Interestingly Dull Numbers

Mathematicians are fond of Edwin Beckenbach's (1945) argument:

A. If some integer is not interesting, then there is a least such integer.

B. If some integer is the first uninteresting integer, then that fact makes the integer interesting.

C. Therefore, all integers are interesting.

The sophism has attracted no philosophical commentary because of a trivializing resemblance to Berry's paradox and the sorites.

The publication date of Beckenbach's sophism suggests that he was actually inspired by the surprise test paradox -- which was discovered by the Swedish mathematician Lennart Ekbom in 1943-4 (Sorensen 1988, 253). The surprise test paradox was itself suspected of being old news, though more for its kinship with the liar paradox than for its similarity to the sorites. The surprise test paradox was only emerged as an instructive, novel paradox when `surprise' was interpreted in terms of knowledge (or idealized belief). I shall attempt a parallel rescue of Beckenbach's paradox by interpreting `interest' in terms of attention worthiness. Just as the surprise test paradox acted as a goad and a guide to the development of epistemic logic, Beckenbach's sophism stimulates a logic of interest.

Shoring Up the Paradox

Philosophers who doubt that there are numbers propose substitutes such as sets or logical paraphrases (Field 1980). Beckenbach's sophism can reformulated in their terms. All he needs is a list of items such that the first break in interest would itself be interesting. To successfully mimic numbers, the substitutes must mimic this feature too. So I shall stick with Beckenbach's original formulation.

I agree with Beckenbach that some numbers are interesting. Indeed, I believe there are infinitely many interesting integers. Prime numbers, for instance, become more interesting as they become rarer and rarer down the number line. Since there are infinitely many prime numbers and at some point they become scarce enough to become interesting, there must be infinitely many interesting integers.

But I also think that there are infinitely many uninteresting integers. Since the dull integers must start somewhere, there must be a first one -- even if the vagueness of `dull' makes it impossible to specify which it is. The dullness of a number is typically difficult to establish because it involves "proving a negative": that there are no sufficiently interesting facts about the number. The vagaries of assessing interest magnify this logical hurdle.

Beckenbach's sophism is not stopped by the relativity of `interest'. When Hercules cut a head off the nine-headed hydra, two new heads would sprout in its place. Dividing `interest' into interest_1 and interest_2 generates an interesting_1 sophism and an interesting_2 sophism. The old problem has just branched into the two "new" problems of identifying the first uninteresting_1 number and the first uninteresting_2 number.

Beckenbach bases premise B on the fact that any instance of D will imply E:

D. It is interesting that *n* is the first uninteresting integer.

E. Therefore, *n* is an interesting integer.

Instances of this schema are necessarily unsound because the conclusion contradicts the premise. Beckenbach tacitly explains this unsoundness as due to the necessary falsehood of any instance of D. After all, the impossibility of a first dull number would entail the truth of B.

My counter-explanation of the unsoundness is that the inference is invalid. Since D contradicts E, this invalidity can be proved by merely showing that there is a true instance of D. The inference appears valid only as long as we assume objects inherit interest from *any* interesting fact they participate in.

Patterns of Interest

Constituents of facts are interesting by virtue of being the potential topic of interesting comments. For instance, six is an interesting number because of the sort of remarks Monte Zerger (2002, 86) makes about six: Six is the first perfect number (both the sum and product of its proper divisors), Six is the diameter of the only sphere whose volume and surface area are the same, Six is the number of possible isometries in R^3 . The interest of six cannot be traced to its role in unremarkable facts such as `Six is even'.

Interesting facts come in patterns: uniqueness claims, symmetries, counterexamples, and so on. `*n* is the first dull number' falls into the superlative genre: 8 is the *largest* cube in Fibonacci series, 50 is the *smallest* number that can be written as the sum of 2 squares in 2 ways,, and so on.

Interest in superlatives is reinforced (and shaped) by a drive to optimize: How can three villages be joined by the least length of road? How can a boat be shaped so as to minimize water resistance? What height-diameter combination maximizes the volume of a cylindrical container?

Biologists understand organisms by understanding their goals. Consequently, the practical mathematics behind optimizing is also of theoretical interest. Even inanimate things get subsumed under the principles of optimization. Inspired by faith in God as Designer, Fermat believed that nature behaved economically. Accordingly, he confidently used the Principle of Least Time to prove the law of refraction.

Iterated Attitudes

`Interestingly uninteresting number' does not collapse into interest or uninterest or force us to distinguish senses of `interest'. The source of this stability was first spotlighted by Benson Mates. According to Rudoph Carnap's principle of exchangeability, a statement retains its truth-value if synonyms are substituted for synonyms. Mates (1950, 125) agrees that the principle appears to work for simple belief attributions:

(M1) Whoever believes lawyers are wealthy believes that lawyers are wealthy.

(M2) Whoever believes lawyers are wealthy believes that attorneys are wealthy.

However, exchangeability fails for iterated attitudes:

(M1.1) Nobody doubts that whoever believes lawyers are wealthy believes that lawyers are wealthy.

