Companion *Mathematica* Notebook for "What is The 'Equal Weight View'?"

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July 30, 2009

The methods used in this notebook are special cases of a more general decision procedure for the probability calculus called **PrSAT**, which is described in this paper **<http://fitelson.org/pm.pdf>**, and which can be downloaded here **<http://fitelson.org/PrSAT/>**.

Table 1 Example:

p	q	P_1^0	P_2^0	P_1^1	P_2^1
T	T	0.1	0.55	a	d
T	F	0.2	0.25	b	e
F	T	0.3	0.15	c	f
F	F	0.4	0.05	1 - a - b - c	1 - d - e - f

Assume *p* & *q* and *p* & *~q* are only peer-propositions. Then, applying (SA), we'll have:

p	q	P_1^0	P_2^0	P_1^1	P_2^1
T	T	0.1	0.55	0.325	0.325
T	F	0.2	0.25	0.225	0.225
F	T	0.3	0.15	??	??
F	F	0.4	0.05	??	??

Table 2 Example: When we apply (SAMC), we end-up with:

p	q	P_1^0	P_2^0	P_1^1	P_2^1
T	T	0.1	0.55	0.325	0.325
T	F	0.2	0.25	0.225	0.225
F	T	0.3	0.15	a	c
F	F	0.4	0.05	b	d

Here is the minimization code that yields the (unique) (SAMC)-solutions for **a**, **b**, **c**, **d**.

```
Minimize[{EuclideanDistance[{0.1, 0.2, 0.3, 0.4}, {0.325, 0.225, a, b}],
  (Plus @@ {0.325, 0.225, a, b}) == 1 && 0 < a < 1 && 0 < b < 1}, {a, b}]
```

```
{0.287228, {a → 0.175, b → 0.275}}
```

```
Minimize[{EuclideanDistance[{0.55, 0.25, 0.15, 0.05}, {0.325, 0.225, c, d}],
  (Plus @@ {0.325, 0.225, c, d}) == 1 && 0 < c < 1 && 0 < d < 1}, {c, d}]
```

```
{0.287228, {c → 0.275, d → 0.175}}
```

Thus, (SAMC) yields the following pair of updated distributions:

p	q	P_1^0	P_2^0	P_1^1	P_2^1
T	T	0.1	0.55	0.325	0.325
T	F	0.2	0.25	0.225	0.225
F	T	0.3	0.15	0.175	0.275
F	F	0.4	0.05	0.275	0.175

Table 3 Example: Now, if we add the further assumption that *p* and *q* are also peer-propositions, we get the following single distribution:

p	q	P_1^0	P_2^0	$P_1^1 = P_2^1$
T	T	0.1	0.55	0.325
T	F	0.2	0.25	0.225
F	T	0.3	0.15	0.225
F	F	0.4	0.05	0.225

But, this solution violates (C), since:

$$P_i^1(q \mid p) = \frac{P_i^1(q\&\, p)}{P_i^1(p)} = \frac{0.325}{0.325+0.225} = 0.590909 \neq \frac{P_1^0(q|p)+P_2^0(q|p)}{2} = \frac{\frac{P_1^0(q\&\, p)}{P_1^0(p)}+\frac{P_2^0(q\&\, p)}{P_2^0(p)}}{2} = \frac{\frac{0.1}{0.1+0.2}+\frac{0.55}{0.55+0.25}}{2} = 0.510417.$$

What if we weaken (SAMC) to only require "approximate straight averaging" (ASAMC)? There are three versions of this sort of rule that we consider in the paper. The first is the weakest: it only requires that each peer get "close to the straight average". First, we define a ≈ b iff |a-b| < ϵ. Then, on the weakest version of the rule, we need to require that each peer ends-up satisfying (P) and (C), and also ends-up "close to the straight average". More precisely, in the example above, we're looking for a pair of vectors **<a,b,c,d>** and **<e,f,g,h>** such that **a** ≈ 0.325, **b** ≈ 0.225, **c** ≈ 0.225, and **d** ≈ 0.225, and (P) and (C) are satisfied by **<a,b,c,d>**, and **e** ≈ 0.325, **f** ≈ 0.225, **g** ≈ 0.225, and **h** ≈ 0.225, and (P) and (C) are satisfied by **<e,f,g,h>**. Moreover, we want both vectors to be *in between* P_1^0 *and* P_2^0.

p	q	P_1^0	P_2^0	P_1^1	P_2^1
T	T	0.1	0.55	a	e
T	F	0.2	0.25	b	f
F	T	0.3	0.15	c	g
F	F	0.4	0.05	d	h

Here, we see that (even on this weakest rendition of ASAMC) any solution requires that $\epsilon > \frac{1}{16}$.

