A Decision Theory for Imprecise Credences

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Abstract

Those who model doxastic states with a set of probability functions, rather than a single function, face a pressing challenge: can they provide a plausible decision theory compatible with their view? Adam Elga (2010) and others claim that they cannot, and that the set of functions model should be rejected for this reason. This paper aims to answer this challenge. The key insight is that the set of functions model can be seen as an instance of the supervaluationist approach to vagueness more generally. We can then generate our decision theory by applying the general supervaluationist semantics to decision-theoretic claims. The result: if an action is permissible according to all functions in the set, it’s determinately permissible; if impermissible according to all, determinately impermissible; and – crucially – if permissible according to some, but not all, it’s indeterminate whether it’s permissible. This proposal handles with ease some difficult cases (including one from Elga (2010)) on which alternative decision theories falter. One reason this view has been overlooked in the literature thus far is that all parties to the debate presuppose that an acceptable decision theory must classify each action as either permissible or impermissible. But I will argue that this thought is misguided. Seeing the set of functions model as an instance of supervaluationism provides a compelling motivation for the claim that there can be indeterminacy in the rationality of some actions.

0. Introduction

How confident are you that someone exactly one hundred and forty-three days younger than you owns exactly nine bicycles? Give me one real number between 0 and 1.

If forced to at gunpoint, no doubt you would be able to come up with one such number. But that would be to represent your doxastic state with artificial precision. Most of us just don’t have perfectly precise credences in such propositions. Moreover, this doesn’t reflect a failure of rationality—even an ideally rational agent with our evidence would lack precise credences.

This is a problem for orthodox Bayesianism, which represents an agent’s doxastic state with a single credence function. The standard fix (promoted in Jeffrey (1983), van Fraassen (1990), Joyce (2005, 2010), and others) is to use a set of functions instead.

Recently, however, this set of functions model has come under attack. It has been claimed, for example in Elga (2010), that there is no acceptable decision theory that could accompany it. The sentiment is echoed in Dorr (2010): “In my view, there is no adequate account of the way unsharp credences should be manifested in decision-making.”

The aim of this paper is to answer this challenge. The key to the solution involves focusing attention on the fact that the set of functions model can be seen as an instance of the supervaluationist approach to vagueness more generally. The standard supervaluationist semantics can then be straightforwardly applied to propositions about which actions are
permissible, required, impermissible, etc. Doing so provides us with a decision theory that avoids the problems faced by other proposed accounts of decision making with imprecise credences. Decision theory is not the Achilles’ heel of the set of functions model.

1. The Supervaluationist Interpretation of the Set of Functions Model

Although the set of functions model is commonly employed, unfortunately there is not much discussion in the literature about how it should be interpreted. What are we saying about an agent when we represent them with a particular set of functions? What is the doxastic state we are recommending to them when we say that a particular set of functions is the rational response to their evidence?

It is not my aim here to survey multiple interpretations, or to take up the question of which one is best. Rather, I will simply describe one particular interpretation, on which the set of functions model is an instance of the supervaluationist approach to vagueness. This is endorsed or suggested by many of the main proponents of the model (including, among others, van Fraassen (1990, 2005, 2006), Hajek (2003), and Joyce (2005, 2010)). As we will see, one advantage of this interpretation is that it yields a novel and attractive decision theory.

The general supervaluationist strategy begins by distinguishing admissible and inadmissible precisifications of some vague term or concept. If a proposition is true according to all admissible precisifications, then it is determinately true. If it is true according to some, but not all, admissible precisifications, then it’s indeterminate whether it’s true. For example, consider a paradigm vague predicate, like “tall.” Each possible precisification specifies one particular height as the cut-off between “tall” and “not-tall.” Suppose a cut-off at 5 feet 7 inches is admissible, as is a cut-off at 5 feet 8 inches, but a cut-off at 8 feet is not. Then someone 8 feet tall is determinately tall, but it’s indeterminate whether someone 5 feet 7 inches is tall.

We can apply this supervaluationist strategy to doxastic imprecision by seeing each function in your set as one admissible precisification of your doxastic state. Functions excluded from your set are inadmissible precisifications. Whatever is true according to all functions in your set is determinately true; if something is true on some, but not all functions in your set, then it’s indeterminate whether it’s true. For example, if all functions in your set have Pr(A) > Pr(B), then it’s determinate that you’re more confident of A than B. If different functions in your set assign different values to some proposition P, then for each such value, it’s indeterminate whether that value is your credence in P.

We can now see how the supervaluationist interpretation answers the two questions with which we began this section. When we represent an agent with a particular set of functions, what we are saying is that propositions true according to all functions in that set are determinately true of the agent, and propositions true according to some but not all functions in the set are indeterminately true of them. When we say that a particular set of functions is the rational response to your evidence, what we are saying is that, if you are rational, then the propositions true on all functions in that set will be determinately true of you, and the

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1 One subtlety concerns the fact that strictly speaking, since each function in your set assigns some precise value to P, the supervaluationist interpretation has it that you do have a precise credence in each proposition—it’s just indeterminate what that credence is. This is analogous to the supervaluationist’s commitment to there being a sharp line between “tall” and “not-tall”—it’s just indeterminate where that line is. Whether this is a feature or a bug is not a question I’ll take up here.
The aim of this section has been to point out that we can see the set of functions model as just one instance of the supervaluationist approach to vagueness in general. In a sense, this is old news—as noted above, the supervaluationist interpretation is presupposed by many of the main proponents of the set of functions model. However, it is typically just noted briefly in passing. It deserves more attention. In fact, I think it is the key insight required to generate the right decision theory for imprecise credences.

2. A Supervaluationist Decision Theory for Imprecise Credences

Decision theory on the single function model is relatively straightforward. An action is rationally permissible if it has the highest expected value, or ties with some other option for highest. It is impermissible if some other option has a higher expected value.

Things are more complicated on the set of functions model, however. It can happen that different functions in your set disagree about whether an action is permissible or not. For example, some action A may have the highest expected value according to one function in your set, while another action B has the highest expected value according to some other function in your set. What should you do in such a case? Is action A permissible? Is action B permissible?

