ACCURACY, COHERENCE AND EVIDENCE

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ABSTRACT. Taking Joyce's (1998; 2009) recent argument(s) for probabilism as our point of departure, we propose a new way of grounding formal, synchronic, epistemic coherence requirements for (opinionated) full belief. Our approach yields principled alternatives to deductive consistency, sheds new light on the preface and lottery paradoxes, and reveals novel conceptual connections between alethic and evidential epistemic norms.

1. SETTING THE STAGE

This essay is about formal, synchronic, epistemic, coherence requirements. We begin by explaining how we will be using each of these (five) key terms.

Formal epistemic coherence requirements involve properties of judgment sets that are logical (and, in principle, determinable *a priori*). These are to be distinguished from other less formal and more substantive notions of coherence that one encounters in the epistemological literature. For instance, so-called "coherentists" like BonJour (1985) use the term in a less formal sense which implies (*e.g.*) that coherence is truth-conducive. While there will be conceptual connections between the accuracy of a doxastic state and its coherence (in the sense we have in mind), these connections will be quite weak (too weak to merit the conventional honorific "truth-conducive"). All of the varieties of coherence to be discussed below will be intimately related to deductive consistency. Consequently, whether a set of judgments is coherent will be determined by (*i.e.*, will supervene on) logical properties of the set of propositions that are the objects of the judgments in question.

Synchronic epistemic coherence requirements apply to the doxastic states of agents at individual times. These are to be distinguished from diachronic requirements (*e.g.*, conditionalization, reflection, *etc.*), which apply to sequences of doxastic states across times. Presently, we will be concerned only with the former.[1]

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This material has been presented in various places over the past several years, and the list of people with whom we've had useful relevant conversations is too long to enumerate here. But, in addition to three anonymous referees who provided very useful written comments, we must single out (in alphabetical order) the following people who have been especially generous with valuable feedback regarding this project: Rachael Briggs, Fabrizio Cariani, Jim Joyce, Ole Hjortland, Hannes Leitgeb, Ben Levinstein, Richard Pettigrew, Florian Steinberger, and Jonathan Weisberg. Branden Fitelson would also like to thank the Alexander von Humboldt (AvH) Foundation for their generous support during his summer (2011, 2012) visits to the Munich Center for Mathematical Philosophy at Ludwig-Maximilians-Universität München, where he presented much of this material in seminars.

[1]See Titelbaum (2013) for an excellent recent survey of the contemporary literature on (Bayesian) diachronic epistemic coherence requirements. Some, *e.g.*, Moss (2013) and Hedden (2013), have argued that there are no diachronic epistemic rational requirements (*i.e.*, that there are only synchronic epistemic rational requirements). We take no stand on this issue here. But, we will assume that there are (some) synchronic epistemic rational requirements of the sort we aim to explicate (see *fn. 7*).

Epistemic requirements are to be distinguished from, *e.g.*, pragmatic requirements. Starting with Ramsey (1928), the most well-known arguments for probabilism as a formal, synchronic, coherence requirement for credences have depended on the pragmatic connection of belief to action. For instance, "Dutch Book" arguments and "Representation Theorem" arguments (Hájek 2008) aim to show that an agent with non-probabilistic credences (at a given time t) must (thereby) exhibit some sort of "pragmatic defect" (at t).[2] Following Joyce (1998; 2009), we will be focusing on non-pragmatic (*viz.*, epistemic) defects implied by the synchronic incoherence (in a sense to be explicated below) of an agent's doxastic state. To be more precise, we will be concerned with two aspects of doxastic states that we take to be distinctively epistemic: (a) how accurate a doxastic state is, and (b) how much evidential support a doxastic state has. We will call these (a) alethic and (b) evidential aspects of doxastic states, respectively.[3]

