- Richard Feldman [3, 4, 5] has been defending a principle whose slogan is: "evidence of evidence is evidence" (EEE).
- He uses the (EEE) principle to support a conciliationist position regarding the epistemology of peer disagreement.

> …even if it is true that the theists and the atheists have private evidence, this does not get us out of the problem. Each may have his or her own special insight or sense of obviousness. But each knows about the other's insight. Each knows that this insight has evidential force. And now I see no basis for either of them justifying his own belief simply because the one insight happens to occur inside of him. **A point about evidence that plays a role here is this: evidence of evidence is evidence. More carefully, evidence that there is evidence for p is evidence for p. Knowing that the other has an insight provides each of them with evidence.**

- Our aims today will be to: (a) briefly survey the recent dialectic on (EEE), and (b) describe a new, higher-order approach.
- My focus will be on recent (first-order) work by myself, Tal & Comesaña [13, 14], Roche [10], and Moretti [9].
- Then: Kevin will describe a new, higher-order approach.

- Two odd features of the extant lit. on (EEE). (1) it doesn't focus on the (motivating) case of peer disagreement. (2) it uses only first-order evidential relations. [Kevin will fix these in his talk.]
- Before we get into the extant literature, some terminology.
- e makes h **firm** for S, just in case $P(h \mid e) > t$, for some threshold $t \geq 1/2$, where $P(\cdot)$ is S's (probabilistic) credence function.
- e **increases the firmness** of h for S, just in case $P(h \mid e) > P(e)$.
- Until now, the literature on (EEE) has focused on (first-order) relations of *increase in firmness*, and it has revolved around the fact that (unlike firmness) it violates Hempel's (SCC).

(SCC) If e increaes the firmness of h and h entails h', then e increaes the firmness of h'.

- More generally, the extant (EEE)-literature focuses on various kinds of failures of the *transitivity of increase in firmness* [12, 2, 11].
- OK, now we're ready to sample the extant dialectic.

- Initial renditions of (EEE) — as discussed by Feldman [5] and myself [6] — were both naïve and ambiguous. For instance:

(EEE) If e is evidence for the claim that S possesses some evidence for h, then E is evidence for h.

- As T&C point out, this initial statement of (EEE) is ambigious between a *de re* reading and a *de dicto* reading.

(EEE_{dr}) If (a) e is evidence for the claim that S possesses e', and (b) e' is evidence for h, then e is evidence for h.

(EEE_{dd}) If e is evidence for: $\exists_{e'}$ such that (a) S possesses e' and (b) e' is evidence for h, then E is evidence for h.

- My proposed counterexample to (EEE) was as follows [6, 13].

> **Card**. John (S) has observed a card that was drawn at random from a standard deck. Let e := the card is black, e' := the card is the ace of spades, and h := the card is an ace.

- **Card** is a counterexample to (EEE_{dr}). But, as T&C point out, **Card** is *not* a counterexample to (EEE_{dd}).

☞ e is *not* evidence for the claim that John possesses *some* h-entailing evidence. That is, e is *not* evidence for

(x') John observed an ace.

- For e is *neutral* regarding x' [$P(x' \mid e) = 1/13 = P(x')$].
- William Roche [10] has shown how to modify **Card**, so as to transform it into a counterexample to (EEE_{dd}).

> **Card***. Just like **Card**, except that, unbeknownst to John, the card will be shown to him *iff* it is the Ace of Spades.

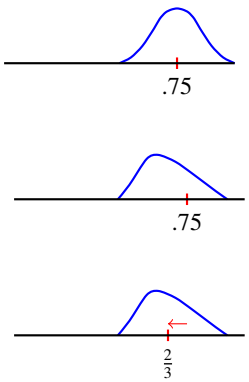
- In **Card***, e *is* evidence for the claim that John possesses *some* h-entailing evidence. That is, now e *is* evidence for x'. This is because, in **Card***, $\Pr(x' \mid e) = 1/26 > 1/52 = \Pr(x')$.
- Roche also shows even (EEE_{dr}) is true *if x screens-off e from h.*
- The literature continues through various iterations of first-order (EEE) principles & counterexamples [9, 14]. I now think this is a degenerating research programme. Time for a new approach…

EEE and Higher-Order Evidence

- Back to peer disagreement. Suppose we share evidence (this is generalizable).
- I think the Butler's probably guilty. Then I learn that you think he's probably not. I should lower my confidence in his guilt. Why?
- Proposal: because your opinion provides me with (higher-order) evidence about what our shared evidence warrants.