Many different answers have been proposed in the literature, but all parties to the debate thus far have shared the following basic presupposition: that every action must be neatly classifiable into one of just two categories: permissible, or impermissible. This presupposition gives rise to another: that we need to supplement the set of functions model with some additional decision rule to handle those cases in which your functions disagree about whether an action is permissible. For example, we might say that an action is permissible if it’s permissible according to at least one function in your set, otherwise impermissible; or we might say that it’s permissible if permissible according to all functions in your set, otherwise impermissible.

The approach I propose differs from all of these in a fundamental way. I do not think we need to supplement the set of functions model with some additional decision rule. Rather, if we interpret this model along supervaluationist lines, everything we need to know about decision making is already contained in the interpretation of the model itself—particularly, in its supervaluationist semantics. This semantics has the result that in some cases it is indeterminate whether an action is permissible.

Our supervaluationist semantics, recall, is as follows: a proposition is true if it’s true according to all functions in your set; it’s indeterminate whether it’s true if true according to some, but not all, functions in your set. We can apply this semantics to the decision-theoretic claims in which we are interested, with the following results. If some option A has the highest expected value (or ties for highest) according to every function in your set, then every function agrees that A is permissible, so it’s determinately true that A is permissible. If, according to

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2 Here I am ignoring the debate between causal and evidential decision theorists. It is orthogonal to the issues with which we are presently concerned.

3 Elga (2010) discusses a wide variety of different decision rules for imprecise credences.

4 Joyce (2010, p. 311) expresses the thought as follows: “What you need is some decision rule that will tell you how to make choices when expected utility assessments are equivocal. Such rules should never recommend one act over another when every member of your committee [i.e. every function in your set] says that the utility of the latter exceeds that of the former. Beyond that, there is no consensus among proponents of the imprecise model about what features a decision rule should have.”
every function, there is some other action with an expected value higher than A’s, then all functions agree that A is not permissible, so it’s determinate that A is impermissible.

What if the functions disagree? That is, what if A has (or ties for) the highest expected value according to some functions—but, according to others, some other option has a higher expected value? Then, according to the former functions, A is permissible; according to the latter functions, A is impermissible. The proposition “A is permissible” is true according to some but not all of the functions in your set, so it is indeterminate whether this proposition is true. This is the crucial aspect of the view—this is what distinguishes it from all the others.

No doubt, some will find it odd to think that there can be cases in which it’s indeterminate whether or not some action is permissible. However, there are at least two reasons why indeterminate permissibility is not objectionable.

First, consider the source of indeterminate permissibility on the supervaluationist interpretation. Recall that on this interpretation, for each function in your set, it’s indeterminate whether that function is your credence function. Indeterminacy in belief is built into the interpretation of the model. It is a platitude that what you should do depends on two things: your beliefs and your values. Given this dependence, it is only natural that indeterminacy in your beliefs would result in indeterminacy in the permissibility of your actions. Indeed, it would be very odd if there were indeterminacy in your beliefs but perfect determinacy in the rationality of all possible actions. So although the idea of an action’s being indeterminately permissible may strike one as initially odd, it is really to be expected, given a view on which there is indeterminacy in your beliefs, and given that the rationality of your actions depends crucially on what you believe. Indeterminacy in beliefs can lead to indeterminacy in the rationality of actions.

More specifically, consider any case that satisfies the following description: There are two doxastic states, B1 and B2, such that it’s indeterminate whether you’re in B1 and it’s indeterminate whether you’re in B2. Moreover, if you were determinately in B1, A would be determinately permissible for you; and if you were determinately in B2, then A would be determinately impermissible for you. In such a case, on the view proposed here, it is indeterminate whether A is permissible. This is because whether A is permissible depends on which belief state you’re in—but it’s indeterminate which belief state you’re in. This is how indeterminacy in doxastic states can lead to indeterminacy in the permissibility of an action.

 Moreover, there are reasons to think that there can be indeterminacy in what is morally permissible. This bolsters the claim that there can be indeterminacy in what is rationally permissible. For example, Miriam Schoenfield (ms) motivates indeterminacy in moral permissibility as follows (she attributes the example to Ian Proops): “Darryl is watching his two year old daughter play in a city park. It is permissible to divert his attention from her for 1 second. It is not permissible to divert his attention from her for 5 minutes. Is it permissible to divert his attention for 30 seconds? 31? 32? Plausibly, we can create a sorites series, admitting of borderline cases of permissibility, out of a series of diversions whose lengths differ by a second.”

Similarly, Tom Dougherty (forthcoming) writes: “It seems that you may save your friend at the expense of saving a few strangers, but not at the expense of saving very many strangers. How many is very many? No precise number appears to fix a threshold. The boundary seems fuzzy, and correspondingly some actions, e.g. saving your friend rather than thirteen strangers, are borderline cases of a morally permissible action. Thus the term “morally permissible” is vague. Similarly, there seems no precise number of milliseconds of pleasure that would
compensate someone for a broken pinky toe; nor does there seem to be an exact maximal number of cents that you must spend on a taxi in order to keep your promise to meet someone at a restaurant at 7 pm.” Cristin Constantinescu (forthcoming) makes a detailed case for the claim that “moral predicates display all the standard symptoms of vagueness: (a) imprecise gradability; (b) boundarilessness; (c) borderlineness; (d) tolerance; (e) soriticality” and concludes that “Predicates like “just,” “happy,” “cruel,” “generous,” “good,” “permissible,” etc. are vague in much the same way as “bald,” “tall,” “thin,” “red,” “heap,” and other paradigms of vagueness.”

However, one might object that, intuitively, it seems that the notion of blameworthiness couldn’t admit of borderline cases. If so, this casts doubt on the claim that permissibility could be indeterminate, because there is a wide class of cases in which blameworthiness and impermissibility go together.⁵

However, the examples used to motivate indeterminacy for permissibility also serve to motivate indeterminacy for blameworthiness. For example, Darryl is not blameworthy for diverting his attention from his daughter for 1 second, nor for 2. He would clearly be blameworthy for diverting his attention for 5 minutes. However, not all diversions in between are clearly classifiable as either blameworthy or blameless. There does not seem to be any N such that, plausibly, N-second diversions are clearly blameworthy, while N – 1 second diversions are clearly blameless. Sometimes it is simply indeterminate whether an action is blameworthy.