(M2.1) Nobody doubts that whoever believes lawyers are wealthy believes that attorneys are wealthy.

Consider Bates, a man who doubts (M2). Bates worries that there is someone who believes that lawyers are wealthy and yet who fails to believe that attorneys are wealthy. Perhaps

Bates believes this individual does not realize that `lawyer' and `attorney' are synonyms. Bates' belief, even if false and confused, is enough to make (M2.1) false.

Mates' counterexample shows that some indubitable propositions are not indubitably indubitable. (M2) is trivial because substitution of a synonym makes it equivalent to the patently trivial (M1). However, (M2) is not trivially trivial because prefacing it with `no one doubts that' yields the falsehood (M2.1).

Doubling up propositional attitude switches the issue from cognition to meta-cognition. On the first day of class, an answer can be obvious to everyone in a classroom without it being obvious that it is obvious to all. Obvious facts need not be obviously obvious.

Interesting Dullness

In the midst of the turmoil leading to the overthrow of the Shah, Iranian revolutionaries instructed citizens to take a day off from their protests. Both sides took an interest in the spectacle of residents of Tehran quietly taking out their trash, commuting to work, and attending soccer matches. Spokesmen for the Shah portrayed these unremarkable events as a sign of waning support for the uprising. The revolutionaries hailed the dullness of the day as a sign of their control over the masses; they could switch social disorder on and *off*.

The structural rationale for the interest of dullness is the relative amount of information in a dullness attributions. An attribution of interest is merely a feeble existential generalization (`There are at least some sufficiently interesting facts about *n*') while an attribution of dullness is a sweeping universal generalization (`There are no sufficiently interesting facts about *n'*).

This difference in logical form creates an asymmetry in proof. A proof of interest is *deductive*; one example can settle the issue. A proof of dullness is *inductive*. An induction may always be overturned by new evidence. The memorable debates about whether p is interesting tend to be decisive refutations of dullness attributions. But from a logical point of view, attributions of interest are *duller* than attributions of interest.

`**I***p* → **II***p*' is invalid (read `**I***p*' as `It is interesting that *p* '). An uninteresting fact can embed an interesting fact. (For instance, it is interesting that the coastline of Norway is longer than the coastline of the United States but it is not interesting that this fact is interesting.) The case for D centers on the dual of this embedding principle: An interesting fact can incorporate a dull fact.

Indeed, it can be interesting that a fact is dull: **I**~**I***p*. `873 is the difference between the squares of two consecutive integers' *looks* interesting. But actually this fact is not interesting; *any* odd number greater than 1 is the difference between the squares of two consecutive integers. The dullness of `873 is the difference between the squares of two consecutive integers' is interesting because this dullness is explained by an interesting generality.

Undistinctiveness is just one genre of instructive dullness. The monotony of `The decimal expansion of 1/9 is .111….' is a sign that it is a non-terminating fraction. The enervating patternlessness of the decimal expansion of π is a sign that it is a transcendental number.

In an early example of computer program, Alan Turing analyzed chess into a sequence subtasks. The more menial he made the sub procedures, the more interest he added to the overall effect. Turing's chess programs breathed new life into homuncular models of psychological processes.

The robustness of interesting dullness is manifested by the sheer volume of commentary on boredom. Philosophers marvel at the power of this motivational vacuum. Stimulus-response behaviorists are chagrinned by how an absence of stimulus (silence, solitude, darkness) generates responses (pacing, eating, sleep). Social scientists agree that just as there can be sober studies of inebriation, there can be interesting studies of tedium, repetition, and apathy.

Triviality results

Consider Martin Gardner's interest in the triviality of Wilhelm Fliess's The Rhythm of Life: Foundations of an Exact Biology. Fleiss believed that 28 is the female period and 23 is the male period. He uses the formula 23x + 28y to explain cycles ranging from the cell to the solar system. Gardner points out that "if any two positive integers that have no common divisor are substituted for 23 and 28 in his basic formula, it is possible to express *any positive integer whatever*." (1965, 155) This interesting criticism does not revive interest in Fleiss' theory.

A triviality results show that some criterion for F makes its application absurdly broad or narrow. The limit case for broadness is a tautology. Often we fail to recognize the triviality because of an equivocation: "An untruism is an ambiguous sentence which taken in one sense states a dull truism -- an analytical or a platitudinous truth -- and taken in another sense makes a statement that is interesting but either certainly or probably false or at least of uncertain truth-value.'" (Barnes and Richardson 1972: 189) Pointing out the trivial readings of `Only the fittest survive' shepherds us toward more fruitful interpretations.

Conditional triviality results function as friendly warnings. These interpretive constraints help clarify an elusive insight. Friends and foes an hypothesis can hail a triviality result as important news. Consider the lovely hypothesis that the probabilities of conditionals are conditional probabilities. David Lewis (1976) demonstrated that various precisifications of this elegant equation entail the triviality of the probability functions. This has proven to be a core constraint on subsequent analyses of conditionals.