$$\text{FindInstance}\Big[0.325-\epsilon < a < 0.325+\epsilon \,\&\&\, 0.1 < a < 0.55 \,\&\&\, 0.225-\epsilon < b < 0.225+\epsilon \,\&\&\, 0.2 < b < 0.25 \,\&\&\, 0.225-\epsilon < c < 0.225+\epsilon \,\&\&\, 0.15 < c < 0.3 \,\&\&\, 0.225-\epsilon < d < 0.225+\epsilon \,\&\&\, 0.05 < d < 0.4 \,\&\&\, 0 < \epsilon < \frac{1}{15} \,\&\&\, \frac{a}{a+b} == \frac{\frac{0.1}{0.1+0.2}+\frac{0.55}{0.55+0.25}}{2} \,\&\&\, a+b+c+d == 1 \,\&\&\, 0.325-\epsilon < e < 0.325+\epsilon \,\&\&\, 0.1 < e < 0.55 \,\&\&\, 0.225-\epsilon < f < 0.225+\epsilon \,\&\&\, 0.2 < f < 0.25 \,\&\&\, 0.225-\epsilon < g < 0.225+\epsilon \,\&\&\, 0.15 < g < 0.3 \,\&\&\, 0.225-\epsilon < h < 0.225+\epsilon \,\&\&\, 0.05 < h < 0.4 \,\&\&\, 0 < \epsilon \leq \frac{1}{15} \,\&\&\, \frac{e}{e+f} == \frac{\frac{0.1}{0.1+0.2}+\frac{0.55}{0.55+0.25}}{2} \,\&\&\, e+f+g+h == 1 \,\&\&\, a \neq e \,\&\&\, b \neq f \,\&\&\, c \neq g \,\&\&\, d \neq h,\ \{a, b, c, d, e, f, g, h, \epsilon\}\Big]$$

```
{{a → 0.259486, b → 0.248895, c → 0.246386, d → 0.245234,
  e → 0.260062, f → 0.249447, g → 0.268738, h → 0.221752, ϵ → 0.0660904}}
```

```
FindInstance[0.325 - ϵ < a < 0.325 + ϵ && 0.1 < a < 0.55 && 0.225 - ϵ < b < 0.225 + ϵ && 0.2 < b < 0.25 &&
  0.225 - ϵ < c < 0.225 + ϵ && 0.15 < c < 0.3 && 0.225 - ϵ < d < 0.225 + ϵ && 0.05 < d < 0.4 && 0 < ϵ < 1/15 &&
  a/(a + b) == (0.1/(0.1+0.2) + 0.55/(0.55+0.25))/2 && a + b + c + d == 1 && 0.325 - ϵ < e < 0.325 + ϵ && 0.1 < e < 0.55 &&
  0.225 - ϵ < f < 0.225 + ϵ && 0.2 < f < 0.25 && 0.225 - ϵ < g < 0.225 + ϵ && 0.15 < g < 0.3 &&
  0.225 - ϵ < h < 0.225 + ϵ && 0.05 < h < 0.4 && 0 < ϵ ≤ 1/16 && e/(e + f) == (0.1/(0.1+0.2) + 0.55/(0.55+0.25))/2 &&
  e + f + g + h == 1 && a ≠ e && b ≠ f && c ≠ g && d ≠ h, {a, b, c, d, e, f, g, h, ϵ}]

{}
```

The strongest version of this “approximate“ strategy forces the two peers to end-up in *exact* agreement on all peer-propositions, and to be “close to splitting the difference“ as well. For instance, let $\epsilon = 0.05$. Then, the question is this (for the strongest rendition). Is there a vector **<a,b,c,d>** such that **a** ≈ 0.325, **b** ≈ 0.225, **c** ≈ 0.225, and **d** ≈ 0.225, and (P) and (C) are satisfied by the resulting vector? Moreover, we want these values to be *in between* P_1^0 *and* P_2^0.

p	q	P_1^0	P_2^0	$P_1^1 = P_2^1$
T	T	0.1	0.55	a
T	F	0.2	0.25	b
F	T	0.3	0.15	c
F	F	0.4	0.05	d