One might respond that we should not countenance borderline blameworthiness, because blamelessness is the default option. An agent should be considered blameless unless she has committed a clearly blameworthy action. When considering a particular case, if there is any unclarity about whether the action is blameworthy, then the agent is blameless—not indeterminately blameworthy.

However, as Tom Dougherty has pointed out to me in conversation, this view fails to do justice to all the moral contours of such situations. For example, suppose Robert is an avid birder. He wants to go find the rare European Golden Plover that was recently sighted, but if he goes, his daughter must come with him, and she always gets carsick. If the Golden Plover is just a 3-minute drive away, Robert would not be blameworthy for going, and taking his daughter. If, however, the Golden Plover is a 27-hour drive away, he would be blameworthy for making his daughter put up with that much misery just so he can chase birds. However, there is no precise number of seconds that constitutes a sharp line between the clearly blameworthy drives and the clearly blameless ones.

Suppose that, given the facts of the situation (the severity of the carsickness, the intensity of Robert’s desire, etc.), a drive of 57 minutes seems to be a borderline case. On the view currently under consideration, if Robert were to go ahead with a drive of this length, he would be blameless in doing so, because the action is not clearly blameworthy, and blamelessness is the default. However, this view fails to do justice to the considerable discomfort experienced by his daughter. This discomfort pulls in the direction of blameworthiness; his love for birds pulls in the direction of blamelessness; if we treat blamelessness as the default, we are privileging his love for birds over her discomfort. A more accurate account of the situation acknowledges the importance of both, treats neither blameworthiness nor blamelessness as default, and admits that sometimes an action is neither determinately one nor the other.

3. Applying the View to Problem Cases

⁵ Thanks to an anonymous reviewer for suggesting that I discuss this objection.
The decision theory proposed here can be tested by trying it out on a particularly difficult case—an example from Elga (2010). Suppose that your credence in H is the interval [.1, .8]. You are offered a series of two bets, one right after the other, and your doxastic attitude towards H does not change in between.

Here is the payoff matrix:

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>~H</th>
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<tbody>
<tr>
<td>Bet A</td>
<td>$15</td>
<td>-$10</td>
</tr>
<tr>
<td>Bet B</td>
<td>-$10</td>
<td>$15</td>
</tr>
</tbody>
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Notice that, if you accept both bets, you are guaranteed to gain $5, no matter what. (If H is true, you will win $15 on bet A and lose $10 on bet B, for a net gain of $5; if H is false, you will lose $10 on bet A and win $15 on bet B, again for a net gain of $5.) Of course, if you reject both bets, you are guaranteed to win nothing. So, says Elga, it would be irrational to reject both bets, since that would amount to giving up a guaranteed $5.

Elga argues that no plausible decision theory for the set of functions model is able to accommodate this fact. For example, let Liberal be the decision rule according to which it is permissible for you to perform any action that is permissible according to at least one of the functions in your set. There is some function in your set on which rejecting bet A is permissible. For example, consider the function with P(H) = .1. According to this function, the expected value of accepting bet A is ($15)(.1) + (-$10)(.9) = -$7.5. This is less than 0, the expected value of rejecting bet A, so rejecting bet A is permissible. Since there is at least one function in your set on which rejecting bet A is permissible, according to Liberal it is permissible for you to reject bet A.

Now consider bet B. There are other functions in your set on which rejecting bet B is permissible. For example, consider the function with Pr(H) = .8. On this function, the expected value of accepting bet B is (-$10)(.8) + $15(.2) = -5, which is less than 0, the expected value of rejecting bet B. So according to that function, rejecting bet B is permissible. Since there is at least one function in your set on which it is permissible to reject bet B, according to Liberal, rejecting bet B is permissible.

According to Liberal, then, it is permissible to reject bet A, and it is permissible to reject bet B. Elga concludes that, according to Liberal, it is permissible to reject both bets. But this would be the wrong result, since rejecting both bets amounts to passing up a guaranteed $5.

What if we follow the view proposed here, and simply apply our supervaluationist semantics? Rejecting bet A is permissible according to some functions in your set but impermissible according to others, so it is indeterminate whether it’s permissible to reject bet A. Rejecting bet B is also permissible according to some but not all functions, so it is likewise indeterminate whether rejecting bet B would be permissible. So far, so good—nothing commits us to the problematic result that rejecting both bets would be permissible.

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6 One’s interval for H includes a number r just in case there is some function in one’s set with Pr(H) = r. Note that this use of interval terminology is entirely compatible with the claim made in Joyce (2010) that the interval alone may not constitute a complete characterization of one’s doxastic attitude towards H. One’s entire set of functions may be needed for this.

7 Terminology from White (2010).

8 Liberal is a popular view; see Moss (forthcoming) for one recent defense.
Ideally, though, we would like more—we would like our view to entail the correct result that it would be determinately impermissible to reject both bets. Happily, the view delivers. Let’s evaluate the *compound* action “reject both bet A and bet B.” Every function in your set agrees that this action is impermissible, because every function in your set agrees that there is some alternative action with a higher expected value—namely, the compound action of *accepting* both bets. According to every function, the expected value of *accepting* both bets is $5, while the expected value of *rejecting* both bets is $0. So, according to every function in your set, it would be impermissible to reject both bets. So, according to the supervaluationist decision theory proposed here, it is determinate that rejecting both bets is impermissible. This is exactly the result we were looking for!

However, seeing how we get this result should make us reconsider Elga’s argument against Liberal. Recall that, according to Liberal, it is permissible to perform an action if and only if there is at least one function in your set on which that action is permissible. But as we have just seen, there is no function in your set on which the compound action of rejecting both bets is permissible. It follows that, *contra* Elga, Liberal too yields the correct result that rejecting both bets is impermissible!

Why did Elga think otherwise? Well, as explained above, it’s true that, according to Liberal, it is permissible to reject bet A; and, according to Liberal, it is permissible to reject bet B. Elga concluded that, according to Liberal, it is permissible to reject both bets. However, this follows only if we adopt the following principle:

**Export for Permissibility:**

If action A is permissible, and action B is permissible, then the compound action A and B is permissible.