The Paradox of Analysis

The stability of interesting triviality suggests a concessive solution to C. H. Langford's paradox of analysis. This dilemma targets G. E. Moore's meta-philosophy. Moore claimed that the main job of the philosopher was to analyze concepts (as opposed to erecting alternatives to common sense). For instance, Moore thought epistemologists had instructively dismantled knowledge into justified true belief. He agreed with the utilitarians' definition of `right' as the optimization of consequences. Langford does not understand how conceptual analysis could be informative:

> Let us call what is to be analyzed the analysandum, and let us call that which does the analyzing the analysans. The analysis then states an appropriate relation of equivalence between the analysandum and the analysans. And the paradox of analysis is to the effect that, if the verbal expression representing the analysandum has the same meaning as the verbal expression representing the analysans, the analysis states a bare identity and is trivial; but if the two verbal expressions do not have the same meaning, the analysis is incorrect. (1968, 323)

In defense of conceptual analysis, I say the dullness of an identity statement often promotes the interest of the analysis. Consider contested identities. Students at first deny 1 = .999…. The teacher then points out that 1/3 = .333…. and 1/3 x 3 = 1. This conjunction of trivial truths makes most students regard 1 and .999…. as alternate numeric representations of the same number. They stop viewing 1 = .999…. as a near miss and start regarding it as trivially true. In the final analysis, the interesting fact is not that 1 = .999…. Just the opposite! The interesting fact is that `1 = .999….' is dull.

All "informative" identity statements involve recognition of hidden dullness. Our linguistic system presumes that distinct singular terms have distinct referents (just as our visual system presumes that there is a single light source). Astronomers had to override this labeling reflex to explain away the coincidences involving Hesperus and Phosphorus. Despite this conscious correction, `Hesperus is Phosphorus' will always sound more informative than `Hesperus is Hesperus'. Thanks to the modular nature of the mind, the default assumption of distinctness is only muffled, never silenced.

The astronomer experiences the same double-think when he realizes that a lunar mountain is really a crater (figure 1).

Figure 1

Given the default assumption that the scene is illuminated from above, the shading is interpreted as a shadow cast by a convex object. The illusion can be reversed by turning the image upside down. But since the visual system is modular, this demonstration never educates the homunculi into accepting the concave interpretation (when the scene is restored to its original orientation). The astronomer must just remember to consciously override what his eye reflexively represents.

When astronomers explain away coincidences with an identity hypothesis, loss of wonder is experienced as insight. Demystification is a sign of explanatory progress. Astronomers were embarrassed at ever having been impressed with the coincidences between

Hesperus and Phosphorus (same size, same plane of motion, and so on). Wisdom is nourished by humble pie.

I once amazed a classmate with the following trick: Write down some digits to form a number. Re-arrange its digits to form another number. Subtract the smaller from the larger. The difference will always be divisible by 9. Intrigued, she confirmed this algorithm for several sums. We then did the algebra behind the trick. I could tell the explanation was sinking in because she lost interest in doing the sums. The magic had evaporated from the relationship.

Several philosophers regard boredom as a metaphysical insight. According to Blaise Pascal, boredom exposes the emptiness of life without God. Martin Heidegger is more synoptic and secular: "Profound boredom, drifting here and there in the abysses of our existence like a muffling fog, removes all things and men and oneself along with it into a remarkable indifference. This boredom reveals being as a whole." Boredom forces you to face Time. And when you are bored, you embody the indifference of the universe.

Satiation differs from boredom in that you can exit. The gorged gourmand just leaves the restaurant. But the dishwasher is obliged to stay. World weary, the dishwasher can only escape into daydreams and diversions.

Boredom correlates with understanding. So there is some temptation to compress Heidegger into two lines: To understand everything is to be bored by everything. So everything is boring.

But boredom can also be produced by incomprehension or a slight distraction (too small to be recognized as the true cause of one's inability to focus). The laggard is too far behind to make sense of the lesson. The prodigy is too far ahead to find the lesson stimulating. The interested student lies in between, challenged but not overwhelmed.

Prudent students monitor their boredom to check whether they have deviated from this balance. The vain misconstrue the boredom of incompetence as the boredom of mastery.

Aesthetics of boredom

Andy Warhol's 1963 movie "Sleep" is devoted to a man sleeping for eight hours. Henry Geldzahler writes

> what appears boring is the elimination of incident, accident, story, sound, and the moving camera. . . . As less and less happens on the screen, we become satisfied with almost nothing and find the slightest shift in the body of the sleeper the least movement of the camera interesting enough. (1970, 301)

Standards can be lowered to make a boring event less boring than another boring event. But a lesser degree of boredom is not sufficient for being interesting. Contrary to Geldzahler, "Sleep" is not merely apparently boring. It is boring and is presented as boring. Warhol encourages this common sense reaction by adopting a passive, self-effacing persona: "I like boring things", "I am a deeply superficial person", and so on. Warhol commends the bland, the soporific, and the monotonous.

