The No Alternatives Argument

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Abstract

Scientific theories are hard to find, and once scientists have found a theory H, they often believe that there cannot be many distinct alternatives to H. But is this belief justified? What should scientists believe about the number of alternatives to H, and how should they change these beliefs in the light of new evidence? These are some of the questions that we will address in this paper. We also ask under which conditions failure to find an alternative to H confirms the theory in question. This kind of reasoning (which we call the *no alternatives argument*) is rather popular in science and therefore deserves a careful philosophical analysis.

1 Introduction

We typically confirm or disconfirm a scientific hypothesis with a piece of empirical evidence. For example, the observation of a black raven confirms the hypothesis that all ravens are black, and certain clicks in a particle detector confirm the existence of the top quark. However, there are situations where crucial empirical evidence is unattainable over long periods of time. Such situations arise with particular force in contemporary high energy physics, where the characteristic empirical signatures of theories like grand unified theories or string theory must be expected to lie many orders of magnitude beyond the reach of present day experimental technology. They are entirely common also in scientific fields such as palaeontology or anthropology, where scientists must rely on the scarce and haphazard empirical evidence they happen to find in the ground. Interestingly, scientists are at times quite confident regarding the adequacy of their theories even when empirical evidence is largely or entirely absent. Trust in a theory H in such cases must be based on what we want to call non-empirical evidence for H, i.e. evidence that is neither deductively nor probabilistically predicted by H.

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It is often assumed that arguments relying on non-empirical evidence constitute mere speculation and do not contribute to actual theory confirmation. Therefore they are regarded as being void of objective scientific weight. This essay challenges this claim. To do so, we first look at arguments in science that rely on non-empirical evidence. As we will see, scientists at times develop a considerable degree of trust in a theory H based on the observation that no alternatives to H have been found despite considerable efforts to do so (Sect. 2). We call this argument the *no alternatives argument* (NAA). In order to formalize this argument, we introduce the concept of the number of possible alternative theories and study how our beliefs about the number of alternatives respond to empirical evidence (Sect. 3). On that basis, we construct a probabilistic model of the NAA and show the possibility of non-empirical theory confirmation (Sect. 4).

For the NAA to have significant impact on scientific reasoning, we have to argue that it confers substantial confirmation upon a scientific theory. We discuss under which circumstances the NAA becomes significant, and tentatively sketch a *meta-inductive argument* (MIA) that may make a case for the relevance of NAA (Sect. 5). Finally, we put our findings into context and briefly look at applications in epistemology and philosophy of science (Sect. 6). Throughout this paper, we operate in the framework of Bayesian epistemology.[1]

2 The conceptual framework

In order to understand the problem of non-empirical theory confirmation, we first contrast it with its empirical counterpart. We call some evidence E empirical evidence for H if and only if (i) H predicts E and (ii) E is observed. The evidence E can be observed directly (as we observe a black raven) or by means of measurement instruments (as typically in modern science). If T denotes the statement that hypothesis H is empirically adequate, then this amounts to $P(\mathrm{E}|\mathrm{T}) > \mathrm{P}(\mathrm{E}|\neg\mathrm{T})$, or in a more familiar form, $P(\mathrm{T}|\mathrm{E}) > \mathrm{P}(\mathrm{T})$. Bayesian epistemologists use this inequality as a criterion for whether E confirms H.

Non-empirical evidence F for a theory H is evidence that is neither deductively nor probabilistically predicted by H. In other words, F exemplifies evidence that does not fall into the intended domain of H or a related scientific theory. Then, how is it possible at all that F is evidence for H? Does F qualify as evidence in an argument from ignorance (Walton 1995, Hahn and Oaksford 2007, Sober 2009)?

The most plausible way to solve this problem is to deploy a two step process. First, we find a statement that does predict evidence of the type F. Then, we show that this statement is positively relevant for the empirical adequacy of H. In the case of the NAA, our non-empirical evidence $\mathrm{F_A}$ consists in the fact that scientists have not found any alternatives to a specific solution of a research problem, despite looking for them with considerable energy and for

[1]Recent surveys of Bayesian epistemology are Hájek and Hartmann (2010) and Hartmann and Sprenger (2010).

a long time. Then it is straightforward to identify a natural candidate for a statement that predicts F_A, namely that *the number of alternative theories is small.* If there were only very few alternatives to H, then this would render F_A more likely than a scenario where a huge number of possible alternative theories can be constructed: in the latter case, one might expect that scientists would have found at least one of them already. Thus, F_A is rendered more likely by the hypothesis that there are very few (or perhaps just one) possible scientific theories which can account for the available data.

The number k of possible scientific theories which can account for a certain set of data is in turn relevant for the probability of the empirical adequacy of H. We assume that scientists who develop a theory in accordance with available data do not have a perfectly reliable method to select the true theory if false theories can be constructed which also reproduce the available data. This assumption seems to be fairly plausible in science: scientists often come up with an incorrect, but fruitful theory when they begin to investigate a new field (Bohr's model of the atom is a good example for this claim).

Based on the above reasoning, we introduce a random variable Y measuring the number of alternatives to H, and the set of propositions $\mathrm{Y_k} := \{Y = k\}$ expressing that there are k adequate and distinct alternatives which can account for the available data E. We will later show that, via its effect on the $\mathrm{Y_k}$, the non-empirical evidence F_A confirms H under plausible conditions.[2]

Note that any inference about the number of alternatives to a theory H requires an account of what counts as an alternative to a given theory and how scientific theories are individuated. Such an account will depend on the specific scientific context and scientists typically have a good grip on what counts as a distinct theory. There are, however, two conditions that are worth stressing and that are important for the following discussion.

First, different theories make different predictions. If two theories make exactly the same predictions, then we consider them to be identical. For example, we consider the De Broglie-Bohm version and the Copenhagen version of quantum mechanics as representing the same theory (Cushing 1994). As a consequence, we are only interested in arriving at empirically adequate theories, and not in the more ambitious goal of finding true theories (cf. van Fraassen 1980).