Coherence requirements are global and wide-scope. Coherence is a global property of a judgment set in the sense that it depends on properties of entire set in a way that is not (in general) reducible to properties of individual members of the set. Coherence requirements are wide-scope in Broome's (2007) sense. They will be expressible using "shoulds" (or "oughts") that take wide-scope over some logical combination(s) of judgments. As a result, coherence requirements will not (in general[4]) require specific attitudes toward specific individual propositions. Instead, coherence requirements will require the avoidance of certain combinations of judgments. We use the term "coherence" — rather than "consistency" — because (a) the latter is typically associated with classical deductive consistency (which, as we'll see shortly, we do not accept as a necessary requirement of epistemic rationality), and (b) the former is used by probabilists when they discuss analogous requirements for degrees of belief (*viz.*, probabilism as a coherence requirement for credences). Because our general approach (which was inspired by Joycean arguments for probabilism) can be applied to many types of judgment — including both full belief and partial belief[5] — we prefer to maintain a common parlance for the salient requirements in all of these settings.

Finally, and most importantly, when we use the term "requirements", we are talking about necessary requirements of ideal epistemic rationality.[6] The hallmark of a necessary requirement of epistemic rationality N is that if a doxastic state

[2]We realize that there are "depragmatized" versions of these arguments (Christensen, 1996). But, even these versions of the arguments trade essentially on the pragmatic role of doxastic attitudes (in "sanctioning" bets, *etc.*). In contrast, we will only be appealing to epistemic connections of belief to truth and evidence. Our arguments do not explicitly rely upon connections between belief and action.

[3]The alethic/evidential distinction is central to the pre-Ramseyan debate between James (1896) and Clifford (1877). Roughly speaking, "alethic" considerations are "Jamesian", and "evidential" considerations are "Cliffordian". We will be assuming for the purposes of this article that alethic and evidential aspects exhaust the distinctively epistemic properties of doxastic states. But, our approach could be generalized to accommodate additional dimensions of epistemic evaluation.

[4]There are two notable exceptions to this rule. It will follow from our approach that (a) rational agents should never believe individual propositions ($\perp$) that are logically self-contradictory, and (b) that rational agents should never disbelieve individual propositions ($\top$) that are logically true.

[5]In fact, the framework can be applied fruitfully to other types of judgment as well. See (Fitelson & McCarthy 2013) for an application to comparative confidence, which leads to a new foundation for comparative probability. For a survey of applications of the general framework, see (Fitelson 2014).

[6]Here, we adopt Titelbaum's (2013, chapter 2) locution "necessary requirement of (ideal) rationality" as well as (roughly) his usage of that locution (as applied to formal, synchronic requirements).

S violates N, then S is (thereby) epistemically irrational. However, just because a doxastic state S satisfies a necessary requirement N, this does not imply that S is (thereby) rational. For instance, just because a doxastic state S is coherent (*i.e.*, just because S satisfies some formal, epistemic coherence requirement), this does not mean that S is (thereby) rational (as S may violate some other necessary requirement of epistemic rationality). Thus, coherence requirements in the present sense are (formal, synchronic) necessary conditions for the epistemic rationality of a doxastic state.[7] Our talk of the epistemic (ir)rationality of doxastic states is meant to be evaluative (rather than normative[8]) in nature. To be more precise, we will (for the most part) be concerned with the evaluation of doxastic states, relative to an idealized[9] standard of epistemic rationality. Sometimes we will speak (loosely) of what agents "should" do — but this will (typically) be an evaluative sense of "should" (*e.g.*, "should on pain of occupying a doxastic state that is not ideally epistemically rational"). If a different sense of "should" is intended, we will flag this by contrasting it with the idealized/evaluative "should" that features in our rational requirements. Now that the stage is set, it will be instructive to look at the most well-known "coherence requirement" in the intended sense.

2. Deductive Consistency, The Truth Norm, and The Evidential Norm

The most well-known example of a formal, synchronic, epistemic coherence requirement for full belief is the (putative) requirement of deductive consistency.

(CB) All agents S should (at any given time t) have sets of full beliefs (*i.e.*, sets of full belief-contents) that are (classically) deductively consistent.