Barbara Rose characterized all popular art as involving an aesthetics of boredom. Many landmark paintings in the transition to Pop art conform to Rose's startling generalization. In 1951, the minimalist Robert Rauschenberg's presented "White Painting", a repetition of seven adjoining panels of bare white canvas. In 1957 Jasper Johns unveiled "Grey Numbers" a matrix of random, single digits.

Figure 2

Previous artists presented numbers with special significance in their paintings. Consider Albrecht Dürer's depiction of frustrated creativity “Melancholia”:

Figure 3

The picture incorporates a magic square.

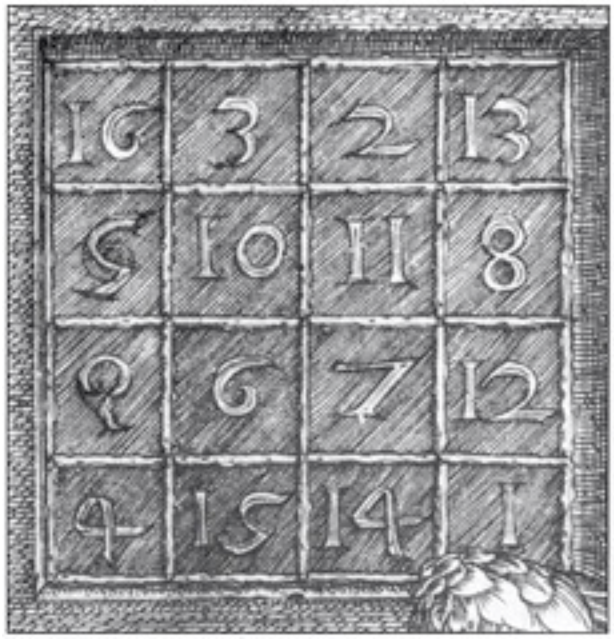

Figure 4

The sum of 34 can be reached by adding the numbers in any row, column, diagonal, or quadrant; the four center squares; the four corner squares; the four numbers clockwise from the corners; or the four counterclockwise. Moreover, the two numbers in the middle of the bottom row give the date of the engraving: 1514.

Geldzahler struggles to find something interesting in “Sleep” because artists typically present their work to be appreciated. The painting or sculpture comes with the implicit invitation “Look at me!”. If this is construed as an essential requirement, then the aim of an artwork cannot be to inspire indifference. In an effort to satisfy the requirement (and deflect the objection that Pop art is anti-art), some suggest that the Pop artist is just challenging our preoccupation with creation and production and redirecting attention to *re-creation* and *re-production* of media images (such as Warhol’s seminal paintings of Campbell soup cans – all thirty two varieties).

As earlier mentioned, I take the primary bearers of interest to be facts, not things. I do not require that the artist aim at producing an *object* of aesthetic interest. Indeed, I allow that the absence of objects can be aesthetically interesting. The sculptor Doris Salcedo's "Shibboleth 2007" is a 167 meter crack in the Tate Gallery on London's South Bank. The crack symbolizes borders, as experienced by immigrants and victims of segregation. A crack is a thin hole, not an object.

An interesting fact can be constituted by a dull object. The dullness of the object can contribute to the interest of the fact. Warhol aims at interestingly boring artwork.

Inheritance rules

Beckenbach's inference from D to E seems valid because we assume that objects always inherit the interest of the facts they help constitute:

D. It is interesting that `*n* is the first uninteresting integer'.

E. Therefore, *n* is an interesting integer.

However, interest is sometimes intercepted by an intermediate heir. It is interesting that 6 = 9 when turned upside down. But that is a symmetry between representations that does not extend to what they represent. A dull number can be denoted by an interesting numeral. In hexadecimal (base 16), 570005 is denoted by DEAD.

The distinction between numbers and numerals alerts us to subtler misattributions. Is 153 made interesting by the interesting fact that its binary representation is a palindrome? Is 1/97 made interesting by the fact that its decimal expansion endlessly repeats a sequence of 96 digits? Once we become sensitized to the distinction between using an expression and merely mentioning it, we become more discriminating about the means by which a number can inherit interest from a fact.

To share interest with its representation, a number must bear a richer relationship with its notation than that afforded by brute labeling. In the self-descriptive number 6210001000 the digit 0 occurs 6 times, the digit 1 occurs 2 times, the digit 2 occurs 1 time, the digit 3 occurs 0 times, and so on. Although this is only mildly interesting, it is on a continuum leading up to Godel-numbering.

The relationship need not be self-referential. A cyclic number is an integer of *n* digits whose cyclic order is preserved when multiplied by 1 through *n*:

142857 x 1 = 142857	142857 x 4 = 571428
142857 x 2 = 285714	142857 x 5 = 714285
142857 x 3 = 428571	142857 x 6 = 857142

When expressed in decimal notation, the string 142857 behaves like a string of beads that has had a segment cut and reattached at the beginning. Many mathematical tricks exploit this uniformity of 142857. No other number behaves this way in decimal notation (except for the trivial case of 1). Other bases have several cyclic numbers. Duodecimal has three, base 24 has four, octal has five. Thus 142857 is made remarkable by its uniquely uniform behavior in a decimal environment.

