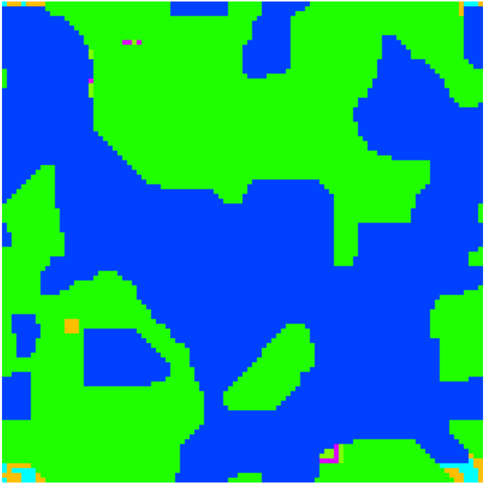

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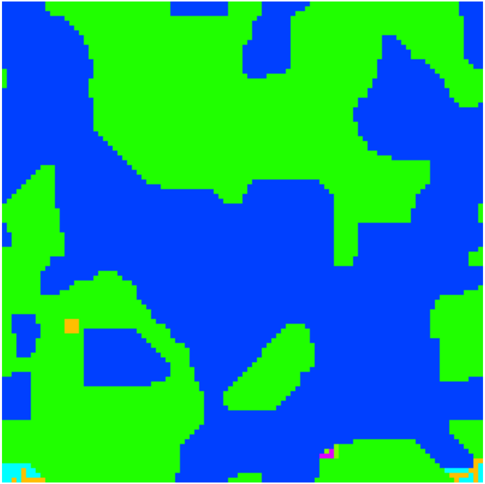

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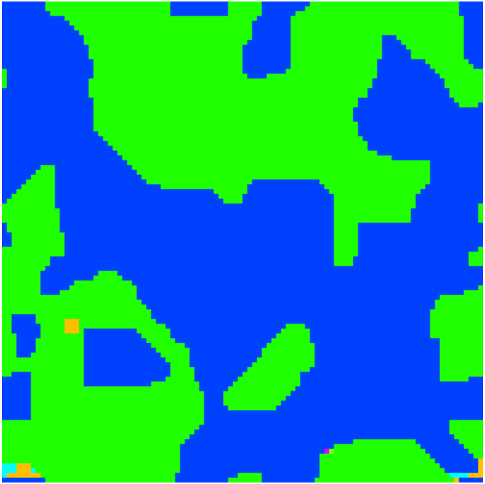

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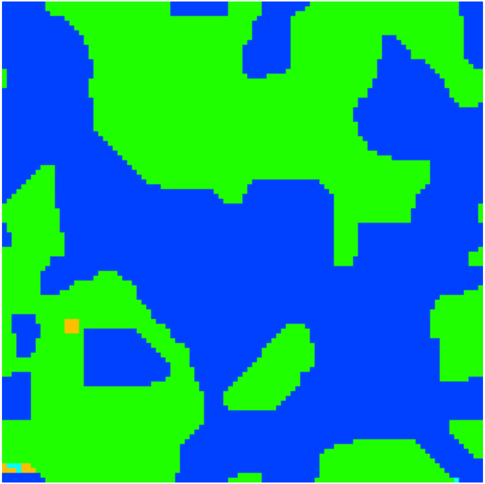

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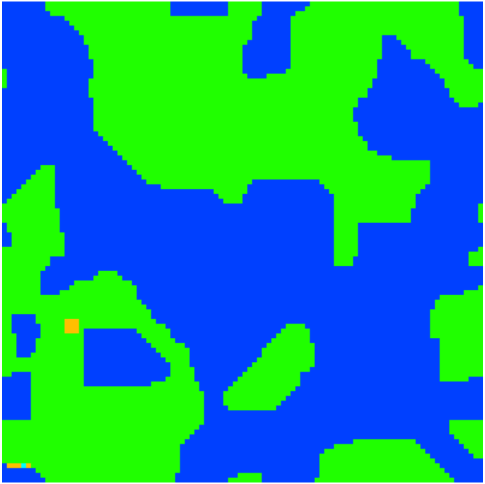

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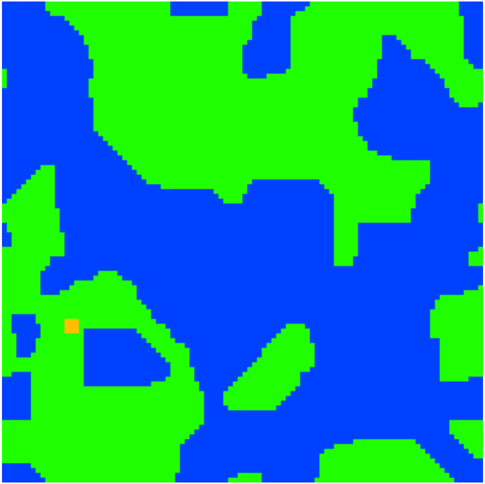