Second, different theories provide different solutions to a given scientific problem. That is, theories which only differ in a detail, say in the value of a parameter, do not count as different theories. For example, the Higgs model in particle physics is treated as *one* theory, although the hypothesized (and perhaps finally discovered) Higgs particle could have different mass values. What is at stake here is the general adequacy of the Higgs model as a theoretical mechanism that can explain particle masses with the help of a scalar field.

This condition makes it plausible that the number of alternatives to a given theory is finite. If it were enough to slightly modify the value of a certain pa-

[2]Throughout this paper we follow the convention that propositional variables are printed in italic script, and that the instantiations of these variables are printed in roman script. See Bovens and Hartmann (2003).

rameter in order to arrive at a new theory, then coming up with new theories would be an easy and not very creative task. However, inventing a novel mechanism, or telling a new story of why a certain phenomenon came about is much harder. It is not so plausible that there is an infinite number of such distinct stories. This brings us to the next question: what can empirical evidence tell us about the (probable) number of alternatives to a given theory H?

3 Assessing the number of alternative theories

It is a truism of philosophy of science that there are, for each set of data, infinitely many theories that agree with those data. A particularly well-known case is the curve-fitting problem (Forster and Sober 1994): for any number of points in a coordinate system, there is an infinite number of families of curves that passes through them. This observation seems to undercut any confirmatory argument that relies on the number of alternatives to a theory. If that number is *a priori* infinite, then no evidence, whether empirical or non-empirical, can change this fact, and our envisioned road to non-empirical theory confirmation is blocked.

However, in our scenario, additional constraints on what counts as an adequate solution to a scientific problem have entered the game. For instance, the demands for coherence with other theories can substantially narrow down the space of possible solutions. Theories are asked to do more than just tracking past data, and theory individuation can be quite coarse. In such circumstances, it makes sense to assume that the number of adequate alternatives to H is finite.

If an agent is convinced that the number of alternatives Y to a theory H is finite, then a particularly interesting belief structure may arise, namely when she also asserts that the *expected number of alternatives to H* is infinite. This sounds paradoxical at first sight, but it is perfectly coherent. Formally, we can express this tension as follows (proof in appendix A):

Proposition 1. *For any $k \in \mathbb{N}$ and any $\varepsilon > 0$, an agent may coherently believe that $P(Y \leq k) \geq 1 - \varepsilon$ while at the same time believing that*

$$\langle Y \rangle = \sum_{k=1}^{\infty} k \, P(\mathrm{Y_k}) = \infty. \tag{1}$$

In this notation, $\langle Y \rangle$ denotes the *expectation value* of Y. In other words, an agent might consider it probable that there are few alternatives to H, and yet retain the belief that our best guess regarding the number of alternatives to H is "infinitely many" or "greater than any number that we can imagine". This phenomenon is well-known from paradoxes of decision theory, such as the valuation of the St. Petersburg Game, but to our knowledge, its epistemic variant has not been explored before. In other words, Proposition 1 points out the possibility of a strong epistemic tension within a single agent regarding the number of alternatives to a theory H. This tension transfers to the agent's assessment of the problem of theoretical underdetermination: she might believe that H is fundamentally underdetermined by evidence (because our best guess for the number of alternatives is infinity), but at the same time be quite strongly convinced that elimination of a finite set of alternatives will eventually lead us to the empirically adequate theory.[3]

Let us now study whether such a belief structure is responsive to evidence E, be it empirical or non-empirical. First, we ask under which circumstances evidence E *lowers* the expected number of alternatives. In answer to this question, we can demonstrate the following theorem (proof in appendix A):

Theorem 1. *Let* Y_k^+ *denote the proposition that there are at least k alternatives to theory H, and let* Y_k^- *denote the proposition that there are at most* $k-1$ *alternatives to H. Then, if* $P(\mathrm{E}|\mathrm{Y}_\mathrm{k}^+) \leq \mathrm{P}(\mathrm{E}|\mathrm{Y}_\mathrm{k}^-)$ *for all* $k \in \mathbb{N}$ *and* $P(\mathrm{E}|\mathrm{Y}_\mathrm{k}^+) < \mathrm{P}(\mathrm{E}|\mathrm{Y}_\mathrm{k}^-)$ *for at least one* $k \in \mathbb{N}$*, it will also be the case that* $\langle Y \rangle > \langle Y \rangle_\mathrm{E}$*, the latter expression denoting the expectation value of* Y *under* $P(\cdot|\mathrm{E})$*.*

In other words, if a decrease in the number of alternatives raises the probability of E, then the expected number of alternatives to H will be smaller *a posteriori* than it was *a priori*.

The condition of the theorem can be satisfied by empirical as well as non-empirical evidence. For non-empirical evidence such as $\mathrm{F_A}$:= "the scientists have not yet found an alternative to H", it is evident that this evidence is the more likely the less alternatives there are. (See condition **A3** in the next section.) Also, the required condition seems very plausible with respect to *contrastive* empirical evidence E predicted by theory H. The more alternative theories with diverse predictions are possible, the less likely it is that the data which we observe are correctly predicted by H, but not by its competitors.

Second, we ask the following question: Can an agent who believes that $\langle Y \rangle = \infty$ come to the belief that $\langle Y \rangle_\mathrm{E} < \infty$? Indeed, she can. The following theorem characterizes that case by stating four different sufficient conditions for such a belief change (proof in appendix A).