Many philosophers have assumed that (CB) is a necessary requirement of ideal epistemic rationality. That is, many philosophers have assumed that (CB) is true, if its "should" is interpreted as "should on pain of occupying a doxastic state that is not ideally epistemically rational". Interestingly, in our perusal of the literature, we haven't been able to find many (general) arguments in favor of the claim that (CB) is a rational requirement. One potential argument along these lines takes as its point of departure the (so-called) *Truth Norm* for full belief.[10]

(TB) All agents S should (at any given time t) have full beliefs that are true.[11]

Presumably, there is some sense of "should" for which (TB) comes out true, *e.g.*, "should on pain of occupying a doxastic state that is not perfectly accurate" (see *fn.* 11). But, we think most philosophers would not accept (TB) as a rational requirement.[12] Nonetheless, (TB) clearly implies (CB) — in the sense that all agents who satisfy (TB) must also satisfy (CB). So, if one violates (CB), then one must also violate (TB). Moreover, violations of (CB) are the sorts of things that one can (ideally, in principle) be in a position to detect *a priori*. Thus, one might try to argue that (CB) is a necessary requirement of ideal epistemic rationality, as follows. If one is (ideally, in principle) in a position to know *a priori* that one violates (TB), then one's doxastic state is not (ideally) epistemically rational. Therefore, (CB) is a rational requirement. While this (TB)-based argument for (CB) may have some *prima facie* plausibility, we'll argue that (CB) itself seems to be in tension with another plausible epistemic norm, which we call the *Evidential Norm* for full belief.

(EB) All agents S should (at any given time t) have full beliefs that are supported by the total evidence.

For now, we're being intentionally vague about what "supported" and "the total evidence" mean in (EB), but we'll precisify these locutions in due course.[13]

[7]For simplicity, we will assume that there exist some (synchronic, epistemic) rational requirements in the first place. We are well aware of the current debates about the very existence of rational requirements (*e.g.*, coherence requirements). Specifically, we are cognizant of the salient debates between Kolodny (2007) and others, *e.g.*, Broome (2007). Here, we will simply adopt the non-eliminativist stance of Broome *et al.* who accept the existence of (ineliminable) rational requirements (*e.g.*, coherence requirements). We will not try to justify our non-eliminativist stance here, as this would take us too far afield. However, as we will explain below, even coherence eliminativists like Kolodny should be able to benefit from our approach and discussion (see *fn.* 45). As such, we (ultimately) see our adoption of a non-eliminativist stance in the present context as a simplifying assumption.

[8]Normative principles support attributions of blame or praise of agents, and are (in some sense) action guiding. Evaluative principles support classifications of states (occupied by agents) as "defective" *vs* "non-defective" ("bad" *vs* "good"), relative to some evaluative standard (Smith, 2005, §3).

[9]Deductive consistency and the other formal coherence requirements we'll be discussing are highly idealized rational epistemic requirements. They all presuppose a standard of ideal rationality which is insensitive to semantic and computational (and other) limitations of (actual) agents who occupy the doxastic states under evaluation. While this is, of course, a strong idealization (Harman 1986), it constitutes no significant loss of generality in the present context. This is because our aims here are rather modest. We aim (mainly) to do two things in this paper: (a) present the simplest, most idealized version of our framework and the (naïve) coherence requirements to which it gives rise, and (b) contrast these new requirements with the (equally simple and naïve) requirement of deductive consistency. Owing to the idealized/evaluative nature of our discussion, we will typically speak of the (ir)rationality of states, and not the (ir)rationality of agents who occupy them. Finally, we will sometimes speak simply of "rational requirements" or just "requirements". It is to be understood that these are shorthand for the full locution "necessary requirements of ideal epistemic rationality".

[10]We will use the term "norm" (as opposed to "requirement") to refer to local/narrow-scope epistemic constraints on belief. The Truth Norm (as well as the Evidential Norm, to be discussed below) is local in the sense that it constrains each individual belief — it requires that each proposition believed by an agent be true. This differs from the rational requirements we'll be focusing on here (*viz.*, coherence requirements), which are global/wide-scope constraints on sets of beliefs. Moreover, the sense of "should" in norms will generally differ from the evaluative/global sense of "should" that we are associating with rational requirements (see *fn.* 13).