*Export for Permissibility* is highly plausible. But it must be rejected by a defender of Liberal. As we have just seen, applying Liberal’s criterion directly to each action (the two simple actions and the compound action) yields the result that Export for Permissibility is false. According to Liberal, it is permissible to reject bet A, and permissible to reject bet B, but impermissible to reject both bets.

Liberal’s incompatibility with Export for Permissibility is a significant cost of the view. But the point remains that Elga’s case against Liberal is not as strong as it might at first have seemed. Liberal does not entail that it can be permissible to pass up a guaranteed monetary gain. Rather, Liberal requires giving up the principle that if each of two actions is individually permissible, then their compound action is permissible.

Although the case against Liberal is not as strong as it might at first have seemed, I still think Liberal should be rejected. This is because we have a superior alternative: the supervaluationist decision theory proposed here, which we might call “Moderate.”

One respect in which Moderate is superior to Liberal is that it entails, rather than conflicts with, Export for Permissibility. To see why, first, suppose the antecedent of Export for Permissibility is true: action A is permissible, and action B is permissible. On the standard supervaluationist semantics, if a claim is true, then it is determinately true. So action A is determinately permissible, and action B is determinately permissible. Assuming Moderate, this means that action A is permissible according to every function in your set, and action B is also permissible according to every function in your set. If A is permissible according to some function Pr, and B is also permissible according to the same function Pr, then the compound
action A and B is permissible according to Pr. So, since A and B are both individually permissible according to every function in your set, the compound action is permissible according to every function in your set. So, according to Moderate, the compound action is determinately permissible. I have shown that Moderate, when combined with the antecedent of Export for Permissibility, entails the consequent of Export for Permissibility; so, Moderate entails Export for Permissibility.

At this point, a defender of Liberal might point out that, although Moderate entails Export for Permissibility, it conflicts with the following principle:

Export for Indeterminate Permissibility:

If it’s indeterminate whether action A is permissible, and indeterminate whether action B is permissible, then it’s indeterminate whether the compound action A and B is permissible.

This is false according to Moderate, because Moderate says that it’s indeterminate whether rejecting bet A is permissible, and indeterminate whether rejecting bet B is permissible, but determinate that rejecting both bets is impermissible. In the next section, I will argue that Moderate’s rejection of Export for Indeterminate Permissibility is not problematic, even though Liberal’s rejection of Export for Permissibility is.  

4. Comparing Moderate and Liberal

We saw in the previous section that Moderate, our supervaluationist decision theory, yields the correct result that the compound action reject bet A and reject bet B is determinately impermissible. However, we then noticed that, contra Elga, Liberal—the most popular decision theory for imprecise credences—also yields this result. This revealed that Liberal is incompatible with Export for Permissibility, the principle that if two actions are individually permissible, their compound action is also permissible. Moderate, on the other hand, entails Export for Permissibility, but is incompatible with Export for Indeterminate Permissibility. According to Moderate, each of the two component actions (reject bet A and reject bet B) is indeterminately permissible.

In this section I will argue that Moderate’s rejection of Export for Indeterminate Permissibility is not problematic, even though Liberal’s rejection of Export for Permissibility is. The crucial difference is that Moderate is grounded in a body of theory—supervaluationism—that has the resources to provide a deeper explanation for why Export for Indeterminate Permissibility should be rejected but Export for Permissibility retained. Liberal’s rejection of Export for Permissibility, on the other hand, is ad hoc and unmotivated.

I will then consider an objection to Moderate that also applies to Liberal. Once again, we will see that Moderate can defuse the objection by drawing on its supervaluationist backdrop; Liberal, however, has no such resources. Finally, I will point out that any defender of Liberal owes us an alternative interpretation of the set of functions model. I consider a few possibilities,

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9 Hedden (forthcoming) defends a view on which compound actions are not assessable for rationality. This includes ordinary actions like making tea, driving to work, etc. If Hedden is right, then both Export for Permissibility and Export for Indeterminate Permissibility embody a category mistake. A thorough consideration of this view is beyond the scope of this paper.
but none can provide Liberal with a theoretical home of the kind that supervaluationism provides for Moderate. How does supervaluationism help motivate Moderate’s acceptance of Export for Permissibility in tandem with its rejection of Export for Indeterminate Permissibility? Consider the following two principles:

**Export for Truth:**
If A is true, and B is true, then their conjunction is true.

**Export for Indeterminate Truth:**
If it’s indeterminate whether A is true, and it’s indeterminate whether B is true, then it’s indeterminate whether their conjunction is true.

These two principles are analogous to Export for Permissibility and Export for Indeterminate Permissibility, respectively. According to supervaluationism, Export for Truth is true but Export for Indeterminate Truth is false. Why is this? Since truth entails determinate truth, if A is true and B is true, each is determinately true, so their conjunction is determinately true, and hence true. So supervaluationism entails Export for Truth. However, supervaluationism entails that Export for Indeterminate Truth is false. Suppose “S is red” and “S is not red” are both indeterminately true (because, for example, S is on the borderline between red and orange). Nevertheless, their conjunction “S is red and not red” is determinately false. (It is determinately false because it is false according to every admissible precisification of “is red.”) These examples provide a helpful model for understanding why it makes sense to reject Export for Indeterminate Permissibility while endorsing Export for Permissibility. Thus, Moderate’s rejection of Export for Indeterminate Permissibility is theoretically motivated, whereas Liberal’s rejection of Export for Permissibility is ad hoc.

In response, a defender of Liberal might try to motivate their rejection of Export for Permissibility by attempting to undermine the analogous principle for moral permissibility:

**Export for Moral Permissibility:**
If action A is morally permissible, and action B is morally permissible, then the compound action A and B is morally permissible.

Consider the following case. It would, let us suppose, be morally permissible for you not to visit your parents this weekend. Alternatively, it would also be morally permissible for you to call on Friday and tell your parents that you plan to visit them this weekend. However, it would not be morally permissible to perform the compound action: call your parents on Friday, tell them that you will visit, and then not visit. Cases like this may seem to show that Export for Moral Permissibility is false.