Theorem 2. *Assume that* $\langle Y \rangle = \infty$*. Then any of the following conditions on evidence E with* $P(\mathrm{E}) \neq 0$ *is* sufficient *for* $\langle Y \rangle_\mathrm{E} < \infty$*.*

1. *The sequence* $(k \cdot P(\mathrm{E}|\mathrm{Y_k}))_{\mathrm{k}\in\mathbb{N}}$ *is bounded.*

2. *There are* $\alpha, \beta > 0$ *be such that* $\alpha + \beta > 2$*, and that* $(k^\alpha P(\mathrm{E}|\mathrm{Y_k}))_{\mathrm{k}\in\mathbb{N}}$ *and* $(k^\beta P(\mathrm{Y_k}))_{\mathrm{k}\in\mathbb{N}}$ *are bounded.*

3. $\sum_{k=1}^\infty P(\mathrm{E}|\mathrm{Y_k}) < \infty$ *and there is a* $N_0 \in \mathbb{N}$ *such that* $(P(\mathrm{Y_k}))_{\mathrm{k}\in\mathbb{N}}$ *is, for all* $k \geq N_0$*, monotonically decreasing.*

4. $P(\mathrm{E}|\mathrm{Y_k}) \to 0$ *and there is an* $\alpha > 0$ *such that*

$$\limsup_{k\to\infty} k^{2+\alpha} \left| P(\mathrm{E}|\mathrm{Y_k}) - \mathrm{P}(\mathrm{E}|\mathrm{Y_{k-1}}) \right| < \infty.$$

[3]When we are talking about theoretical underdetermination, we do not have in mind Forster and Sober's conception in their 1994 paper, but rather the existence of several *autonomous*, content-wise distinct theories for a given set of data.

These four conditions correspond to different rationales. All of them constrain the rate of decline of $P(\mathrm{E}|\mathrm{Y_k})$ as k increases, that is, E is more unlikely if more alternatives are around. The first and second condition could also be expressed as $P(\mathrm{E}|\mathrm{Y_k}) \in \mathcal{O}(k^\alpha)$ for a suitable exponent α. The third condition makes a similar constraint by demanding that $\sum_{k=1}^{\infty} P(\mathrm{E}|\mathrm{Y_k})$ converges, and the fourth condition controls the differences between the values of $P(\mathrm{E}|\mathrm{Y_k})$ for neighboring values of k.

Note that only the second condition makes an assumptions about the values of $P(\mathrm{Y_k})$. This is in line with the idea that we have little grip on the rational beliefs about the number of empirically adequate alternatives, whereas we might be in a better position to assess how our evidence E is affected by the number of alternatives.

Most empirical evidence that we encounter in practice will be related to some test or corroboration of H. In that case, it is reasonable to assume that $P(\mathrm{E}|\mathrm{Y_k}) \to 0$: if there are more and more alternatives, why should H, instead of an unconceived alternative (Stanford 2006), survive empirical tests? Moreover, the conditions in Theorem 2 make constraints on the *speed* of convergence. If $P(\mathrm{E}|\mathrm{Y_k})$ falls "fast enough" (e.g. $\mathcal{O}(1/k)$), or if the precise value of $\mathrm{Y_k}$ for large k makes a very small difference to the likelihood of E, then we will abandon the belief that the expected number of alternatives is infinite.

Conversely, we may also ask the question: can an agent who believes that Y takes finite values only, but who believes that $\langle Y \rangle < \infty$ come to the belief that $\langle Y \rangle_{\mathrm{E}} = \infty$ for any evidence E? The answer to this question is a no. No empirical evidence is able to overturn the verdict that the expected number of alternatives to H is finite (proof in appendix A):

Proposition 2. *If $\langle Y \rangle < \infty$ then for any evidence E (empirical or non-empirical) with $P(\mathrm{E}) \neq 0$, $\langle Y \rangle_{\mathrm{E}} < \infty$.*

This means that the belief that the expected number of alternatives is finite is *not* responsive to empirical evidence: once you believe it, you will always believe it, independently of which evidence you receive. This points to an interesting asymmetry: evidence can change the belief that there are infinitely many alternatives, but it cannot change the belief that there are finitely many alternatives. The asymmetry between Theorem 2 and Proposition 2 confirms a suspicion that Theorem 1 has already prompted, namely that empirical evidence usually lowers the expected number of alternatives, contributing to the convergence of scientific inquiry. In the following section, we investigate whether non-empirical evidence can lower the expected number of alternatives and indirectly confirm the currently best theory.

4 The no alternatives argument

Having investigated the belief dynamics for the number of alternatives to a theory H, we now proceed to a formal analysis of the no alternatives argument (NAA). In this case, the non-empirical evidence consists in the observation that the scientists have not yet found an alternative to H. In accordance with

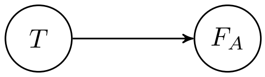

Figure 1: The Bayesian Network representation of the two-propositions scenario.

our previous analysis, this observation is taken to indicate that, in some sense, there actually are not too many alternatives to H. Focusing on the case of string theory, Dawid (2006) called this the *argument of no choice*.

Following this line of reasoning, we will reconstruct the no alternatives argument based on the notion that there exists a specific but unknown number k of possible scientific theories. These theories have to be compatible with a set of constraints $\mathcal{C}$ – whose nature is left to the scientific community, cf. Sect. 2 –, to be compatible with the existing data $\mathcal{D}$, and to give distinguishable predictions for the outcome of some set $\mathcal{E}$ of future experiments. We will then show that failure to find an alternative to H raises the probability of H being empirically adequate and thus confirms H.

To do so, we introduce two binary propositional variables, already briefly encountered in Sect. 2. T has the values: T: "The hypothesis H is empirically adequate", and $\neg$T: "The hypothesis H is not empirically adequate". The propositional variable F_A has the values: $\mathrm{F_A}$: "The scientific community has not yet found an alternative to H that fulfills $\mathcal{C}$, explains $\mathcal{D}$ and predicts the outcomes of $\mathcal{E}$", and $\neg\mathrm{F_A}$: "The scientific community has found an alternative to H that fulfills $\mathcal{C}$, explains $\mathcal{D}$ and predicts the outcomes of $\mathcal{E}$". Note that "explains" may be replaced by "may be expected to explain", as in the case of string theory.