[11]Our statement of (TB) is (intentionally) somewhat vague here. Various precisifications of (TB) have been discussed in the contemporary literature. See Thomson (2008), Wedgwood (2002), Shah (2003), Gibbard (2005) and Boghossian (2003) for some recent examples. The subtle distinctions between these various renditions of (TB) will not be crucial for our purposes. For us, (TB) plays the role of determining the correctness/accuracy conditions for belief (*i.e.*, it determines the alethic ideal for belief states). In other words, the "should" in our (TB) is intended to mean something like "should on pain of occupying a doxastic state that is not entirely/perfectly correct/accurate". In this sense, the version of (TB) we have in mind here is perhaps most similar to Thomson's (2008, Ch. 7).

[12]Some philosophers maintain that justification/warrant is factive (Littlejohn 2012; Merricks 1995). In light of the Gettier problem, factivity seems plausible as a constraint on the type of justification required for knowledge (Zagzebski 1994; Dretske 2013). However, factivity is implausible as a constraint on (the type of justification required for) rational belief. As such, we assume that "is supported by the total evidence" (*i.e.*, "is justified/warranted") is not factive. This assumption is kosher here, since it cross-cuts the present debate regarding (CB). For instance, Pollock's defense of (CB) as a coherence requirement does not trade on the factivity of evidential support (see *fn.* 15).

[13]The evidential norm (EB) is [like (TB)] a local/narrow-scope principle. It constrains each individual belief, so as to require that it be supported by the evidence. We will not take a stand on the precise content of (EB) here, since we will (ultimately) only need to make use of certain (weak) consequences of (EB). However, the "should" of (EB) is not to be confused with the "should" of (TB). It may be useful (heuristically) to read the "should" of (EB) as "should on pain of falling short of the Cliffordian ideal" and the "should" of (TB) as "should on pain of falling short of the Jamesian ideal" (see *fns.* 3 & 10).

Versions of (EB) have been endorsed by various "evidentialists" (Clifford 1877; Conee & Feldman 2004). Interestingly, the variants of (EB) we have in mind conflict with (CB) in some ("paradoxical") contexts. For instance, consider the following example, which is a "global" version of the Preface Paradox.

> **Preface Paradox**. Let **B** be the set containing all of *S*'s justified first-order beliefs. Assuming *S* is a suitably interesting inquirer, this set **B** will be a very rich and complex set of judgments. And, because *S* is fallible, it is reasonable to believe that some of *S*'s first-order evidence will (inevitably) be misleading. As a result, it seems reasonable to believe that some beliefs in **B** are false. Indeed, we think *S* herself could be justified in believing this very second-order claim. But, of course, adding this second-order belief to **B** renders *S*'s overall doxastic (full belief) state deductively inconsistent.

We take it that, in (some) such preface cases, an agent's doxastic state may satisfy (EB) while violating (CB). Moreover, we think that (some) such states need not be (ideally) epistemically irrational. That is, we think our Preface Paradox (and other similar examples) establish the following key claim:

(†) (EB) does not entail (CB). [*i.e.*, the Evidential Norm does not entail that deductive consistency is a requirement of ideal epistemic rationality.]

We do not have space here to provide a thorough defense of (†).[14] Foley (1992) sketches the following, general "master argument" in support of (†).

> . . . if the avoidance of recognizable inconsistency were an absolute prerequisite of rational belief, we could not rationally believe each member of a set of propositions and also rationally believe of this set that at least one of its members is false. But this in turn pressures us to be unduly cautious. It pressures us to believe only those propositions that are certain or at least close to certain for us, since otherwise we are likely to have reasons to believe that at least one of these propositions is false. At first glance, the requirement that we avoid recognizable inconsistency seems little enough to ask in the name of rationality. It asks only that we avoid certain error. It turns out, however, that this is far too much to ask.