However, this appearance is deceiving. In fact, this case does not constitute a counterexample to Export for Moral Permissibility. This is because it is not true that not visiting your parents is morally permissible, simpliciter. Whether this individual action is permissible depends in part on whether or not you called them and told them you would visit. If you did, then the action is not morally permissible. So, what we have here is not a case in which action A and action B are both individually morally permissible, but their compound action is not. Rather, what we have is a case in which whether or not action B is permissible depends on whether or
not action A is performed. So this case does not constitute a counterexample to Export for Moral Permissibility. This principle, along with Export for Permissibility, remains in good standing, and Liberal’s rejection of the latter remains unmotivated.

I will now discuss an objection that applies to both Moderate and Liberal. Once again, we will see that Moderate can disarm the objection by drawing on analogies with other applications of supervaluationism. No such strategy is available to Liberal.

Notice that according to both Moderate and Liberal, the following can come apart: (1) action X is determinately impermissible; and (2) performing action X would make it the case that you have performed some determinately impermissible action. For example, suppose I offer you bet A, and you reject it. I then go on to offer you bet B. In this context, rejecting bet B would make it the case that you have performed some determinately impermissible action—namely, the compound action of rejecting both bets. However, as we have seen, according to Liberal, it is still permissible for you to reject bet B. Moreover, according to Moderate, it is still indeterminate whether rejecting bet B is permissible. So on neither view does it follow that rejecting bet B would be determinately impermissible.

This might seem problematic (Elga (2010) thinks so). If I know that performing a certain action X would make it the case that I’ve performed some determinately irrational action, doesn’t it follow that it would be determinately irrational to perform X? According to both Liberal and Moderate, the answer is no.

In the case of Moderate, however, we can situate this feature in the broader supervaluationist framework in a way that renders it unproblematic. Suppose I assert, of some borderline red shade S, “S is red.” On supervaluationism, it is indeterminate whether what I said is true. I then go on to assert “S is not red.” It is also indeterminate whether this is true. However, in making the second assertion, I make it the case that I have committed myself to the truth of something determinately false, namely, “S is red and S is not red.” This conjunction is determinately false because it’s false no matter where we place the cut-off between “red” and “not-red,” and so false according to every admissible precisification of “red.” When I asserted “S is not red,” what I asserted was indeterminate; however, under the circumstances, asserting it made it the case that I committed myself to the truth of something determinately false. (I assume that if one asserts A and then asserts B one is thereby committed to the truth of their conjunction.) This is analogous to Moderate’s result that it can be indeterminate whether some action is rational, even though performing that action would make it the case that you have performed some determinately irrational action. Seen in this light, this result seems unproblematic. But Liberal has no broader theoretical resources to draw on—it can give no deeper explanation for why an action might be permissible, even though performing it would mean that one has done something determinately impermissible.

Moreover, any defender of Liberal has an unpaid debt: they owe us an alternative interpretation of the set of functions model. As we have seen, the supervaluationist interpretation entails Moderate. Since Liberal and Moderate conflict, this means that Liberal is incompatible with the supervaluationist interpretation. I think part of the reason why Liberal has enjoyed such relative popularity as a decision rule for imprecise credences is that this incompatibility has heretofore gone unnoticed.

Are there other plausible interpretations of the set of functions model? Unfortunately, as mentioned earlier, there has not been much attention in the literature on the interpretation of the model. The supervaluationist interpretation used here has been explicitly endorsed by several proponents of the model (including van Fraassen (1990, 2005, 2006) and Hajek (2003)). Others
(e.g. Joyce (2010)) also give this same interpretation, but don’t explicitly refer to it as supervaluationist.

There is one suggestion that may seem to constitute an alternative to the supervaluationist interpretation. On this suggestion, an agent’s doxastic state consists must fundamentally of a set of judgments. For example, the agent may judge that A is more likely than B; that C is certain; that E has probability .3; etc. The set of functions model is then seen as just a convenient way to represent these judgments: an agent’s set is comprised of all and only those functions compatible with the agent’s judgments. (For example, functions compatible with the above-mentioned judgments have \( \Pr(A) > \Pr(B) \), \( \Pr(C) = 1 \), and \( \Pr(E) = .3 \).)

Moreover, it may seem that this suggestion provides an independent motivation for Liberal. On this suggestion, the functions in the agent’s set are all and only those not ruled out by her judgments. It may seem that anything not ruled out by the agent’s judgments is permissible for her. So, any function in the agent’s set, since not ruled out by the agent’s judgments, is a permissible option for the agent, in the sense that it would be rationally permissible for her to have that function as her credence function. But it’s natural to think that, if it would be permissible for an agent to have a function \( \Pr \) as her credence function, then it would be permissible for her to perform any action that is permissible according to \( \Pr \). Combining these very natural claims, we get the result that, if an action is permissible according to some function in the agent’s set, then that action is permissible for the agent. This is Liberal.

Before we can evaluate this line of thought, we need to get clearer about the suggested interpretation of the set of functions model. I will argue that on the most literal way of understanding this suggestion, it faces serious problems. There is a less literal way of understanding it that avoids these problems, but on this version, the line of thought in favor of Liberal does not go through.

On the most literal way of understanding the suggestion, to say, for example, that an agent judges that A is more likely than B is to say that the agent takes a certain propositional attitude—namely, judgment—toward the proposition that A is more likely than B. To say that the agent judges that C is certain is to say that she takes that same proposition attitude (namely, judgment) toward the proposition that C is certain. And so forth. However, it is problematic to think that the totality of an agent’s doxastic state consists of a set of judgments of this kind.

One problem concerns how to understand the propositions toward which the agent supposedly takes the attitude of judgment. For example, what is meant by “more likely” in “A is more likely than B”? It is common to recognize multiple possible disambiguations: likelihood may be interpreted as objective chance; or subjective level of confidence; or rational level of confidence (given a body of evidence). However, there are problems regardless of which of these we employ. Suppose we go with objective chance. Then the agent’s set will contain all and only those functions compatible with her judgments about objective chance. But we wanted the set of functions to represent the agent’s attitudes toward all propositions, not just those she takes to have objective chances. Suppose, then, we go with subjective level of confidence. But then the set will represent only the agent’s judgments about her own subjective levels of confidence (or, worse, someone else’s). We wanted it to also represent the agent’s first-order attitudes toward all propositions (after all, her judgments about her attitudes may be mistaken or incomplete). If we go with rational level of confidence, then the set will represent the agent’s judgments about the rationality of various attitudes. But, again, what we are trying to represent are the agent’s actual attitudes toward all propositions, not simply her views on rationality.