We would like to explore under which conditions $\mathrm{F_A}$ confirms H, i.e. when

$$P(\mathrm{T}|\mathrm{F_A}) > P(\mathrm{T}) \ . \qquad (2)$$

This equation suggests a direct influence of T on F_A. See Figure 1 for a Bayesian Network representation of this scenario. But since such a direct influence is blocked by the non-empirical nature of F_A, we have to propose a third variable Y which mediates the connection between T and F_A. Like in the previous section, Y has values in the natural numbers, and Y_k corresponds to the proposition that there are exactly k hypotheses that fulfill $\mathcal{C}$, explain $\mathcal{D}$ and predict the outcomes of $\mathcal{E}$. The respective alternatives cannot be distinguished now, but they make different predictions and can therefore (in principle) be told apart.

We should also note that the value of F_A – that the scientists find/do not find an alternative to H – does not only depend on the number of available alternatives, but also on the the complexity of the problem, the cleverness of the scientists, or the available computational, experimental, and mathematical tools. Call the variable that models contextual influence A, and let it take values in the natural numbers, with $\mathrm{A_j} := \{\mathrm{A} = \mathrm{j}\}$ and $a_j := P(\mathrm{A_j})$. The higher the values of A, the more complex the problem. It is clear that A has no direct influence on Y and T (or vice versa). So the only relevant link in the graph goes from A into F_A. It is natural to assume that the more complex a problem

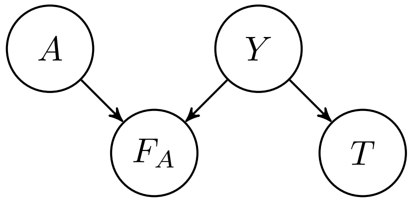

Figure 2: The Bayesian Network representation of the four-propositions scenario.

is, the more likely it is that the scientists did not find an alternative to H, but we do not need such an assumption in order to show the validity of the NAA. We therefore restrict ourselves to the following five assumptions.

A1. The variable T is conditionally independent of F_A given Y:

$$T \perp\!\!\!\perp F_A | Y \tag{3}$$

Hence, the following holds: Once we know that there are exactly k possible alternatives to H that account for the given data, we will not change our belief in T if we learn that the scientific community has not yet found an alternative to H. Whether or not the scientists have already found one or more alternative to H does not matter once we know that there are k alternatives. In our opinion, this assumption is eminently sensible.

Figure 2 shows the corresponding Bayesian Network. To complete it, we have to specify the prior distribution over the remaining root node Y and the conditional distributions over the other two nodes, given the values of their parents. This is done in the following four assumptions.

A2. The prior probabilities

$$y_k := P(\mathrm{Y_k}) \tag{4}$$

are smaller than 1, i.e. $0 \leq y_k < 1$.

This assumption reflects the fact that we do not know the number of possible theories a priori.

A3. The conditional probabilities

$$f_{kj} := P(\mathrm{F_A}|\mathrm{Y_k A_j}) \tag{5}$$

are monotonically decreasing in k for all $j \in \mathbb{N}$ and monotonically increasing in j for all $k \in \mathbb{N}$.

The decrease in the first argument reflects the intuition that the more alternative theories there are, the more likely it is that the scientists find at least one of them, given a certain level of complexity. The increase in the second argument reflects the intuition that the more complex a problem gets, the less likely are scientists to find an alternative to H, for a fixed number of actual alternatives.

A4. The conditional probabilities

$$t_k := P(\mathrm{T}|\mathrm{Y_k}) \tag{6}$$

are monotonically decreasing in k.

This assumption reflects the intuition that the more alternative theories there are, the less likely it is that scientists have found the right one.

A5. There is at least one pair (i, j) with $j > i$ for which (i) $y_i\, y_j > 0$, (ii) $f_{ik} > f_{jk}$ for some $k \in \mathbb{N}$, and (iii) $t_i > t_j$.

The occurrence of instances of $f_{ik} > f_{jk}$ and $t_i > t_j$ seems plausible based on the reasoning given in Sect. 2 and repeated in the context of A3 and A4 above.[4]

With these assumptions, we can show that (proof in appendix B):

Theorem 3. *If assumptions A1 to A5 hold, then* $\mathrm{F_A}$ *confirms* T, *that is,* $P(\mathrm{T}|\mathrm{F_A}) > P(\mathrm{T})$.

We have therefore shown that $\mathrm{F_A}$ confirms the hypothesis in question under rather weak (and plausible) assumptions. Note that $\mathrm{F_A}$ does not confirm H if we believe, for example, with probability 1 that the number of alternatives is infinite. How much $\mathrm{F_A}$ confirms H depends of course on the various parameter assignments (for details, see appendix B).

5 The significance of NAA

We have seen that the no alternatives argument provides theory confirmation under plausible conditions. The question remains, however, whether this confirmation is of significant strength and whether using NAA in a specific situation is justified.