We think Foley is onto something important here. As we'll see, Foley's argument dovetails nicely with our approach to grounding coherence requirements for belief.

So far, we've been assuming that agents facing Prefaces (and similar paradoxes of deductive consistency) may be opinionated regarding the (inconsistent) sets of propositions in question (*i.e.*, that the agents in question either believe or disbelieve each proposition in the set). In the next section, we consider the possibility that the appropriate response to the Preface Paradox (and other similar paradoxes) is to suspend judgment on (some or all) propositions implicated in the inconsistency.

[14] Presently, we are content to take (†) as a *datum*. However, definitively establishing (†) requires only the presentation of one example (preface or otherwise) in which (CB) is violated, (EB) is satisfied, and the doxastic state in question is not (ideally) epistemically irrational. We think our Preface Paradoxes suffice. Be that as it may, we think Christensen (2004), Foley (1992), and Klein (1985) have given compelling reasons to accept (†). And, we'll briefly parry some recent philosophical resistance to (†) below. One might even want to strengthen (†) so as to imply that satisfying (EB) sometimes requires the violation of (CB). Indeed, this stronger claim is arguably established by our Preface Paradox cases. In any event, we will, in the interest of simplicity, stick with our weaker rendition of (†).

3. Suspension of Judgment to the Rescue?

Some authors maintain that opinionation is to blame for the discomfort of the Preface Paradox (and should be abandoned in response to it). We are not moved by this line of response to the Preface Paradox. We will now briefly critique two types of "suspension strategies" that we have encountered.

It would seem that John Pollock (1983) was the pioneer of the "suspension strategy". According to Pollock, whenever one recognizes that one's beliefs are inconsistent, this leads to the "collective defeat" of (some or all of) the judgments comprising the inconsistent set. That is, the evidential support that one has for (some or all of) the beliefs in an inconsistent set is defeated as a result of the recognition of said inconsistency. If Pollock were right about this (in full generality), it would follow that if the total evidence supports each of one's beliefs, then one's belief set must be deductively consistent. In other words, Pollock's general theory of evidential support (or "warrant"[15]) must entail that (†) is false. Unfortunately, however, Pollock does not offer much in the way of a general argument against (†). His general remarks tend to be along the following lines (Pollock 1990, *p*. 231).[16]

> The set of warranted propositions must be deductively consistent.
> . . . If a contradiction could be derived from it, then reasoning from some warranted propositions would lead to the denial (and hence defeat) of other warranted propositions, in which case they would not be warranted.

The basic idea here seems to be that, if one (knowingly) has an inconsistent set of (justified) beliefs, then one can "deduce a contradiction" from this set, and then "use this contradiction" to perform a "*reductio*" of (some of) one's (justified) beliefs.[17] Needless to say, anyone who is already convinced that (†) is true will find this general argument against (†) unconvincing. Presumably, anyone who finds themselves in the midst of a situation that they take to be a counterexample to (EB) ⇒ (CB) should be reluctant to perform "*reductios*" of the sort Pollock seems to have in mind, since it appears that consistency is not required by their evidence. Here, Pollock seems to be assuming a closure condition (*e.g.*, that "is supported by the total evidence" is closed under logical consequence/competent deduction) to provide a *reductio* of (†). It seems clear to us that those who accept (†) would/should reject

[15] Pollock uses the term "warranted" rather than "supported by the total evidence". But, for the purposes of our discussion of Pollock's views, we will assume that these are equivalent. This is kosher, since, for Pollock, "*S* is warranted in believing *p*" means "*S* could become justified in believing *p* through (ideal) reasoning proceeding exclusively from the propositions he is objectively justified in believing" (Pollock 1990, *p*. 87). Our agents, like Pollock's, are "idealized reasoners", so we may stipulate (for the purposes of our discussion of Pollock's views) that when we say "supported by the total evidence", we just mean whatever Pollock means by "warranted". Some (Merricks 1995) have argued that Pollock's notion of warrant is factive (see *fn.* 12). This seems wrong to us (in the present context). If warrant (in the relevant sense) were factive, then Pollock wouldn't need such complicated responses to the paradoxes of consistency — they would be trivially ruled out, *a fortiori*. This is why, for present purposes, we interpret Pollock as claiming only that (EB) entails the consistency of (warranted) belief sets [(CB)], but not necessarily the truth of each (warranted) belief [(TB)].