10 Thanks to an anonymous reviewer for suggesting that I consider this view.
upshot is that, regardless of how we understand likelihood, if we take the agent’s set to include all and only those functions compatible with her judgments about likelihood, it will fail to represent the totality of her actual doxastic state.\textsuperscript{11}

Another potential problem is that judgments, like beliefs, may come in degrees. If so, then we would want our formalism to reflect these differences in degree. But all judgments are treated in the same way: functions compatible with a judgment are allowed into the agent’s set, regardless of the strength of that judgment. But then the set of functions model would be blind to any differences in degree of judgment—potentially important features of the agent’s doxastic state. (In the background here is the question of whether judgment is the same as belief. If so, then this suggestion amounts to the claim that the agent’s doxastic state consists solely of a set of beliefs—ananthema to those sympathetic to a broadly Bayesian picture. If not, then we are owed an account of this new propositional attitude.)

I conclude that it is not plausible to think of the totality of an agent’s doxastic state as consisting of a set of judgments, if these are thought of as takings of a certain propositional attitude—namely, judgment—toward propositions such as “A is more likely than B.” However, we can avoid these problems if we take the suggestion in a less literal way. For example, we can understand “the agent judges that A is more likely than B” to mean simply “the agent is more confident of A than B.” “The agent judges that C is certain” can be taken as “the agent is certain of C.” And so forth. Instead of a set of judgments, then, what we have is a set of true propositions about the agent’s doxastic state. A function will be included in the agent’s set just in case, for each true proposition P about the agent’s doxastic state, P is true according to that function. (For example, “the agent is more confident of A than B” is true according to functions with Pr(A) > Pr(B); “the agent is certain of C” is true according to functions with Pr(C) = 1; etc.)

This way of reading the suggestion avoids the problems that confront the more literal reading. However, clarification on some further points is needed. First, is it the case that for each proposition P about the agent’s doxastic state, there is a determinate fact of the matter about whether P is true—or, is it sometimes indeterminate whether P is true? If we allow for indeterminacy, then the suggestion is perfectly compatible with the supervaluationist interpretation, on which there is determinacy when all functions in the set agree, and indeterminacy when they disagree.

In order to make sure we have a genuine alternative to the supervaluationist interpretation, then, henceforth I will take the suggestion to essentially involve commitment to the following determinacy assumption: for every proposition P about the agent’s doxastic state, there is a determinate fact of the matter about whether P is true. Next I turn to the trichotomy thesis. I will show that, when combined with the determinacy assumption, it follows from the trichotomy thesis that all rational agents have perfectly precise credences! So any proponent of the suggestion must reject the trichotomy thesis. But if so, then on the resulting view, the motivation for Liberal does not go through.

The trichotomy thesis\textsuperscript{12} says that for each pair of propositions A and B, one is either more confident of A than B, or equally confident of A and B, or less confident of A than B. I will now show why, when combined with the determinacy assumption, it follows from the trichotomy thesis that every rational agent has a perfectly precise credence in every proposition. Let B[r] be

\textsuperscript{11} Suppose we consider an expressivist interpretation of judgment, on which, for example, to judge that A is more likely than B is simply to express one’s greater confidence in A than B. This proposal is equivalent in all relevant respects to the less literal reading of the suggestion, which I discuss below.

\textsuperscript{12} I take this terminology from Chang (2002), who uses it as a name for an analogous principle about value.
the proposition that a coin with bias $r$ toward heads will land heads on the next toss. For every $r$, every rational agent has a perfectly precise credence in $B[r]$, namely, $r$. Let $A$ be an arbitrary proposition. We are assuming a determinate fact of the matter about all propositions of the forms “the agent is more confident of $B[r]$ than $A$,” “the agent is equally confident of $B[r]$ and $A$,” and “the agent is less confident of $B[r]$ than $A.”” Let $MC$ be the set of numbers $r$ such that the agent is more confident of $B[r]$ than $A$. It follows from the trichotomy thesis that, for all $r$ outside $MC$, either the agent is equally confident of $B[r]$ and $A$, or the agent is less confident of $B[r]$ than $A$. It can’t be the case that, for all $r$, either $r$ is inside $MC$ (i.e., the agent is more confident of $B[r]$ than $A$), or the agent is less confident of $B[r]$ than $A$. So, there will be some $r$ outside $MC$ for which the agent is equally confident of $B[r]$ and $A$. If so, then the agent has a perfectly precise credence in $A$, namely, $r$.

So, given the determinacy assumption, any proponent of this suggestion must reject the trichotomy thesis. Interestingly, the resulting view turns out to be equivalent to an alternative interpretation of the set of functions model that already exists in the literature. It is found in Schoenfield (2012) and Kaplan (2010).

Let’s briefly review some of the dialectic. We began considering a suggested alternative interpretation—on which the functions in your set are those compatible with your judgments—because this suggestion seemed to allow for a line of thought motivating Liberal. Before assessing this line of thought, we needed to get clear on the suggestion. A literal reading turned out to be problematic. We determined that a less literal reading, which avoids these problems, must involve the determinacy assumption and the denial of the trichotomy thesis. The resulting view is just the Schoenfield/Kaplan interpretation.

So, the crucial question now is this: does the above-described line of thought in favor of Liberal go through on the Schoenfield/Kaplan interpretation? I will argue that the answer is no.

The original line of thought went roughly as follows. Whatever is not ruled out by the agent’s judgments is permissible. So, each function in the agent’s set is a permissible doxastic option for her, since any such function is compatible with her judgments. But then any action permissible according to any function in the set is permissible for the agent.

A key part of this line of thought is that any function in the agent’s set is a permissible doxastic option for her. This seems natural when we take the suggestion literally—thinking of the functions in the set as those compatible with the agent’s judgments. But as we saw, the literal reading is problematic. On the less literal reading (i.e. the Schoenfield/Kaplan interpretation) it is no longer plausible to think that any function in the agent’s set is a permissible doxastic option for her. Suppose, for example, that the agent’s set contains functions with $Pr(A) > Pr(B)$, $Pr(A) = Pr(B)$, and $Pr(A) < Pr(B)$.