EEE and Higher-Order Evidence

- I should be .75 confident of *guilt*. But I should have **higher-order doubt** about whether that opinion is warranted by my evidence.
- Learning your opinion provides evidence that our evidence *didn't* warrant that much confidence in *guilt*.
- If receiving evidence that my evidence doesn't support *guilt* provides evidence against *guilt* (EEE), I should lower my confidence in *guilt*.



Probabilistic Epistemic Logic

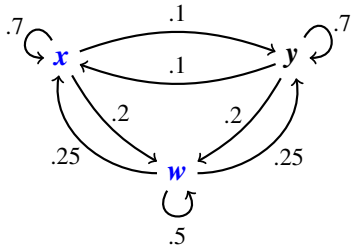
- Assume my evidence warrants having some probability function. Let P be that function—whatever it is.
- My evidence warrants uncertainty, i.e. having a non-extreme opinion in *guilt*: $0 < P(\textit{guilt}) < 1$. Suppose $P(\textit{guilt}) = .75$.
- My evidence also warrants **higher-order uncertainty**, i.e. having a non-extreme opinion about which opinion my evidence warrants having in *guilt*: $0 < P(P(\textit{guilt}) = .75) < 1$.
  - So P cannot be introspective: $[P(h) = t] \nRightarrow [P(P(h) = t) = 1]$
- How to tractably model this sort of higher-order uncertainty?

Probabilistic Epistemic Logic

- A **probability frame** $\langle W, \mathcal{P} \rangle$ consists of a (finite) set of worlds W and a function $\mathcal{P}$ from worlds w to probability functions $\mathcal{P}_w$ over W.
- Propositions are modeled as subsets of W: h true at w iff $w \in h$. If $h, e \subseteq W$, $\neg h := (W - h)$, $(h \wedge e) := (h \cap e)$, $(h \rightarrow e) := (\neg h \cup e)$, etc.
- We use $\mathcal{P}$ to define propositions about probabilities, e.g. $\mathbf{[P(h) = t]} := \{w | \mathcal{P}_w(h) = t\}$.
  - Since claims about what your evidence warrants (i.e. claims about P) are sets of worlds, the model automatically encodes higher-order opinions.

Example: Disagreement

- Let $guilt = \{w, x\}$
- $[P(guilt) = .75] = \{w\}$,
  $[P(guilt) = .9] = \{x\}$, and
  $[P(guilt) = .3] = \{y\}$.
- Supposing w is actual, $P(guilt) = .75$ but
  $P(P(guilt) = .9) = .25$,
  $P(P(guilt) = .75) = .5$, and
  $P(P(guilt) = .3) = .25$.
- Suppose learning that you have low credence in *guilt* rules out $[P(guilt) \geq .9]$.
- $P(guilt|disagree) = \frac{.5}{.75} = \frac{2}{3} < .75 = P(guilt)$
- **Question:** Can we generalize this effect?



EEE Principles

- Using this framework and the firm/increasing distinction, we can formulate simple versions of EEE principles.
- Let $\boldsymbol{P_e} := P(\cdot|e)$ be the credence function—whatever it is—that my evidence warrants having conditional on e.
- On simplest version:
  - e makes h firm $\Leftrightarrow [P_e(h) \geq t]$
  - e increases h's firmness $\Leftrightarrow [P_e(h) > P(h)]$
- Two simple EEE principles:
  - **Simple F-F-F:** $[P_e(P_e(h) \geq t) \geq t] \rightarrow [P_e(h) \geq t]$
  - **Simple I-F-I:** $[P_e(P_e(h) \geq t) > P(P_e(h) \geq t)] \rightarrow [P_e(h) > P(h)]$
- These are too strong. How to show?