Notational systems are themselves mathematical entities. A number may inherit interest from how it relates to its notation.

In mathematics, inheritance is restricted to internal relations. The interest of `92 is the number of different arrangements of 8 non-attacking queens on an 8 x 8 chessboard' is assigned to chess rather than 92 because chess is an alien relatum.

People relate interestingly to numbers but the interest of these relationships attaches to people. The interest of `The grandmaster Bobby Fisher died at 64, the number of squares on a chessboard' attaches to Fisher, not 64.

What role do human beings play in defining what is interesting? My answer takes a cue from Alvin Goldman's (1992) account of the virtues, both moral and epistemological. Inquiry-specific interest is just a subset of evidence and so is conditioned by human psychology as much as evidence is. Inquiry-generic interest fits the two-stage model that R. M. Hare advocated for ethics. At the utilitarian *critical* stage, we select traits that make facts helpful to inquirers: generality, recursivity, symmetry, and so on. Once these virtues are established, we streamline our interest attributions by just checking for these traits and ignoring consequences. The predictions were only used to "fix the reference" of what is generically interesting, not to give the truth conditions for `x is interesting'. Recall Saul Kripke's example of `heat'. The sensation of heat is used in a reference fixing description of heat, but this description is "off the record". Once the natural kind is designated, the description retires. Consequently, heat is objective even though it was picked out with a mere feeling. Generic interest is also objective. Numbers would be interesting even if there were no human beings (even though the whole point of introducing `interesting' is to allocate attention in a way that promotes human inquiry).

The founding members of the mathematic community had special influence over the extension of interesting mathematics. Ironically, Pythagorean austerity played a role in filtering out many of the motives that generated their own study of numbers.

Plutarch remarks that "The Pythagoreans also have a horror for the number 17, for 17 lies exactly halfway between 16, which is a square, and the number 18, which is the double of a square, these two, 16 and 18, being the only two numbers representing areas for which the perimeter equals the area". This is an interesting fact about *Pythagoreans*. Their horror does not add to the interest of *17* (though 17 may accrue interest from the mathematical relationship that troubled the Pythagoreans).

The Pythagoreans had a warmer relationship with amicable pairs, couples of integers such that one was the sum of the proper divisors of the other. For instance, the proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110, which add up to 284; and the proper divisors of 284 are 1, 2, 4, 71, and 142, which add up to 220. In 1636, Pierre de Fermat discovered the pair (17296, 18416). In 1638, Rene Descartes found (9363584, 9437056). In a large scale feat of high altitude exploration, Leonard Euler discovered *sixty* amicable pairs! This climax was followed by an anti-climax (Dickson 2005, 47). In 1866, sixteen year old B. Nicolò I. Paganini found the small amicable pair (1184, 1210). The great masters had overlooked it!.

Paganini's amicable pair is interesting in that it partly answers `Which are the amicable pairs?'. But it is more interesting as evidence for the *psychological* question `How reliable were the great mathematicians?'. Mathematicians are reluctant to credit a pair of numbers with the interest that attaches to *contingent* facts about it.

Another obstacle to crediting interest to Paganini's numbers 1184 and 1210 is that they are interesting *as a pair*. Interest in a pair need not pass down to the individuals comprising the pair (just as a husband and wife can each be bores and yet be interesting as a couple).

The Fibonacci sequence inherits interest from the interesting facts it participates in. But this interest does not trickle down to the numbers composing that sequence. To avoid spreading attention too thinly, interest attributions must be applied sparingly – only to the level that shows promise of rewarding study.

Interest is Relative to a Question

Interesting facts are evidence. They promote inquiry as clues, either directly by confirming an answer or correctively, by refuting a presupposition of the question. (A presupposition of a question is a necessary condition for the question having a true, direct answer.)

But not all evidence is interesting. The evidence may be too weak to merit attention. And *misleading* evidence ought to be ignored.

Evidence need not be possessed, so some interesting facts may be unknown. Either it is interesting that there is always a prime number between *n* and 2*n* or it is interesting that there are exceptions. Number theorists knew this disjunction was true before they knew which disjunct made it true. (It is the first disjunct. "Chebyshev said it, but I'll say it again; There's always a prime between *n* and 2*n*." -- N. J. Fine couplet commemorating Pafnuty Chebyshev's 1850 proof.)

Detective stories pose a clear question: Who did it?. The audience easily orients to what is interesting. When the underlying question becomes murkier so does the criterion for what is interesting.

Obscurity is greatest for the standing curiosity we manifest as collectors of interesting facts. Instead of having any specific question in mind, we accumulate facts that have signs of being interesting for future inquiries or potential inquiries. The attics of our minds store facts that might come in handy later (either for ourselves or other inquirers). Premise B of Beckenbach's sophism avoids clear counterexamples partly because the relevant interest stems from *standing* curiosity.