The Bayesian network representation of NAA given in Figure 2 suggests that such significance is difficult to attain by NAA on its own without further supportive reasoning. According to Figure 2, $\mathrm{F_A}$ may confirm A (limitations to the scientists' capabilities) as well as Y (limitations to the number of possible theories). From the Bayesian Network depicted in Figure 1, it is easy to see that for $l \in \mathbb{N}$,

$$P(\mathrm{A_l}|\mathrm{F_A}) = \frac{P(\mathrm{A_l}, \mathrm{F_A})}{P(\mathrm{F_A})} = \frac{a_l \cdot \sum_k y_k\, f_{kl}}{\sum_{j,k} a_j\, y_k\, f_{kj}}\,.$$

Hence,

$$\frac{P(\mathrm{A_l}|\mathrm{F_A})}{P(\mathrm{A_l})} = \frac{\sum_k y_k\, f_{kl}}{\sum_{j,k} a_j\, y_k\, f_{kj}}\,. \tag{7}$$

We cannot give fully general conditions for when this expression is greater than 1. However, we observe that (7) is monotonically increasing in l since the

[4]Compare our remark in the third paragraph of Sect. 3. Assumption A5 is particularly credible if we assume that the scientists' degrees of belief are *regular*: that is, they assign probability zero only to logically impossible propositions, or those that have been proven wrong by experience.

f_{kl} are monotonically increasing in l for fixed k (see assumption **A3**). That is, the degree of confirmation F_A lends to A_l, as expressed by the ratio measure of degree of confirmation, increases with l. Thus, $\mathrm{F_A}$ typically confirms the claim that the problem at hand is rather complicated (i.e., that it has a high rank l) and typically disconfirms claims that it is not particularly complicated (i.e., that it has a low rank l). This conclusion depends, however, on the precise values of a_j and y_k, which we left unrestricted.

To accentuate the resulting problem, note that the situation could be such that A_l is, for sufficiently large l, confirmed much more than T. While we acknowledge that failure to find an alternative confirms the empirical adequacy of H, this failure would then confirm, to a stronger degree, the hypothesis that the problem is just too complex for the scientific means we have at our disposal, weakening the significance of the NAA.

Usually, if two theories are not distinguished by evidence collected up to some point, it is possible to distinguish between them by collecting further empirical evidence. The strategy of NAA, on the contrary, is incapable of distinguishing between A_l (for high l) and T. We could wait for 1000 years and the scientists' failure to find alternatives could still be taken to support both A_l and T. So it is unclear whether we can ever speak of *contrastive* confirmation of T vis-à-vis A_l.

There is another question regarding the strength of the NAA. The prior probabilities of the Y_k – the number of alternatives to H – play a crucial role in determining the posterior probability of T, and thus, in determining the degree to which F_A confirms T. In other words, the strength of the NAA depends on what exactly the priors of the Y_k look like. Given that the priors reflect the subjective judgments of a scientist, different scientists may come to very diverse judgments on the significance of the NAA.

Therefore, it would be desirable to have some procedure that ensures agreement on those prior probabilities. This would allow scientists to agree on how strong a no alternative argument really is. Here we tentatively explore whether a *meta-inductive argument* (MIA) can serve that purpose.

The gist of the argument is best illustrated with a special case. It is notoriously difficult to find precisely the one theory that makes the correct *predictions*, rather than just accommodating existing data (Kahn et al. 1992, Hitchcock and Sober 1994). But remarkably, scientists have often succeeded at identifying that theory. Now, if there are a lot of alternative solutions to a given problem, then there is no reason to assume that scientists identified the one which will prevail in the future. Thus, repeated predictive success within a particular research program seems to justify the assumption that there may be few available alternatives in that theoretical context.

Now, assume that a novel theory H shows similarities to theories H_1, H_2, etc., in the same research program. The joint feature of these theories may be a certain theoretical approach, a shared assumption, or any other relevant characteristic. Let us assume that a substantial share of the theories to which H is similar have been empirically confirmed. Assume further that for those theories, we have empirically grounded posterior beliefs about the number of alternatives. Then, it seems reasonable to use these posteriors as priors for the number of alternatives to H. After all, H is quite similar to H_1, H_2, etc. Statisticians routinely use this way of determining "objective" prior beliefs and refer to it as the *empirical Bayes method* (Carlin and Louis 2000).

If this move is accepted, then one is in a much better position to appreciate the significance of NAA, due to agreement on the prior probabilities of the Y_k. Of course, this account of MIA remains informal and does not provide a rigorous justification for the practical significance of NAA. Future work is required to formalize this approach, and to strengthen the link between both arguments.

6 Conclusions

In this paper, we have completed three tasks: (i) we have studied how beliefs about the number of alternatives to a theory change in the light of new empirical or non-empirical evidence and pointed out a paradoxical tension in beliefs about theoretical underdetermination; (ii) we have formalized the no alternatives argument (NAA) and explored under which conditions non-empirical evidence confirms a scientific theory H; and (iii) we have sketched a meta-inductive argument (MIA) that allows us to assess the number of alternatives to H before empirical evidence in favor or against H is found.

In future work, we plan to relate the formal account given in this paper more closely to case studies from science. Here we are particularly interested in the case of string theory and the reasoning strategies employed in fields such as palaeontology and anthropology where contingent evolutionary details have to be reconstructed based on scarce and highly incomplete evidence. We will explore what role the NAA plays in these fields, and how good the argument actually is.

There are also two philosophical applications which we would like to point out. First, *Inference to the Best Explanation* (Douven 2011, Lipton 2004) can, to a certain extent, be explicated in terms of the NAA. In as much as the notion "best explanation" is understood as "the only genuinely satisfactory explanation", the fact that no other genuinely satisfactory explanation has been found can play the role of the claim of no alternatives in our argument, supporting the empirical adequacy of the currently best explanation. (Incidentally, nothing fundamental hinges on the fact that precisely one satisfactory theory has been found. If two different theories were found, the fact that no others have turned up constituted a less striking but still relevant observation in our sense.)

Second, one may ask whether the no alternatives argument could also play a role in confirming general philosophical theories. The reputation of a philosophical theory is often based on the understanding that no other consistent answer has been found or is perhaps not even conceivable. Can reasoning of this kind be supported by the NAA? In principle, the answer to this question is yes, but there is a problem: philosophical theories do not have a record of empirical testing. Thus, we will be unable to quantify the significance of NAA with empirical data. Philosophy thus provides us with a neat example of the promises and limits of non-empirical theory confirmation beyond scientific contexts.

