What is the ‘Equal Weight View’?

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We’ll be talking about a pair of (Bayesian) agents $S_1$ and $S_2$ who are epistemic peers, regarding some propositions $P$.

We’ll call such $p$’s in $P$ peer-propositions (for $S_1$ and $S_2$).

We will not assume any general “laws” of the logic of peer-proposition-hood (but, we will presuppose that ‘$p$ & $q$’ is a peer-proposition $\neq$ ‘$p$’ is a peer-proposition).

We’ll be exploring various peer-update rules (PURs) for such pairs of Bayesian agents. These PURs are rules for updating credences, upon learning (exactly) the information $[D(P)]$ that $S_1$ and $S_2$ disagree on some set of peer propositions $P$.

The idea behind Equal Weight Rules (EWRs) is that, when $S_1$ and $S_2$ learn $D(P)$, they should adopt consensus credences on $P$ that “roughly split the difference” [1] in credence on $P$.

We will begin with the simplest EWR. Then, using various constraints from the literature on Bayesian judgment aggregation [5], we will develop more sophisticated EWRs.

Setup SA & P SAMC, C & PCI ASAMC U & IA IA & Summary Refs

Let $\Pr_i^0(p)$ be the credence $S_i$ assigns to $p$ at $t_0$, and let $\Pr_i^1(p)$ be the credence $S_i$ assigns to $p$ at $t_1$, where $t_1 > t_0$.

We will assume that $S_1$ and $S_2$ learn exactly (important caveat for any update rule!) the following between $t_0$ and $t_1$:

$$[D(P)] \quad \text{For each } p \in P, \Pr_i^0(p) \neq \Pr_i^0(p).$$

[Note: we do mean to assume here that $D(P)$ includes the numerical values of the $\Pr_i^0(p)$, but that information is only relevant to our PUR’s to the extent that it informs about the disagreement qua disagreement. If the $\Pr_i^0(p)$ are also relevant (in the context) to the determination of $\Pr_i^1(p)$ for other reasons, our PUR’s will ignore these other relevancies.]

A PUR will just be a rule, which, for each $p \in P$, prescribes how the credences of $S_1$ and $S_2$ should be updated, so as to properly respond to credal disagreements $D(P)$.

For simplicity, we’ll assume that $S_1$ and $S_2$ share a sentential language $L$ with just two atomic sentences $A$ and $B$.

Our first PUR is straight averaging (“split the difference”):

$$\Pr_{\text{SA}}^1(p) = \frac{\Pr_1^0(p) + \Pr_2^0(p)}{2}$$

This naive, exact “split the difference” PUR (SA) may sound appealing, but it is under-specified, as it stands. Example:

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$\Pr_1^0(\cdot)$</th>
<th>$\Pr_2^0(\cdot)$</th>
<th>$\Pr_{\text{SA}}^1(\cdot)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\top$</td>
<td>$\top$</td>
<td>0.1</td>
<td>0.55</td>
<td>0.325</td>
</tr>
<tr>
<td>$\top$</td>
<td>$\bot$</td>
<td>0.2</td>
<td>0.25</td>
<td>0.225</td>
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<tr>
<td>$\bot$</td>
<td>$\top$</td>
<td>0.3</td>
<td>0.15</td>
<td>??</td>
</tr>
<tr>
<td>$\bot$</td>
<td>$\bot$</td>
<td>0.4</td>
<td>0.05</td>
<td>??</td>
</tr>
</tbody>
</table>

Here, $A$ & $B$ and $A$ & $\neg B$, are peer-props (bold), but $\neg A$ & $B$ and $A$ & $\neg B$ are not. So, (SA) prescribes new credences for $A$ & $B$ and $A$ & $\neg B$, but not for $\neg A$ & $B$ and $A$ & $\neg B$.

Neither $S_1$ nor $S_2$ can keep their old credences in both $\neg A$ & $B$ and $A$ & $\neg B$ — on pain of synchronic incoherence!
Probabilism (P). $\Pr_1^A(\cdot)$ and $\Pr_2^A(\cdot)$ should be probabilities.

- (SA) must be revised, so as to tell agents what to do when (P) + (SA) forces changes to credences on non-peer $p$’s.
- Informal Idea: revise (SA) to (SAMC), which recommends that (in such cases) each agent makes minimal (forced) changes to their credences on non-peer propositions.

(SAMC) Upon learning (exactly) $D(P)$, $S_1$ and $S_2$ should (i) obey (SA) for peer-propositions $P$, and (ii) if (P) should force additional revisions, then each agent should revise their credences by moving to a closest probability function compatible with both (SA) and (P).

- In our example above, (SAMC) entails these unique $\Pr_1^A(\cdot)$’s (assuming a Euclidean distance metric [3] on credence $f$’s):

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$\Pr_1^A(\cdot)$</th>
<th>$\Pr_2^A(\cdot)$</th>
<th>$\Pr_1^B(\cdot)$</th>
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<td>0.225</td>
<td>0.225</td>
</tr>
<tr>
<td>$\bot$</td>
<td>$\top$</td>
<td>0.3</td>
<td>0.15</td>
<td>0.175</td>
<td>0.275</td>
</tr>
<tr>
<td>$\bot$</td>
<td>$\bot$</td>
<td>0.4</td>
<td>0.05</td>
<td>0.275</td>
<td>0.175</td>
</tr>
</tbody>
</table>

Suppose $A$, $B$, and $A\&B$ are peer-propositions for $S_1$ and $S_2$ (at both $t_0$ and $t_1$), and that $B$ remains a peer-proposition (at $t_0$) on the supposition that $A$ is true. Then, our example entails a unique (SAMC)-distribution for both agents at $t_1$:

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$\Pr_1^A(\cdot)$</th>
<th>$\Pr_2^A(\cdot)$</th>
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First, let’s calculate the $\Pr_{i}^{0\rightarrow A}(B)$ values for the two agents:

- $\Pr_{1}^{0\rightarrow A}(B) = \Pr_{1}^{0}(B \mid A) = \frac{\Pr_{1}^{0}(A \& B)}{\Pr_{1}^{0}(A)} = \frac{0.1}{0.3} = 0.3333$
- $\Pr_{2}^{0\rightarrow A}(B) = \Pr_{2}^{0}(B \mid A) = \frac{\Pr_{2}^{0}(A \& B)}{\Pr_{2}^{0}(A)} = \frac{0.55}{0.8} = 0.6875$

Finally, let’s calculate the value of $\Pr_{i}^{0}(B \mid A)$. This can be done uniquely here, since — in this example — (SAMC) entails a unique (SAMC)-distribution for both agents, at $t_1$:

- $\Pr_{1}^{0}(B \mid A) = \frac{\Pr_{1}^{0}(B \& A)}{\Pr_{1}^{0}(A)} = \frac{\Pr_{1}^{0}(B \& A)}{\Pr_{1}^{0}(A)} = 0.325 \div 0.55 = 0.5909$

This is a counterexample to (C) for any PUR that exactly “splits the difference” on $P$ — including (SA) and (SAMC).

Moreover, this is also an (SAMC)-counterexample to:

- Preservation of Conditional (In)dependencies (PCI): $\Pr_{1}^{0}(\cdot)$ and $\Pr_{2}^{0}(\cdot)$ should neither reverse initially agreed-upon assessments of conditional (in)dependence, nor force new disagreements about relations of conditional (in)dependence, among the set of peer-propositions $P$.
On either reading of (ASAMC), we have the following:  

**Theorem.** (ASAMC) is compatible with (P), (C), and (PCI). That is, we can always find consensus credence functions that obey (ASAMC) as well as (P), (C), and (PCI). [The only question will be how large a value of $\epsilon$ will be required.]

- Sometimes, non-trivially “large” values of $\epsilon$ are required in order to yield (ASAMC)-updates satisfying (P), (C), and (PCI).
- *E.g.*, in our last example above (Table 3 on slide 7), there do exist (ASAMC) updates satisfying (P), (C), and (PCI), but they all require a threshold value of $\epsilon > \frac{1}{14}$, for all $p \in P$.
- In the MATHEMATICA notebook for this paper [4] (which has all technical results), we have examples in which $\epsilon > \frac{1}{14}$ is forced by (ASAMC) in order to ensure (P), (C) & (PCI).
- We will not take a stand here on how large $\epsilon$ should be allowed to get, in various contexts (or whether different $p$’s in $P$ should require different $\epsilon$’s, depending on context, etc.).
- Our aim here is merely to survey the landscape of EWRs.

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**Approximate SAMC (ASAMC):** Upon learning (exactly) $D(p)$:

$$Pr_1^1(p) = Pr_2^1(p) \approx \frac{Pr_1^0(p) + Pr_2^0(p)}{2},$$

where $Pr_1^1(p)$ is strictly between $Pr_1^0(p)$ and $Pr_2^0(p)$.

And, where the update is done so as to satisfy (P) and (C). If changes to non-peer credences are forced in order to ensure (P) and (C), then the other changes should be made so as to minimize the distance of $Pr_1^1(\cdot)$, $Pr_2^1(\cdot)$ from $Pr_1^0(\cdot)$, $Pr_2^0(\cdot)$, while maintaining (P) and (C). Finally, if the satisfaction of further constraints C (e.g., PCI) is desired, then these should be added to the constraint satisfaction problem (in both the initial and “minimal change” steps), so that the update process respects these additional constraints as well.

- Note: $x \approx y \equiv |x - y| < \epsilon$. Other definitions could be used. We assume a single $\epsilon$ for all $p \in P$. This could be relaxed.
- Note: we require that $Pr_1^1(p)$ be strictly between $Pr_1^0(p)$ and $Pr_2^0(p)$, so as to rule-out a dictatorial update, which adopts one of the agent’s credence in $p$ as the “consensus” value.
- It is worth noting that all of the PUR's we discuss satisfy the following constraint, which is strictly weaker than (A): Unanimity (U): \( \Pr_1^0(p) \) and \( \Pr_1^1(p) \) should not force new point-wise disagreements about credence values concerning peer propositions on which \( S_1 \) and \( S_2 \) already agree (at \( t_0 \)).
- (U) is perhaps the most basic of all desiderata for PUR's.
- Another constraint that people have often discussed in the historical literature on judgment aggregation [7] is:

  **Irrelevance of Alternatives (IA):** \( \Pr_1^0(p) \) and \( \Pr_1^1(p) \) should each be functions of \( \Pr_1^0(p) \) and \( \Pr_1^1(p) \). That is, for each peer-proposition \( p \), \( \Pr_1^1(p) = f_1[\Pr_1^0(p), \Pr_1^1(p)] \), and \( \Pr_2^1(p) = f_2[\Pr_2^0(p), \Pr_2^1(p)] \), for some functions \( f_1 \) and \( f_2 \).

- While (IA) may make some sense in a full belief/inference context (as in traditional judgment aggregation [7]), it makes much less sense in a probabilistic/Bayesian context.

- E.g., we conjecture that any remotely plausible EWR/PUR which satisfies (IA) must fail to satisfy either (P) or (C) [2].

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**References**