Subordinate Interest

Alfred North Whitehead declares "it is more important that a proposition be interesting than it be true. The importance of truth is, that it adds to interest" (1978, 259). A failure to attract attention dooms a proposition to be ignored. A proposition can only promote inquiry by promoting itself (as being noteworthy).

Whitehead concedes "false propositions have fared badly, thrown into a dustheap, neglected" (1978, 259). His explanation is that actions based on true propositions are more likely to be successful. And "the contemplation of truth has an interest of its own." (1933, 313)

I prefer the simpler explanation that *basic* interest does entail truth – as revealed by the prefix test (put `It is interesting that' in front of the proposition). To be interested in *p* is to believe that it is interesting that *p* is true.

Interest in The Teachings of Don Juan: A Yaqui Way of Knowledge collapsed after anthropologists discovered that many of the lessons were fabricated by Carlos Castaneda in the UCLA library. The emotion of interest is like anger. Anger that `Kittens are being grown in bottles as a form of decoration` collapsed after www.BonsaiKitten.com was exposed as a prank web site. Contrary to Whitehead, truth does not merely add to interest; interest presumes truth just as anger or joy or regret presume truth.

A falsehood can be interesting *as a hoax* or *as a thought experiment*. We may even chide ourselves for attaching too much weight to the tale being a "true story" (Fischer 1995). But then the interesting proposition is the superordinate proposition that embeds the falsehood. It is metonymy to say `$\pi^2 = 10$ is interesting' on the strength of `$\pi^2 = 10$ is interesting as an approximation' (by virtue of it being within 1.3% of the true value). This ellipsis is harmless when the "as an F" qualification is tacitly heeded. But if we overlook the qualifications, our inquiry will be polluted by false clues. In How the Laws of Physics Lie Nancy Cartwright agrees that the laws of nature are interesting – but only as idealizations.

Many philosophical propositions are only interesting as false consequences of plausible premises: solipsism, skepticism, nihilism, and so forth. We study these traps to avoid them.

Other philosophical propositions are interesting as interpretive tools. We do not need to believe the divine command theory of morality to find of it of service in understanding religious people and those who base their moral nihilism on atheism.

In mathematics some refuted conjectures retain interest as illustrations. In 1919 George Polya conjectured that over half of the natural numbers less than any given number have an odd number of prime factors. In 1958 C. B. Haselgrove proved that there exists a counterexample. He estimated its location at around $1{,}845 \times 10^{361}$. Polya's conjecture illustrates the principle that a seriously researched generalization can hold for a long sequence of numbers and yet still be false.

A false conjecture can be an interesting as a point of comparison. Someone once sent Augustin-Louis Cauchy a purported proof that $a^3 + b^3 + c^3 = d^3$ has no solution in integers. This thesis resembles the first recalcitrant instance of Fermat's Last Theorem, which denies that $a^3 + b^3 = c^3$ has a solution in integers. (There are solutions for $a^2 + b^2 = c^2$, for instance, $3^2 + 4^2 = 5^2$. The general form of Fermat's last Theorem is that $a^n + b^n = c^n$ lacks a solution in integers for any *n* greater than 2.) Fermat's Last Theorem was notoriously hard to prove or disprove. So it would have been interesting that $a^3 + b^3 + c^3 = d^3$ has no solution in integers. However, Cauchy returned the manuscript with a single comment: $3^3 + 4^3 + 5^3 = 6^3$.

Null hypothesis methodology tells us to first work out the implications of the hypothesis that the "cause" is ineffectual. These hypothetical predictions establish a baseline of expectation against which attributions of efficacy can be contrasted.

A falsehood (or even a meaningless statement) can be interesting as a near-miss of a counterexample. For it may suggest an actual counterexample. Feedback permits the searcher to close in.

If the near counterexample is not grounded by an actual counterexample, then it might still draw interest from methodological principles. After all, teachers exhibit fallacious proofs and miscalculations to illustrate hazards. In a 1998 episode of the cartoon comedy "The Simpsons", $3987^{12} + 4365^{12} = 4472^{12}$ is written on Homer Simpson's basement blackboard. This would be a counterexample to Fermat's Last Theorem. Mathematicians are familiar with many near-misses, the smallest and best known being $10^3 + 9^3 = 12^3 + 1 = 1729$. But Homer's counterexample is confirmed by an ordinary calculator!

It is trick though. One of the script writers, Bernard X. Cohen, programmed a computer to find pseudo-counterexamples to Fermat's last theorem (ones that exploit rounding errors).

Upward Inheritance of Interest

Objects and relations inherit their interest from the facts they constitute. They are isolated by an act of selective attention. We cannot pry 26 out of facts such as `26 is the only number to be directly between a square and a cube'.

Questions inherit interest from the propositions in their answer sets. Typically, the question is interesting only if the correct answer is interesting. So there is a risk that the interest is only apparent. But some riddles are safe. They can be known to be interesting without knowing their correct answers. For instance, `Is there an odd perfect number?' is interesting either way it is answered; each answer is interesting if true and one of the answers must be true.