A Proof of the results in Section 3

Proof of Proposition 1: The proof proceeds by construction. For instance, let $P(Y \leq k) = 1 - \varepsilon$, let $P(\mathrm{Y_j}) = \mathrm{C/j^2}\ \forall \mathrm{j} > \mathrm{k}$, and choose C such that $\sum_{j\in\mathbb{N}} P(\mathrm{Y_j}) = 1$. (The series $\sum_j 1/j^2$ converges.) Then, it is easy to check that

$$\begin{aligned}\langle Y \rangle &= \sum_{j=1}^{k} j\, P(\mathrm{Y_j}) + \sum_{j=k+1}^{\infty} j\, P(\mathrm{Y_j}) \\ &\geq C \sum_{j=k+1}^{\infty} \frac{1}{j} \\ &= \infty. \quad \Box\end{aligned}$$

Proof of Theorem 1: Let us define

$$\mathrm{Y_k^+} := \{\mathrm{Y} \geq k\} \qquad\qquad \mathrm{Y_k^-} := \{\mathrm{Y} < k\}$$

We have assumed that $P(\mathrm{E}|\mathrm{Y_k^+}) \leq P(\mathrm{E}|\mathrm{Y_k^-})\, \forall k \in \mathbb{N}$. Since $\mathrm{Y_k^+}$ and $\mathrm{Y_k^-}$ are an exhaustive partition of the probability space, this entails that Y_k^+ and E are negatively relevant to each other, and that

$$P(\mathrm{Y_k^+}|\mathrm{E}) \leq P(\mathrm{Y_k^+})\ \forall k \in \mathbb{N}, \tag{8}$$

with inequality for at least one k. Since $P(\mathrm{Y_k}) = P(\mathrm{Y_k^+}) - P(\mathrm{Y_{k+1}^+})$, we obtain

$$\begin{aligned}\langle Y \rangle &= k \sum_{k=1}^{\infty} P(\mathrm{Y_k}) \\ &= \sum_{k=1}^{\infty} k\, (P(\mathrm{Y_k^+}) - P(\mathrm{Y_{k+1}^+})) \\ &= P(\mathrm{Y_1}) + \sum_{k=2}^{\infty} kP(\mathrm{Y_k^+}) - (k-1)P(\mathrm{Y_k^+}) \\ &= \sum_{k=1}^{\infty} P(\mathrm{Y_k^+}), \end{aligned} \tag{9}$$

and similarly

$$\langle Y \rangle_{\mathrm{E}} = \sum_{k=1}^{\infty} P(\mathrm{Y_k^+}|\mathrm{E}). \tag{10}$$

Combining (9) and (10), we conclude

$$\langle Y \rangle_{\mathrm{E}} = \sum_{k=1}^{\infty} P(\mathrm{Y_k^+}|\mathrm{E}) > \sum_{k=1}^{\infty} P(\mathrm{Y_k^+}) = \langle Y \rangle \tag{11}$$

because of $P(\mathrm{Y}_\mathrm{k}^+|\mathrm{E}) \leq P(\mathrm{Y}_\mathrm{k}^+)\,\forall k \in \mathbb{N}$ (see (8)), and because we have assumed inequality for at least one $k \in N$. □

Proof of Theorem 2: Proof of the first statement. Assume that $(k \cdot P(\mathrm{E}|\mathrm{Y}_\mathrm{k}))_{k\in\mathbb{N}}$ is bounded, that is, there is a $C > 0$ such that $k \cdot P(\mathrm{E}|\mathrm{Y}_\mathrm{k}) < C$. Then it will be the case that

$$\begin{aligned} \langle \mathrm{Y} \rangle_\mathrm{E} &= \frac{1}{P(\mathrm{E})} \sum_{k=1}^{\infty} k\, P(\mathrm{Y}_\mathrm{k}) P(\mathrm{E}|\mathrm{Y}_\mathrm{k}) \\ &\leq C \cdot \frac{1}{P(\mathrm{E})} \sum_{k=1}^{\infty} P(\mathrm{Y}_\mathrm{k}) \\ &< \infty, \end{aligned}$$

proving the sufficiency of the first condition.

Related to this is the case that $k^\alpha \cdot P(\mathrm{E}|\mathrm{Y}_\mathrm{k}) \leq \mathrm{A}_\alpha$ and $k^\beta \cdot P(\mathrm{Y}_\mathrm{k}) \leq \mathrm{A}_\beta$ for all $k \in \mathbb{N}$ and some constants $A_\alpha, A_\beta > 0$, with the additional constraints $\alpha, \beta > 0$ and $\alpha + \beta > 2$. Then we have

$$\begin{aligned} \langle Y \rangle_\mathrm{E} &= \frac{1}{P(\mathrm{E})} \sum_{k=1}^{\infty} k^{1-\alpha-\beta}\, (k^\alpha\, P(\mathrm{E}|\mathrm{Y}_\mathrm{k})) \left(k^\beta\, P(\mathrm{Y}_\mathrm{k})\right) \\ &\leq \frac{1}{P(\mathrm{E})} A_\alpha A_\beta \sum_{k=1}^{\infty} k^{1-(\alpha+\beta)} \\ &< \infty \end{aligned}$$

because by assumption, $1 - (\alpha + \beta) < -1$, ensuring the convergence of the series. In the remainder of the proof we will focus on the properties of the series

$$\sum_{k=1}^{\infty} k\, P(\mathrm{Y}_\mathrm{k}) P(\mathrm{E}|\mathrm{Y}_\mathrm{k}) \tag{12}$$

which is sufficient for examining the convergence properties of $\langle \mathrm{Y} \rangle_\mathrm{E}$.