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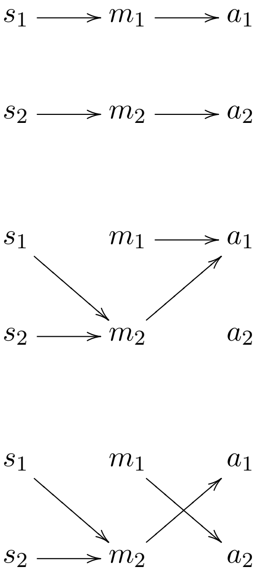
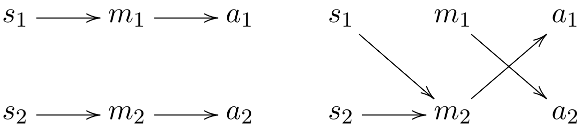


$P(s_1) \neq P(s_2)$ – analysis

Once again, the action happens along the frontiers . . .

Case 1:			Case 2:			Case 3:		
S	S	*S*	S	S	S	H	S	S
H	**H**	S	S	*S*	S	H	*S*	S
H	H	S	H	**H**	H	**H**	H	H

$P(s_1) \neq P(s_2)$ – analysis



Let $p(s_1) = \alpha$ and $p(s_2) = 1 - \alpha$.

An interior half babbler earns an average payoff of α.

A frontier signaler earns 1 when interacting with another signaler, 0 when sending to a half babbler, and $1 - \alpha$ when receiving from a half babbler.

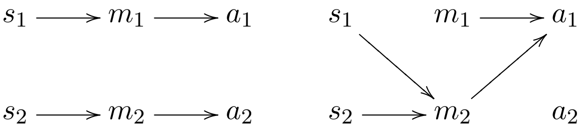
$P(s_1) \neq P(s_2)$ – analysis

Case 1:			Case 2:			Case 3:		
S	S	*S*	S	S	S	H	S	S
H	**H**	S	S	*S*	S	H	*S*	S
H	H	S	H	**H**	H	**H**	H	H

S's payoff $= \frac{15-\alpha}{16}$ — $\frac{13-3\alpha}{16}$ — $\frac{11-5\alpha}{16}$

$\alpha > \frac{15}{17} \approx .88$ — $\alpha > \frac{13}{19} \approx .69$ — $\alpha > \frac{11}{21} \approx .53$

$P(s_1) \neq P(s_2)$ – analysis



Let $p(s_1) = \alpha$ and $p(s_2) = 1 - \alpha$.

An interior babbler earns an average payoff of α.

A frontier signaler earns 1 when interacting with another signaler, α when sending to a half babbler, and $1 - \alpha$ when receiving from a half babbler.

$P(s_1) \neq P(s_2)$ – analysis

Case 1:	Case 2:	Case 3:
S S *S*	S S S	B S S
B **B** S	S *S* S	B *S* S
B B S	B **B** B	**B** B B
S's payoff $= \frac{15}{16}$	$\frac{13}{16}$	$\frac{11}{16}$
$\alpha > \frac{15}{16} \approx .94$	$\alpha > \frac{13}{16} \approx .81$	$\alpha > \frac{11}{16} \approx .69$

$P(s_1) \neq P(s_2)$ – analysis

When can non-signaling system regions be stable?

- $p(s_1) > .69$, case 2 regions of half babblers can be stable
- $p(s_1) > .81$, case 2 regions of babblers can be stable
- $p(s_1) > .88$, case 1 regions of half babblers can be stable
- $p(s_1) > .94$, case 1 regions of babblers can be stable

Almost spite.

Communicating on regular graph structures

What about other sorts of graph structures? Are these results mere artifacts of the Moore-8 toroidal lattice?

Communicating on regular graph structures

Theorem (1)

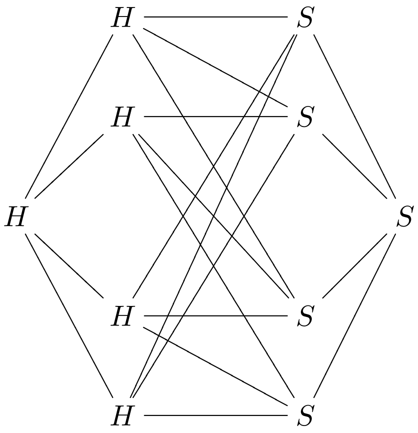
If agents that update their strategies according to the imitate-the-best dynamic play a sender-receiver game with their neighbors on a regular graph of degree k, then regions of agents using babbling strategies can be stable only if either $p(s_1) > \frac{k+1}{2k}$ or $p(s_2) > \frac{k+1}{2k}$.