Testing EEE Principles

- Higher-order uncertainty intuitions are untrustworthy. Need a theory.
- Proposal: allow as much higher-order uncertainty as possible while still guaranteeing the **guidance-value of evidence** (follow I. J. Good [8]).
  - Can be stated as a formal constraint on frames:

    Value: No matter your decision problem (options and values), the expected value of (i) choosing a particular option is no higher than that of (ii) choosing the option—whatever it is—that maximizes expected value by P's lights.
- Geanakoplos [7] and Dorst [1] explore the consequences and permissions of this constraint, showing it to permit large amounts of higher-order uncertainty.

Simple F-F-F is too strong

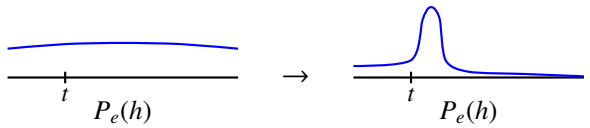
Simple F-F-F: $[P_e(P_e(h) \geq t) \geq t] \rightarrow [P_e(h) \geq t]$

- $\Rightarrow$ If $P(h) = t$, the probability that $[P(h) > t]$ can be no higher than the probability of h.
- So Simple F-F-F fails when there are more possibilities that warrant higher credence in h than possibilities that make h true. This is compatible with Value.

Simple I-F-I is too strong

Simple I-F-I: $[P_e(P_e(h) \geq t) > P(P_e(h) \geq t)] \rightarrow [P_e(h) > P(h)]$

- $\Rightarrow$ Even if increase probability of $[P_e(h) \geq t]$ by drastically *lowering* the expectation of $P_e(h)$, still increase probability of h.



- So Simple I-F-I can fail when $P_e(h)$ matches P_e's expectation of $P_e(h)$. This is compatible with (but not required by!) Value.

Refined EEE Principles

- The problem with Simple F-F-F is that it demands *too much* transference from higher-order opinions.
  - **Simple F-F-F:** $[P_e(P_e(h) \geq t) \geq t] \rightarrow [P_e(h) \geq t]$
  - **Refined F-F-F:** $[P_e(P_e(h) \geq t) \geq s] \rightarrow [P_e(h) \geq \boldsymbol{ts}]$
- The problem with Simple I-F-I is that it places too few constraints on *how else e* should shift your opinion in $P_e(h)$.
  - **Simple I-F-I:** $[P_e(P_e(h) \geq t) > P(P_e(h) \geq t)] \rightarrow [P_e(h) > P(h)]$
  - **Refined I-F-I:** If $[P(h) \geq t]$ and $[P(h) < t]$ screen off e from h, then $[P_e(P(h) \geq t) > P(P(h) \geq t)] \rightarrow [P_e(h) > P(h)]$

Results

Refined F-F-F: $[P_e(P_e(h) \geq t) \geq s] \rightarrow [P_e(h) \geq ts]$
Refined I-F-I: If $[P(h) \geq t]$ and $[P(h) < t]$ screen off e from h, then $[P_e(P(h) \geq t) > P(P(h) \geq t)] \rightarrow [P_e(h) > P(h)]$

Theorem. If a probability frame validates[1] Value, it also validates Refined F-F-F and Refined I-F-I.

Fact. For any *strengthening* F-F-F* of Refined F-F-F, there are probability frames that validate Value but not F-F-F*.

Conjecture. Value requires much stronger I-F-I principles. Example:

- **Fact.** If a probability frame validates Value, then if e is equivalent to a sequence of Jeffrey-shifts from $[P(h) < t_i]$ to $[P(h) \geq t_i]$ for decreasing t_i, then $P_e(h) > P(h)$.

[1] Satisfies constraint at all worlds, for all decision problems (options + utilities).

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