We now proceed to proving the sufficiency of the third condition. We assume that $\sum_{k=1}^{\infty} P(\mathrm{E}|\mathrm{Y}_\mathrm{k}) < \infty$ and that there is a $N_0 \in \mathbb{N}$ such that $P(\mathrm{Y}_\mathrm{k}) \geq P(\mathrm{Y}_{\mathrm{k}+1})$ for all $k \geq N_0$. By Dirichlet's criterion (Knopp 1964, 324), $\sum_{k=1}^{\infty} k\, P(\mathrm{Y}_\mathrm{k}) P(\mathrm{E}|\mathrm{Y}_\mathrm{k})$ converges if (i) $\sum_{k=1}^{\infty} P(\mathrm{E}|\mathrm{Y}_\mathrm{k}) < \infty$ and (ii) $k\, P(\mathrm{Y}_\mathrm{k}) \to 0$ monotonically. The first condition is fulfilled by assumption. The second clause of the criterion can, without loss of generality, be replaced by demanding that for $N_0 \in \mathbb{R}$, $(k\, P(\mathrm{Y}_\mathrm{k}))_{k\in\mathbb{N}}$ be monotonically decreasing for all $k \geq N_0$.

Assume that the second clause of the criterion is not satisfied, and that there is a sequence of natural numbers n_k such that

$$n_k P(\mathrm{Y}_{\mathrm{n}_\mathrm{k}}) < n_{k+1} P(\mathrm{Y}_{\mathrm{n}_{\mathrm{k}+1}}). \tag{13}$$

Then the (sub)sequence $(n_k\, P(\mathrm{Y}_{\mathrm{n}_\mathrm{k}}))_\mathrm{k}$ would not converge to zero, and consequently, $(k\, P(\mathrm{Y}_\mathrm{k}))_k$ would not converge to zero. But we have required that for some $N_0 \in \mathbb{R}$, $P(\mathrm{Y}_\mathrm{k})$ be a monotonically decreasing sequence for all $k \geq N_0$. It is well known that for such sequences, if $\sum_k P(\mathrm{Y_k})$ exists (which is the case here), then also $k\, P(\mathrm{Y_k}) \to 0$ (Knopp 1964, 125). Hence, a subsequence $(n_k\, P(\mathrm{Y_{n_k}}))_\mathrm{k}$ with property (13) cannot exist and the second part of the Dirichlet criterion is satisfied. Thus, the third condition of Theorem 2 is indeed sufficient.

Finally, we demonstrate the joint sufficiency of (i) $P(\mathrm{E}|\mathrm{Y_k}) \to 0$ and (ii) there is an $\alpha > 0$ such that

$$\limsup_{k\to\infty} k^{2+\alpha}\, |P(\mathrm{E}|\mathrm{Y_k}) - P(\mathrm{E}|\mathrm{Y_{k-1}})| < \infty.$$

In particular, there exists a $C > 0$ such that $k^{2+\alpha}\, |P(\mathrm{E}|\mathrm{Y_k}) - P(\mathrm{E}|\mathrm{Y_{k-1}})| \leq C$. Moreover, let $C' := 2C \sum_{k=1}^{\infty} 1/k^{1+\alpha}$.

By Abel's formula (Knopp 1964, 322), we can rewrite the partial sums of the series $\sum_{k=1}^{\infty} k\, P(\mathrm{Y_k})\, P(\mathrm{E}|\mathrm{Y_k})$ in the following way:

$$\begin{aligned}\sum_{k=1}^{N} k\, P(\mathrm{Y_k})\, P(\mathrm{E}|\mathrm{Y_k}) &= \sum_{k=1}^{N} \left(\sum_{j=1}^{k} j\, P(\mathrm{Y_j})\right) (P(\mathrm{E}|\mathrm{Y_k}) - P(\mathrm{E}|\mathrm{Y_{k+1}})) \\ &\quad + \left(\sum_{j=1}^{N} j\, P(\mathrm{Y_j})\right) P(\mathrm{E}|\mathrm{Y_{N+1}}).\end{aligned}$$

Note that the re-ordering of the terms does not affect the convergence properties since (12) has only positive members. It is now sufficient to show that both summands on the right side are uniformly bounded in N since this would mean that (12) has bounded partial sums and is thus convergent.

We begin by showing that the first summand is uniformly bounded:

$$\begin{aligned}&\left|\sum_{k=1}^{N} \left(\sum_{j=1}^{k} j\, P(\mathrm{Y_j})\right) (P(\mathrm{E}|\mathrm{Y_k}) - P(\mathrm{E}|\mathrm{Y_{k+1}}))\right| \\ \leq\ & \sum_{k=1}^{N} \left(\sum_{j=1}^{k} \frac{j}{k}\, P(\mathrm{Y_j})\right) \frac{1}{k^{1+\alpha}} k^{2+\alpha} |P(\mathrm{E}|\mathrm{Y_k}) - P(\mathrm{E}|\mathrm{Y_{k+1}})| \\ \leq\ & C \sum_{k=1}^{N} \left(\sum_{j=1}^{k} P(\mathrm{Y_j})\right) \frac{1}{k^{1+\alpha}} \\ \leq\ & C \sum_{k=1}^{\infty} \frac{1}{k^{1+\alpha}} \\ \leq\ & C',\end{aligned}$$

and the resulting bound is independent of N.