Theorem (2)

When the states of nature are equiprobable, regions of half babbling agents can be stable on some regular graphs of degree k when $k > 3$.

Correlation in graphs

Anti-correlation?



Correlation in graphs

Correlation?

S	S	S
S	**S**	S
B	B	B
B	B	B

The topology of the Moore-8 toroidal lattice guarantees that **S** has five signaling system neighbors.

Correlation in graphs

Correlation of strategy types can lead to cooperation in the prisoner's dilemma.

And on networks, correlation between neighbors along the frontiers appears to increase the value of $p(s_1)$ for which babbling and half babbling become stable.

Correlation in graphs

Graph structures that impose a great deal of correlation between neighbors should be more conducive to the emergence of perfect communication than graph structures that only weekly correlate neighbors.

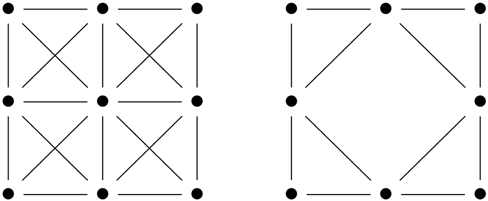
Communication in small-worlds

To study this systematically, we need a measure of correlation between neighbors and a process for generating graphs with a controlled amount of correlation.

Communication in small-worlds

Definition (clustering coefficient)

$$\gamma_v = \frac{|E(\Gamma_v)|}{\binom{k_v}{2}}$$



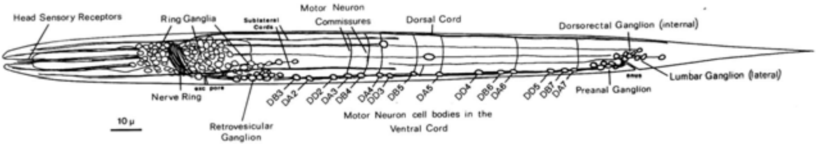
$$\gamma_v = \frac{12}{28} \approx .429$$

Communication in small-worlds

Definition (characteristic path length)

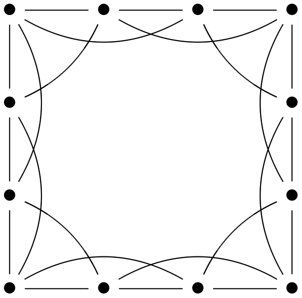
The *characteristic path length* (L) of a graph is the median of the means of the shortest path lengths connecting each vertex $v \in V(G)$ to all other vertices.