Once again, the only way to satisfy this is if $\epsilon > \frac{1}{16}$.

```
FindInstance[0.325 - ϵ < a < 0.325 + ϵ && 0.1 < a < 0.55 && 0.225 - ϵ < b < 0.225 + ϵ &&
  0.2 < b < 0.25 && 0.225 - ϵ < c < 0.225 + ϵ && 0.15 < c < 0.3 && 0.225 - ϵ < d < 0.225 + ϵ &&
  0.05 < d < 0.4 && 0 < ϵ < 1/15 && a/(a + b) == (0.1/(0.1+0.2) + 0.55/(0.55+0.25))/2 && a + b + c + d == 1, {a, b, c, d, ϵ}]

{{a → 0.259486, b → 0.248895, c → 0.246386, d → 0.245234, ϵ → 0.0660904}}

FindInstance[0.325 - ϵ < a < 0.325 + ϵ && 0.1 < a < 0.55 && 0.225 - ϵ < b < 0.225 + ϵ &&
  0.2 < b < 0.25 && 0.225 - ϵ < c < 0.225 + ϵ && 0.15 < c < 0.3 && 0.225 - ϵ < d < 0.225 + ϵ &&
  0.05 < d < 0.4 && 0 < ϵ ≤ 1/16 && a/(a + b) == (0.1/(0.1+0.2) + 0.55/(0.55+0.25))/2 && a + b + c + d == 1, {a, b, c, d, ϵ}]

{}
```

What if we also require (PCI) to be satisfied as well? In the above example, both agents start out agreeing that *p* and *q* are *negatively* dependent. Thus, (PCI) requires that we ensure they continue to agree about this. This adds an additional constraint. But, so long as (C) is satisfied (*i.e.*, if $\epsilon > \frac{1}{16}$), this constraint can also be met. In general, this will be the case (at least, for the 2-atomic-proposition case).

```
FindInstance[0.325 - ϵ < a < 0.325 + ϵ && 0.1 < a < 0.55 && 0.225 - ϵ < b < 0.225 + ϵ && 0.2 < b < 0.25 &&
  0.225 - ϵ < c < 0.225 + ϵ && 0.15 < c < 0.3 && 0.225 - ϵ < d < 0.225 + ϵ && 0.05 < d < 0.4 &&
  0 < ϵ ≤ 1/15 && a/(a + b) == (0.1/(0.1+0.2) + 0.55/(0.55+0.25))/2 && a + b > a/(a + c) && a + b + c + d == 1, {a, b, c, d, ϵ}]
```

{{a → 0.260525, b → 0.249891, c → 0.270779, d → 0.218804, ϵ → 0.0666667}}

```
FindInstance[0.325 - ϵ < a < 0.325 + ϵ && 0.1 < a < 0.55 && 0.225 - ϵ < b < 0.225 + ϵ && 0.2 < b < 0.25 &&
  0.225 - ϵ < c < 0.225 + ϵ && 0.15 < c < 0.3 && 0.225 - ϵ < d < 0.225 + ϵ && 0.05 < d < 0.4 &&
  0 < ϵ < 1/16 && a/(a + b) == (0.1/(0.1+0.2) + 0.55/(0.55+0.25))/2 && a + b > a/(a + c) && a + b + c + d == 1, {a, b, c, d, ϵ}]
```

{}

Here's an example near our Table 3 example which requires $\epsilon > 0.115$:

p	q	P_1^0	P_2^0	$P_1^1 = P_2^1$
T	T	$\frac{168}{6539}$	$\frac{3}{5}$	a
T	F	$\frac{17937}{65390}$	$\frac{2}{10}$	b
F	T	0.3	0.15	c
F	F	0.4	0.05	d

```
test[{e_, f_, g_, h_}, {i_, j_, k_, l_}, ξ_] :=
  FindInstance[(e + i)/2 - ϵ < a < (e + i)/2 + ϵ && If[e > i, i < a < e, e < a < i] && (f + j)/2 - ϵ < b < (f + j)/2 + ϵ &&
    If[j > f, f < b < j, j < b < f] && (g + k)/2 - ϵ < c < (g + k)/2 + ϵ && If[k > g, g < c < k, k < c < g] &&
    (h + l)/2 - ϵ < d < (h + l)/2 + ϵ && If[l > h, h < d < l, l < d < h] && 0 < ϵ ≤ ξ &&
    a/(a + b) == (e/(e+f) + i/(i+j))/2 && a + b + c + d == 1 && e + f + g + h == 1 && i + j + k + l == 1, {a, b, c, d, ϵ}];
```

```
test[{168/6539, 17937/65390, 3/10, 4/10}, {3/5, 2/10, 15/100, 5/100}, 0.115]
```

{}

```
test[{168/6539, 17937/65390, 3/10, 4/10}, {3/5, 2/10, 15/100, 5/100}, 0.116]
```