For the second term, because of $P(\mathrm{E}|\mathrm{Y_k}) \to 0$, there is, for any $k \in \mathbb{N}$, a $N_0(k)$ such that

$$\left(\sum_{j=1}^{k} j\, P(\mathrm{Y_j})\right) P(\mathrm{E}|\mathrm{Y_{N_0(k)}}) \leq C'/2. \tag{14}$$

Then we can calculate

$$\begin{aligned}
& \left(\sum_{j=1}^{k} j\,P(\mathrm{Y_j})\right) P(\mathrm{E}|\mathrm{Y_{k+1}}) \\
\leq\ & \left(\sum_{j=1}^{k} j\,P(\mathrm{Y_j})\right) |P(\mathrm{E}|\mathrm{Y_k}) - P(\mathrm{E}|\mathrm{Y_{k+1}})| + \left(\sum_{j=1}^{k} j\,P(\mathrm{Y_j})\right) P(\mathrm{E}|\mathrm{Y_{k+1}}) \\
\leq\ & \ldots \\
\leq\ & \left(\sum_{j=1}^{k} j\,P(\mathrm{Y_j})\right) \left(\sum_{l=k}^{N_0(k)-1} |P(\mathrm{E}|\mathrm{Y_l}) - P(\mathrm{E}|\mathrm{Y_{l+1}})|\right) + \left(\sum_{j=1}^{k} j\,P(\mathrm{Y_j})\right) P(\mathrm{E}|\mathrm{Y_{N_0(k)}}) \\
\leq\ & \left(\sum_{j=1}^{k} \frac{j}{k}\,P(\mathrm{Y_j})\right) \left(\sum_{l=k}^{N_0(k)-1} \frac{k}{l^{2+\alpha}} \left(l^{2+\alpha}\,|P(\mathrm{E}|\mathrm{Y_l}) - P(\mathrm{E}|\mathrm{Y_{l+1}})|\right)\right) + C'/2 \\
\leq\ & \left(\sum_{j=1}^{k} P(\mathrm{Y_j})\right) \left(\sum_{l=k}^{N_0(k)-1} \frac{C}{l^{1+\alpha}}\right) + C'/2 \\
\leq\ & \left(\sum_{l=1}^{\infty} \frac{1}{l^{1+\alpha}}\right) C + C'/2 \\
\leq\ & C',
\end{aligned}$$

proving the uniform boundedness of the second summand and thereby the sufficiency of the fourth and last condition for $\langle Y \rangle_E < \infty$. □

Proof of Proposition 2: By a straightforward application of Bayes' Theorem:

$$\begin{aligned}
\langle Y \rangle_{\mathrm{E}} &= \sum_{k=1}^{\infty} k\,P(\mathrm{Y_k}|\mathrm{E}) = \frac{1}{P(\mathrm{E})} \sum_{k=1}^{\infty} k\,P(\mathrm{Y_k})\mathrm{P}(\mathrm{E}|\mathrm{Y_k}) \\
&\leq \frac{1}{P(\mathrm{E})} \sum_{k=1}^{\infty} k\,P(\mathrm{Y_k}) = \frac{1}{P(\mathrm{E})} \langle Y \rangle \\
&< \infty. \quad \square
\end{aligned}$$

B Proof of the No Alternatives Argument

$\mathrm{F_A}$ confirms T if and only if $P(\mathrm{T}|\mathrm{F_A}) - P(\mathrm{T}) > 0$, that is, if and only if

$$\Delta := P(\mathrm{T}, \mathrm{F_A}) - P(\mathrm{T})P(\mathrm{F_A}) > 0.$$

We now use the theory of Bayesian Networks to obtain from the Bayesian

Network depicted in Figure 1:

$$\begin{aligned}
P(\mathrm{F_A}) &= \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} P(\mathrm{F_A}|\mathrm{Y_i A_j})P(\mathrm{Y_i A_j}) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_j\, y_i\, f_{ij} \\
P(\mathrm{T}) &= \sum_{k=1}^{\infty} P(\mathrm{T}|\mathrm{Y_k})P(\mathrm{Y_k}) = \sum_{k=1}^{\infty} t_k\, y_k \\
P(\mathrm{T},\mathrm{F_A}) &= \sum_{i=1}^{\infty} P(\mathrm{FT}|\mathrm{Y_i})P(\mathrm{Y_i}) = \sum_{i=1}^{\infty} y_i\, P(\mathrm{F_A}|\mathrm{Y_i})\, P(\mathrm{T}|\mathrm{Y_i}) \\
&= \sum_{i=1}^{\infty} y_i\, t_i \left(\sum_{j=1}^{\infty} P(\mathrm{F_A}|\mathrm{Y_i A_j})\, P(\mathrm{A_j}|\mathrm{Y_i})\right) \\
&= \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_j\, y_i\, t_i\, f_{ij}
\end{aligned}$$

Hence, we obtain, using $\sum_{k\in\mathbb{N}} y_k = 1$,

$$\begin{aligned}
\tilde{\Delta} &= \left(\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_j\, y_i\, t_i\, f_{ij}\right) - \left(\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_j\, y_i\, f_{ij}\right)\left(\sum_{k=1}^{\infty} y_k\, t_k\right) \\
&= \left(\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_j\, y_i\, t_i\, f_{ij}\right)\left(\sum_{k=1}^{\infty} y_k\right) - \left(\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_j\, y_i\, f_{ij}\right)\left(\sum_{j=1}^{\infty} t_k\, y_k\right) \\
&= \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} a_j\, y_i\, y_k\, t_i\, f_{ij} - a_j\, y_i\, y_k\, t_k\, f_{ij} \\
&= \sum_{j=1}^{\infty} a_j \sum_{i=1}^{\infty}\sum_{k\neq i=1}^{\infty} y_i\, y_k\, f_{ij}\,(t_i - t_k) \\
&= \sum_{j=1}^{\infty} a_j \sum_{i=1}^{\infty}\sum_{k>i} y_i\, y_k\, f_{ij}\,(t_i - t_k) + y_k\, y_i\, f_{kj}\,(t_k - t_i) \\
&= \sum_{j=1}^{\infty} a_j \sum_{i=1}^{\infty}\frac{1}{2}\sum_{k\neq i=1}^{\infty} y_i\, y_k\,(f_{ij}\,(t_i - t_k) + f_{kj}\,(t_k - t_i)) \\
&= \frac{1}{2}\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k\neq i=1}^{\infty} a_j\, y_i\, y_k\,(t_i - t_k)\,(f_{ij} - f_{kj}) \\
&> 0
\end{aligned}$$

because of **A3-A5** taken together. □

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