[16] The elipsis in our quotation contains the following parenthetical remark: "It is assumed here that an epistemic basis must be consistent." That is, Pollock gives no argument(s) for the claim that "epistemic bases" (which, for Pollock, are sets of "input propositions" of agents) must be consistent.

[17] Ryan (1991; 1996) gives a similar argument against (†). And, Nelkin (2000) endorses Ryan's argument, as applied to defusing the lottery paradox as a counterexample to ¬(†) [*i.e.*, (EB) ⇒ (CB)].

closure conditions of this sort. We view (some) Preface cases as counterexamples to both consistency and closure of rational belief.[18]

While Pollock doesn't offer much of a general argument for ¬(†), he does address two apparent counterexamples to ¬(†): the lottery paradox and the preface paradox. Pollock (1983) first applied this "collective defeat" strategy to the lottery paradox. He later recognized (Pollock 1986) that the "collective defeat" strategy is far more difficult to (plausibly) apply in the case of the Preface Paradox. Indeed, we find it implausible on its face that the propositions of the (global) Preface "jointly defeat one another" in any probative sense. More generally, we find Pollock's treatment of the Preface Paradox quite puzzling and unpersuasive.[19] Be that as it may, it's difficult to see how this sort of "collective defeat" argument could serve to justify ¬(†) in full generality. What would it take for a theory of evidential support to entail ¬(†) — in full generality — *via* a Pollock-style "collective defeat" argument? We're not sure. But, we are confident that any explication of "supported by the total evidence" (or "warranted") which embraces a phenomenon of "collective defeat" that is robust enough to entail the falsity of (†) will also have some undesirable (even unacceptable) epistemological consequences.[20]

We often hear another line of response to the Preface that is similar to (but somewhat less ambitious than) Pollock's "collective defeat" approach. This line of response claims that there is something "heterogenous" about the evidence in the Preface Paradox, and that this "evidential heterogeneity" somehow undermines the claim that one should believe all of the propositions that comprise the Preface Paradox. The idea seems to be[21] that the evidence one has for the first-order beliefs (in **B**) is a (radically) different kind of evidence than the evidence one has for the second-order belief (*i.e.*, the belief that renders **B** inconsistent in the end). And, because these bodies of first-order and second-order evidence are so heterogenous, there is no single body of evidence that supports both the first-order beliefs and the second-order belief in the Preface case. So, believing all the propositions of the Preface is not, in fact, the epistemically rational thing to do.[22] Hence, the apparent tension between (EB) and (CB) is merely apparent.

We think this line of response is unsuccessful, for three reasons. First, can't we just gather up the first-order and second-order evidential propositions, and put them all into one big collection of total Preface evidence? And, if we do so, why

[18]See (Steinberger 2013) for a incisive analysis of the consequences of the preface paradox for various principles of deductive reasoning [*i.e.*, "bridge principles" in the sense of (MacFarlane 2004)].

[19]We don't have the space here to analyze Pollock's (rather byzantine) approach to the Preface Paradox. Fortunately, Christensen (2004) has already done a very good job of explaining why "suspension strategies" like Pollock's can not, ultimately, furnish compelling responses to the Preface.

[20] For instance, it seems to us that any such approach will have to imply that "supported by the total evidence" is (generally) closed under logical consequence (or competent deduction), even under complicated entailments with many premises. See (Korb 1992) for discussion regarding (this and other) unpalatable consequences of Pollockian "collective defeat".