{{a → 0.196856, b → 0.274294, c → 0.243925, d → 0.284925, ϵ → 0.116}}

- **Code for larger ϵ case search near our Table 3 example**

Reverse engineering a larger ϵ case near Table 3 (for the strongest rendition of ASAMC):

p	q	P_1^0	P_2^0	$P_1^1 = P_2^1$
T	T	x	y	a
T	F	z	u	b
F	T	0.3	0.15	c
F	F	0.4	0.05	d

```
FindExample[x_, y_, z_, u_, a_, b_, c_, d_, ξ_] :=
  FindInstance[x + z + 3/10 + 4/10 == 1 && y + u + 15/100 + 5/100 == 1 && 0 < x < 1 && 0 < y < 1 &&
    0 < z < 1 && 0 < u < 1 && ∀{ϵ},0<ϵ<ξ ! ∃{a,b,c,d},(a|b|c|d)∈Reals ((x + y)/2 - ϵ < a < (x + y)/2 + ϵ &&
        If[y > x, x < a < y, y < a < x] && (z + u)/2 - ϵ < b < (z + u)/2 + ϵ && If[u > z, z < b < u, u < b < z] &&
        (3/10 + 15/100)/2 - ϵ < c < (3/10 + 15/100)/2 + ϵ && 15/100 < c < 3/10 && (4/10 + 5/100)/2 - ϵ < d < (4/10 + 5/100)/2 + ϵ &&
        5/100 < d < 4/10 && a/(a + b) == 1/2 (x/(x + y) + z/(x + u)) && a + b + c + d == 1), {x, y, z, u}, Reals];
```

```
FindExample[x, y, z, 2/10, a, b, 29/100, 2/10, 1/10]
```

```
FindInstance[7/10 + x + z == 1 && 2/5 + y == 1 && 0 < x < 1 &&
  0 < y < 1 && 0 < z < 1 && ∀{ϵ},0<ϵ<1/10 (∀{a,b},(a|b)∈Reals (! ((x + y)/2 - ϵ < a < (x + y)/2 + ϵ &&
        If[y > x, x < a < y, y < a < x] && 1/2 (z + 1/5) - ϵ < b < 1/2 (z + 1/5) + ϵ &&
        If[1/5 > z, z < b < 1/5, 1/5 < b < z] && 1/2 (3/10 + 15/100) - ϵ < 29/100 < 1/2 (3/10 + 15/100) + ϵ &&
        15/100 < 29/100 < 3/10 && 1/2 (4/10 + 5/100) - ϵ < 1/5 < 1/2 (4/10 + 5/100) + ϵ && 5/100 < 1/5 < 4/10 &&
        a/(a + b) == 1/2 (x/(x + y) + z/(x + 1/5)) && a + b + 29/100 + 1/5 == 1))), {x, y, z}, Reals, 2][[2]]
```

```
{x → 168/6539, y → 3/5, z → 17 937/65 390}
```

Verifying the example:

```
test[{e_, f_, g_, h_}, {i_, j_, k_, l_}, ξ_] :=
  FindInstance[(e + i)/2 - ϵ < a < (e + i)/2 + ϵ && If[e > i, i < a < e, e < a < i] && (f + j)/2 - ϵ < b < (f + j)/2 + ϵ &&
    If[j > f, f < b < j, j < b < f] && (g + k)/2 - ϵ < c < (g + k)/2 + ϵ && If[k > g, g < c < k, k < c < g] &&
    (h + l)/2 - ϵ < d < (h + l)/2 + ϵ && If[l > h, h < d < l, l < d < h] && 0 < ϵ < ξ &&
    a/(a + b) == (e/(e + f) + i/(i + j))/2 && a + b + c + d == 1 && e + f + g + h == 1 && i + j + k + l == 1, {a, b, c, d, ϵ}];
```

$$\mathbf{test}\left[\left\{\frac{168}{6539}, \frac{17\,937}{65\,390}, \frac{3}{10}, \frac{4}{10}\right\}, \left\{\frac{3}{5}, \frac{2}{10}, \frac{15}{100}, \frac{5}{100}\right\}, 0.1\right]$$

{}

$$\mathbf{test}\left[\left\{\frac{168}{6539}, \frac{17\,937}{65\,390}, \frac{3}{10}, \frac{4}{10}\right\}, \left\{\frac{3}{5}, \frac{2}{10}, \frac{15}{100}, \frac{5}{100}\right\}, 0.115\right]$$

{}

$$\mathbf{test}\left[\left\{\frac{168}{6539}, \frac{17\,937}{65\,390}, \frac{3}{10}, \frac{4}{10}\right\}, \left\{\frac{3}{5}, \frac{2}{10}, \frac{15}{100}, \frac{5}{100}\right\}, 0.116\right]$$

$\{\{a \to 0.196856,\ b \to 0.274294,\ c \to 0.24393,\ d \to 0.28492,\ \epsilon \to 0.115995\}\}$