A question that lacks a correct answer lacks any direct interest. But it may have indirect interest as a specimen or as a stepping stone to answering a question that really is interesting. When students learn meta-logic, they have trouble following all the intricate rules for constructing well-formed formulas. Instead of fighting this laxness, the teacher will allow many topics to be discussed in garbled form, cleaning up the issues only when there is trouble. "If it doesn't itch, don't scratch."

Arguments may inherit interest from their premises. But this cannot explain the interest of some conditional proofs and *reductio ad absurdums.* They lack premises.

Some of these arguments derive interest from their conclusions. This explains the interest of Euclid's reductio that there are infinitely many prime numbers. But other interesting premiseless proofs lack an interesting conclusion. For instance, there is an interesting premiseless proof that something exists: No premises are needed to derive the tautology (x)(x = x) and that implies (∃x)(x = x). The universal quantifier has existential import in contemporary classical logic. This proof is an anomaly for the principle that logic is ontologically neutral. If we acquiesce to the exception, we will have resolved the question Martin Heidegger famously characterized as the most fundamental issue of philosophy: Why is there is something rather than nothing? Arguments such as these force us to posit another source of interest.

Interesting Conditionals and Academic Appointments

Inferences inherit their interest from the corresponding conditional propositions. It is not interesting that the maximum number of human hairs is less than the number of Londoners. Nor is it interesting that there are two Londoners with the same number of hairs. But it is interesting that this (dull) conclusion follows from the (dull) premise.

The phenomenon of interesting conditionals bears on David Lewis' puzzle about why philosophy departments ignore truth when hiring. Since philosophers aim at knowledge, the more successful ones will be those who begin from beliefs that are closer to the truth. Yet materialists do not say `We should not hire the dualist because dualism is false'. The materialist instead focuses on traits that are *indirectly* conducive to truth such as erudition, diligence, and intelligence.

Lewis considers reasons to trade the advantage of truth for the advantage of cognitive diversity. The tolerant materialist hedges against the risk of materialism being false. Hiring a dualist opens up parallel lines of research. The dualist can prompt the materialist to argue for his beliefs and thereby prevent materialism from becoming dead dogma. The dualist also serves as a specimen; what better way to know your adversary!

These considerations have weight but do not provide enough compensation for heliocentric astronomers to hire a geocentric astronomer or for a continental drift geologist to hire a geologist who believes in stationary continents. Why should philosophers think they are getting a better trade-off?

Lewis' explanation is that *all* knowledge seeking academics operate under a treaty to ignore the truth-value of *controversial* beliefs: the origin of life, the continuum hypothesis, and so on. Since philosophy is a repository of controversial doctrines, the treaty has far more scope.

Lewis' rationale for truth-neutral hiring is in the tradition of John Stuart Mill's utilitarian plea for tolerance in <u>On Liberty</u>. The materialist sacrifices the prospect of complete victory to avoid the risk of complete defeat; he abides by a tacit pact to not to base his judgment directly on the truth.

Lewis' pact postulate is vulnerable to a self-strengthening appeal to ignorance: If there were such a pact, doubters would have been told about it (to curb their partisanship). They have not. So there is no such *modus vivendi*.

Perhaps treaty skeptics are just deaf to the nuances that reveal a *tacit* pact. However, their treaty-skepticism would give believers in the treaty a powerful motive to ignore it. Why should they shoulder the burdens of compliance (which increases exponentially as committee size decreases) with such meager expectation of reciprocity? Right or wrong, treaty skeptics are right – or would make themselves right by out hiring the credulous.

A second reason for doubting the pact postulate is that a treaty is not needed to explain the tolerance of error. The speculative character of philosophy raises the interest of *hypotheticals*. The truth that is being ignored is the truth of the antecedent and the consequent. Materialists agree that dualism is false but are still interested in what follows from this doctrine. Is dualism incompatible with the conservation laws of physics? Can the sharp distinction between mind and body be reconciled with the gradual evolution of *homo sapiens* and the continuous development of a fertilized egg into a man? Must a libertarian be a dualist? Is immortality only possible given dualism?

Right or wrong, dualism has deep roots in common sense. Its branches extend not merely to religion but also into psychology -- and from psychology into neuroscience and from there into medicine. True believers are often the best motivated to figure out the entailments. Materialists accept the cost of the dualist believing his doctrine to secure knowledge of conditionals with dualist antecedents or dualist consequents.

The situation has precedents in the history of mathematics. Prior to Albert Einstein's general theory of relativity, experts in non-Euclidean geometry were hired by mathematicians who believed that space is Euclidean and necessarily so. The appointments were made because the theorems of these hypothetical geometries are interesting even if space is necessarily Euclidean.

Why the Interest of Numbers is Derivative

A list of dull numbers could be interesting and useful. The list could be used to refute conjectures that imply the interest of a member on the list. Mathematicians already use the presence of interesting numbers as a sign of an interesting fact. For instance, $e^{i\pi} + 1 = 0$ combines five of the most interesting numbers into one formula.