On the Schoenfield/Kaplan interpretation, this represents a situation in which the agent is neither more confident of $A$ than $B$, nor equally confident of $A$ and $B$, nor less confident of $A$ than $B$. There is no temptation to say that it would thereby be permissible for the agent to be, say, more confident of $A$ than $B$. Perhaps rationality requires the agent to be just as she is, namely, not more confident of $A$ than $B$. It is not plausible, on the Schoenfield/Kaplan interpretation, that each function in the agent’s set automatically constitutes a permissible doxastic state for that agent.

So the original line of thought in favor of Liberal does not go through on the Schoenfield/Kaplan interpretation. Although the Schoenfield/Kaplan interpretation is not inconsistent with Liberal, it does not provide any special motivation for it. The combination of Moderate with the supervaluationist interpretation is theoretically unified in a way that the combination of Liberal with the Schoenfield/Kaplan interpretation would not be. Moderate is
entailed by supervaluationism, as just one application, among many, of its general semantics. Liberal, however, has no special theoretical connection with the Schoenfield/Kaplan interpretation; it is merely not incompatible with it. (Moderate is also compatible with the Schoenfield/Kaplan interpretation.)

However, our discussion of why the Schoenfield/Kaplan interpretation fails to provide a motivation for Liberal may suggest a quite different alternative interpretation of the set of functions model, which may seem to succeed at motivating Liberal. According to this interpretation, a function is in your set just in case it would be rationally permissible for you to adopt that function as your credence function. Liberal may seem to fit quite naturally with this interpretation. After all, if it would be permissible for you to adopt some function as your credence function, presumably it would be permissible for you to perform an action that would be permissible according that function.

However, (1) this interpretation fails to satisfy a primary purpose of the set of functions model; (2) it seems to presuppose something that defenders of the set of functions model typically reject; and (3) the claim that it motivates Liberal is suspect.

A primary purpose of the set of functions model is to represent the totality of the agent’s actual doxastic state. However, identifying a set of functions, each of which would be rationally permissible for the agent to adopt, fails to do this. For example, if leaves open which (if any) of the functions the agent does in fact adopt. Moreover, the interpretation seems to presuppose that, typically, it would be rationally permissible for an agent to have a precise credence function. However, this is in conflict with one standard motivation for the set of functions model, namely, that rationality often requires imprecise credences.

Finally, the claim that this interpretation would support Liberal is suspect. Suppose that Pr1 and Pr2 are both rationally permissible credence functions for you, and thus (on this interpretation) members of your set. According to Pr1, action A is impermissible; according to Pr2, action A is permissible. Let’s suppose that Pr1 is your actual credence function. Is it so clear that action A would be permissible for you? After all, according to the credence function which you actually (and rationally) have, A is impermissible.

In sum, I am not aware of any interpretation of the set of functions model that could ground Liberal in a broader theoretical context or provide an independent motivation for it. Moderate, on the other hand, falls out as a simple application of the semantics that is an integral part of the supervaluationist interpretation of the set of functions model. Liberal, as we have seen, is forced to reject Export for Permissibility, in what seems like an ad hoc maneuver. Moderate entails Export for Permissibility, and is incompatible with Export for Indeterminate Permissibility; this can be explained as analogous to the way in which supervaluationism entails Export for Truth but is incompatible with Export for Indeterminate Truth. Both Moderate and Liberal have it that an action can fail to be determinately impermissible, even if performing it would make it the case that one has done something determinately impermissible. Once again, supervaluationism provides Moderate with theoretical resources that explain and independently motivate this feature, whereas Liberal has no such resources. For all of these reasons, Moderate is a plausible and tenable decision rule for the set of functions model, whereas Liberal is not.

5 Other Alternative Decision Rules
It is not among the aims of this paper to provide a detailed survey of all the decision rules that have been proposed in the literature. However, in the first part of this section I will briefly discuss two alternative rules. In the course of doing so I will also respond to an objection to Moderate concerning *ought implies can*. Then, in the second part of this section I will consider the idea that whether or not it is permissible to reject bet B depends on whether or not bet A was rejected. We will see that all of these alternative views face serious problems.

After Liberal it is natural to consider a view we might call “Conservative,” on which an action is permissible if permissible according to *all* functions in your set; otherwise, impermissible. As noted in Joyce (2010), a disadvantage of this view is that in some cases, every cell in some partition of your options would be considered determinately impermissible—in violation of *ought implies can*. For example, rejecting bet A would be determinately impermissible, and accepting it would also be determinately impermissible.

This may make one wonder whether Moderate is incompatible with *ought implies can*. After all, according to Moderate it is *indeterminate* whether accepting bet A is permissible, and also indeterminate whether rejecting bet A is permissible. As I will explain below, it may seem that *ought implies can* requires that there always be some determinately permissible option.

Interestingly, this has the potential to constitute an objection to Moderate that could not be handled in the way we’ve dealt with other objections. As we’ve seen, Moderate has a few prima facie odd features, but thus far we’ve always been able to situate them in the broader supervaluationist framework. Seen in this context, they become unobjectionable—just further instances of general phenomena that the supervaluationist is happy to countenance anyway.

However, since *ought implies can* is specific to normativity, if Moderate is incompatible with this maxim, we would not be able to respond by drawing on helpful analogies with the supervaluationist treatment of non-normative concepts like “red” or “tall.” Fortunately, as I explain below, it turns out that Moderate and *ought implies can* are, in fact, perfectly compatible.

Why might one think that *ought implies can* entails that there must be at least one permissible option? Well, suppose that all possible options were impermissible. If all of your options are individually impermissible, then the *disjunction* of all of your options is impermissible. (If you’re not allowed to do A, and not allowed to do B, then you’re not allowed to do A or B.) If the disjunction of all your options is *impermissible*, then the negation of that disjunction is *required*. (In general, the negation of any impermissible act is required.) But the negation of the disjunction of all of your options is impossible! So rationality would require the impossible—a violation of *ought implies can*, which says that rationality can only require what is possible.