Communication in small-worlds



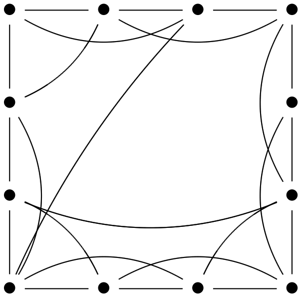
Communication in small-worlds

β-graphs



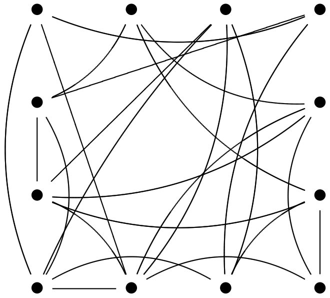
Communication in small-worlds

β-graphs

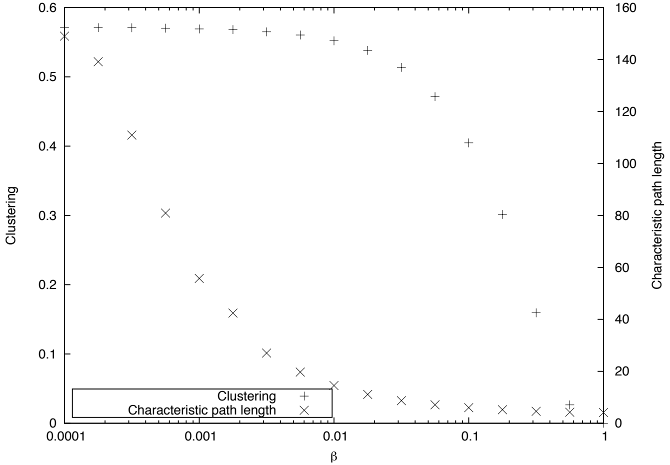


Communication in small-worlds

β-graphs

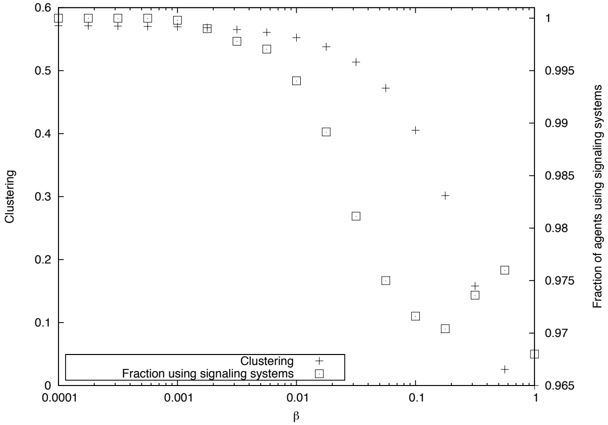


Communication in small-worlds



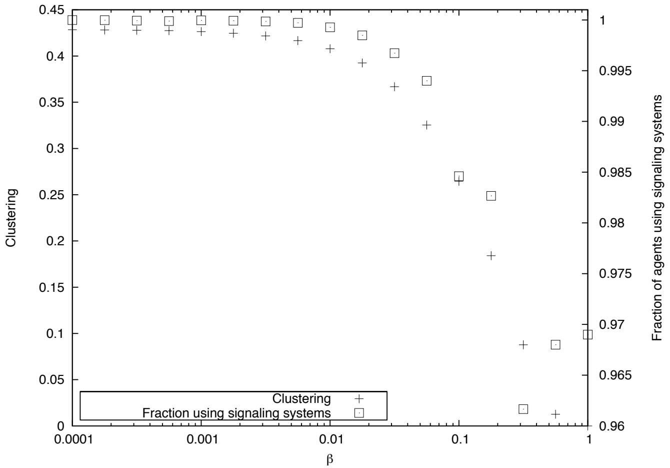
L and γ vs. β for β-graphs with 3,000 vertices and $k = 8$. Each point is an average of 20 realizations of the β-graph process.

Communication in small-worlds



γ and fraction of agents using signaling system strategies vs. β for β-graphs with 10,000 vertices and $k = 8$. $p(s_1) = .88$ and $p(s_2) = .12$. Each point is an average of 20 realizations.

Communication in small-worlds



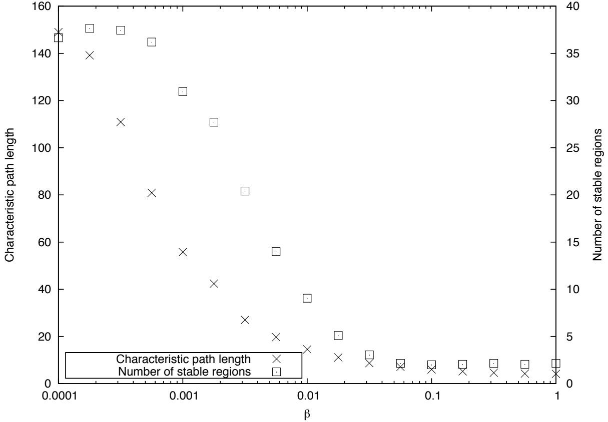
γ and fraction of agents using signaling system strategies vs. β for Moore-8 toroidal lattices rewired using the β-graph process. Each graph has 10,000 vertices and average degree 8. $p(s_1) = .88$ and $p(s_2) = .12$. Each point is an average of 20 realizations.

Communication in small-worlds

Graph structure also has a large influence on the number of regions that form…

- 10,000 vertex Moore-8 toroidal lattice: 9 regions
- 10,000 vertex cycle: 462 regions
- 10,000 vertex wheel: 1 region

Communication in small-worlds



L and number of stables regions vs. β for β-graphs with 3,000 vertices and average degree 8. $p(s_1) = p(s_2) = .5$. Each point is an average of 20 realizations.

Communication in small-worlds

Which graph structures are most hospitable for the emergence of perfect communication?

Small-worlds!

Conclusions

Initial questions...

- Do results (regarding the evolution of communication) obtained with the replicator dynamic generalize to interaction with neighbors?
- Do results obtained with a particular graph topology generalize to other graph topologies?
- Can we describe the general features of a graph's structure that influence the behavior of the system?

Thank you.