If numbers were the primary bearers of interest and facts owed their interest to the numbers, then $e^{i\pi} + 1 = 0$ would be interesting just by virtue of the numeric congregation. But the presence of interesting numbers is only *evidence* of an interesting fact; the numbers do not make the fact interesting. Indeed, there is an algorithm for constructing dull sentences about interesting numbers; just list *m* interesting numbers in a conjunction of the form `n_1 is a number and n_2 is a number and . . . and n_m is a number'.

Hardy's Remarks on Interest

All numbers are interesting in a general way, as *numbers*. In A Mathematician's Apology G. H. Hardy remarks on how numbers are unobscured by the veil of experience:

> A chair or a star is not in the least like what it seems to be; the more we think of it, the fuzzier its outlines become in the haze of sensation which surrounds it; but "2" or "317" has nothing to do with sensation, and its properties stand out the more clearly the more closely we scrutinize it. It may be that modern physics fits best into some framework of idealistic philosophy—I do not believe it, but there are eminent physicists who say so. Pure mathematics, on the other hand, seems to me a rock on which all idealism founds: 317 is a prime, not because we think so, or because our minds are shaped in one way rather than another, but *because it is so*, because mathematical reality is built that way. (1940, 130)

But the conclusion of Beckenbach's sophism attributes more than generic interest to numbers. A mother is interested in each of her children, attributing traits that make each of her children remarkable as individuals. She will deny that her children are interesting only in the way any child is interesting.

Hardy lacked this maternal attitude toward numbers. Hardy was interested in his lack of interest in some facts constituting recreational number theory:

> There are just four numbers, after unity, which are the sums of the cubes of their digits:
>
> $153 = 1^3 + 5^3 + 3^3$, $370 = 3^3 + 7^3 + 0^3$,
>
> $371 = 3^3 + 7^3 + 1^3$, $407 = 4^3 + 0^3 + 7^3$.
>
> These are odd facts, very suitable for puzzle columns and likely to amuse amateurs, but there is nothing in them which appeals to the mathematician. The proofs are neither difficult nor interesting—merely a little tiresome. (Hardy 1940, 105)

This paragraph is not self-defeating. Hardy's interest in the dullness of these facts does not endow them with interest. He stably employs them as data to test hypotheses about which features of mathematical facts make them interesting.

Hardy's corpus of lackluster mathematics encompasses all of ballistics and aerodynamics. They are "intolerably dull" and "repulsively ugly" (1940, 140). According to Hardy, only pure mathematics is interesting and beautiful.

Hardy concedes this is only an accidental regularity. He admits that the discovery of an application for, say, the Chinese remainder theorem, would not mar its beauty or interest.

After all, mathematics is not made interesting by its uselessness. Mathematical theorems are made interesting by “a very high degree of *unexpectedness*, combined with *inevitability* and *economy*.” (Hardy 1940, 113)

Applied mathematicians have proposed counterexamples to Hardy’s purism: “Galileo’s analysis of the ballistic trajectory in a vacuum was a beautiful application of geometry that brought the parabola into sharp focus, to a degree not seen since the time of Apollonius, and harnessed its symmetries to serve the emerging science of motion.” (Groetsch 2000, 15). Norman Levinson argues that coding theory is an elegant formal response to a practical problem. The theory employs some of the very theorems Hardy praises as interesting (such as the law of quadratic reciprocity).

Regardless of who has the correct theory of interest, interest is a genuine property of mathematical facts. Moreover, interest is a holistic property; a fact can be interesting without its elements being interesting.

Conservation of Interest

My main objection is to premise B (`If some integer is the first uninteresting integer, then that fact makes the integer interesting’) of Beckenbach’s sophism rather than to its conclusion C (`All integers are interesting’). However, the implausibility of the conclusion gives us reason to doubt premise B.

Universality precludes distinctiveness:. We cannot think about everything, so standards of interest aim to be discriminative. For human beings, the degree of selectivity is magnified by the sequential nature of consciousness.

Of course, a standard that is elitist in intent might be inadvertently all-inclusive. A wordsmith might wish to focus on just those numerals that share a letter with the succeeding numeral. He is surprised to find that every English numeral for an integer has this property: (one, two), (two, three), (three, four), and so on. Since the wordsmith also treasures *surprising* universality, the frustration of his initial design fills him with delight rather than dismay.

Delight would also be the proper reaction to a cogent proof of Beckenbach's conclusion. *My* objection to premise B does not show there is no such proof.

However, reflection on the competence of mathematicians does suggest that Beckenbach's conclusion is false. When mathematicians aim to adopt a selective standard, they generally succeed. In the rare circumstances in which they fail, they react by raising the standard.

In this sense, mathematicians are victims of their success. They adopt standards that ensure that large finite samples of numbers will be dominated by a dull majority. As mathematicians improve, their aptitude and elitism will only strengthen domination by the dull. Bold souls may extrapolate to Fredrich Nietzsche's lament "Against boredom even the gods struggle in vain." (2005, section 48)

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