The foregoing succeeds in showing that it can’t be the case that all possible options are impermissible. However, given that we are allowing for the possibility of indeterminate permissibility, we *cannot* conclude from this that at least one of your options must be determinately permissible. Rather, all we can conclude is that at least one of your options must fail to be determinately impermissible. This is true if at least one of your options is indeterminately permissible, even if none of them is determinately permissible. So *ought implies can* is compatible with Moderate’s result that, in some cases, each of your options is individually indeterminately permissible.

It is also worth noting that there are cases in which, plausibly, none of your options is determinately *morally* permissible. For example, suppose that Darryl has also brought his dog to the park, along with his daughter. His dog is playing on the other side of the park, so Darryl

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13 One recent proposal, which, it is argued, handles Elga’s cases correctly, can be found in Moss (forthcoming).
can’t watch them both at the same time. His daughter takes precedence over his dog; but he does have some obligations to his dog. Specifically, he is required to watch his dog whenever it is permissible for him to not watch his daughter. Suppose it’s indeterminate whether it’s permissible for him to divert his attention from his daughter for a full 30 seconds. In that case, it’s indeterminate whether he’s required to watch his dog for a full 30 seconds. If he does watch his dog for 30 seconds, he will have diverted his attention from his daughter for 30 seconds, an indeterminately morally permissible act. But if he returns his attention to his daughter before 30 seconds is up, he’s failed to do something indeterminately morally required, namely, watch his dog for the full 30 seconds, and so has done something indeterminately morally permissible. No matter what he does, he will have performed some indeterminately morally permissible act. This completes my response to the objection to Moderate concerning *ought implies can.*

Another alternative decision rule has it that there is one particular function in your set that determines which options are permissible for you and which are impermissible. I agree with Elga (2010) that such a view undermines the original motivation for the set of functions model. Let \( F \) be the special function. On this view, the facts about which actions are permissible for you would be the same, regardless of whether your doxastic state is the set of functions, or just the single function \( F \). But if there is a genuine difference in these two doxastic states—which there should be, if we employ the set of functions model at all—then this difference should manifest itself in the status of at least some claim about the rationality of possible actions.

Finally, I’d like to consider the idea that whether or not it is permissible to reject bet \( B \) depends on whether bet \( A \) was rejected. Here is a modified version of Liberal with this result: an action is permissible just in case it’s permissible according to some function with the following two features: (1) the function is a member of your set; and (2) every action that you have in fact performed in the past was permissible according to that function.\(^{14}\) According to this rule, if you rejected bet \( A \), rejecting bet \( B \) is *not* permissible; if you accepted bet \( A \), rejecting bet \( B \) is permissible.

However, this decision rule conflicts with the plausible principle that the rationality of an action is determined solely by the agent’s beliefs and values. Consider two agents who have exactly the same doxastic attitude towards \( H \): \([.1, .8]\). They also have exactly the same values: both care only about money. Both are offered bet \( B \). Surely if it is permissible for one to reject bet \( B \), then it is permissible for the other to reject bet \( B \). But this does not follow on the decision rule under consideration here. If one, but not the other, just rejected bet \( A \), then this rule has the consequence that rationality requires different things of them with respect to bet \( B \), even though their beliefs and values are exactly the same.\(^{15}\)

In response, one might adopt a different strategy for getting the result that the permissibility of rejecting bet \( B \) depends on whether bet \( A \) was rejected. On this new strategy, we retain Liberal (in its original formulation) as our decision rule, but modify the *updating* rule. Typically, updating on the set of functions model is assumed to proceed by conditionalizing each function in the set on new evidence received. On the modified view now under consideration, the set is still updated in this way, but there is a second way in which updating can occur. Whenever one performs an action, any function according to which that action was impermissible is removed from the set. So, for example, if one rejects bet \( A \), any function on which that action was impermissible is removed. Since there is no single function on which rejecting bet \( A \) and rejecting bet \( B \) are both permissible, this means that anyone who rejects bet

\(^{14}\) Thanks to Ned Hall for encouraging me to discuss this view.

\(^{15}\) Elga (2010) considers a similar rule and gives similar arguments against it.
A will have a set containing only functions on which rejecting bet B is impermissible. So this revised updating rule, when paired with Liberal, has the result that whether rejecting bet B is permissible depends on whether one in fact rejected bet A.

However, this updating rule faces a problem: it conflicts with Evidentialism, the view that the rationality of a doxastic attitude towards a proposition P depends only on the evidence one has that is relevant to P. Consider two agents who initially have exactly the same evidence and exactly the same doxastic attitude towards H: [.1, .8]. They acquire no new relevant evidence, but one accepts bet A while the other rejects it. According to the modified updating rule, rationality now requires them to have different doxastic attitudes towards H, in violation of Evidentialism.

I conclude that we should not adopt the view that the permissibility of rejecting bet B depends on whether one rejected bet A. As we have seen, the first attempt to accommodate this idea was inconsistent with the plausible claim that the rationality of one’s actions depends only on one’s beliefs and values; the second attempt was inconsistent with Evidentialism.

5. Conclusion

The set of functions model of belief can be seen as an instance of supervaluationism, the most popular account of vagueness. Doing so helps us answer one of the most serious objections to this model, namely, that it is not compatible with any tenable decision theory. All we have to do is apply our general supervaluationist semantics to propositions about which actions are permissible, required, impermissible, etc. The resulting view handles with ease certain problem cases, such as one described in Elga (2010), on which other proposals falter.

One notable feature of this view is that in some cases it is indeterminate whether an action is permissible or not. This is, I think, part of the reason why the view has thus far been overlooked: all parties to the debate seem to be operating under the assumption that any acceptable decision theory will provide a definite classification of each action as either permissible or impermissible. But this is misguided when working within a model that already posits indeterminacy in our beliefs. Since the rationality of an action depends in large part on what one believes, it is only to be expected that indeterminacy in belief would lead to indeterminacy in the rationality of some actions.

Acknowledgements

Many thanks to Tom Dougherty, Adam Elga, Luke Elson, Brie Gertler, Andrew Graham, Ned Hall, Brian Hedden, Sarah Moss, Brad Skow, and two anonymous reviewers for Philosophers’ Imprint. Special thanks to Miriam Schoenfield for detailed comments on multiple drafts.

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