[21]We've actually not been able to find this exact line of response to the Preface anywhere in print, but we have heard this kind of line defended in various discussions and Q&A's. The closest line of response we've seen in print is Leitgeb's (2013) approach, which appeals to the "heterogeneity" of the subject matter of the claims involved in the Preface. This doesn't exactly fall under our "evidential heterogeneity" rubric, but it is similar enough to be undermined by our Homogeneous Preface case.

[22]Presumably, then, the rational thing to do is suspend judgment on some of the Preface propositions. But, which ones? As in the case of Pollock's "suspension strategy", it remains unclear (to us) precisely which propositions fail to be supported by the evidence in the Preface Paradox (and why).

wouldn't the total Preface evidence support both the first-order beliefs and the second-order belief in the Preface case? Second, we only need one Preface case in which (EB) and (CB) do genuinely come into conflict in order to establish (†). And, it seems to us that there are "homogeneous" versions of the Preface which do not exhibit this (alleged) kind of "evidential heterogeneity". Here's one such example.

> **Homogeneous Preface Paradox**. John is an excellent empirical scientist. He has devoted his entire (long and esteemed) scientific career to gathering and assessing the evidence that is relevant to the following first-order, empirical hypothesis: (H) all scientific/empirical books of sufficient complexity contain at least one false claim. By the end of his career, John is ready to publish his masterpiece, which is an exhaustive, encyclopedic, 15-volume (scientific/empirical) book which aims to summarize (all) the evidence that contemporary empirical science takes to be relevant to H. John sits down to write the Preface to his masterpiece. Rather than reflecting on his own fallibility, John simply reflects on the contents of (the main text of) his book, which constitutes very strong inductive evidence in favor of H. On this basis, John (inductively) infers H. But, John also believes each of the individual claims asserted in the main text of the book. Thus, because John believes (indeed, knows) that his masterpiece instantiates the antecedent of H, the (total) set of John's (rational/justified) beliefs is inconsistent.

In our Homogeneous Preface, there seems to be no "evidential heterogeneity" available to undermine the evidential support of John's ultimate doxastic state. Moreover, there seems to be no "collective defeat" looming here either. John is simply being a good empirical scientist (and a good inductive non-skeptic) here, by (rationally) inferring H from the total, H-relevant inductive scientific/empirical evidence. It is true that it was John himself who gathered (and analyzed, *etc.*) all of this inductive evidence and included it in one hugely complex scientific/empirical book. But, we fail to see how this fact does anything to undermine the (ideal) epistemic rationality of John's (ultimate) doxastic state. So, we conclude that the "heterogeneity strategy" is not an adequate response to the Preface.[23] More generally, we think our Homogeneous Preface case undermines any strategy that maintains one should never believe all the propositions in any Preface.[24]

We maintain that (adequate) responses to the Preface Paradox need not require suspension of judgment on (any of) the Preface propositions. Consequently, we

[23]We said we rejected the "heterogenous evidence" line of response to the Preface for three reasons. Our third reason is similar to the final worry we expressed above regarding Pollock's "collective defeat" strategy. We don't see how a "heterogeneity strategy" could serve to establish ¬(†) in full generality, without presupposing something very implausible about the general nature of evidential support, *e.g.*, that evidential support is preserved by competent deduction (see *fn.* 20).

[24]This includes Kaplan's (2013) line on the Preface, which appeals to the norms of "what we are willing to say in the context of inquiry". According to Kaplan, "what we are willing to say in the context of inquiry" is governed by a requirement of deductive cogency, which is stronger than (CB). Cogency implies (CB) plus closure (under competent deduction). John (the protagonist of our Homogeneous Preface Paradox) does not seem to us to be violating any norms of "what we are willing to say in the context of inquiry". It seems to us that nothing prevents John from being a perfectly rational scientific inquirer — even if he "says" everything we ascribe to him in the Homogeneous Preface.

would like to see a (principled) response to the Preface Paradox (and other paradoxes of consistency) that allows for (full) opination with respect to the propositions in the Preface agenda. Indeed, we will provide just such a response (to all paradoxes of consistency) below.

Before presenting our framework (and response), we will compare and contrast our own view regarding the Preface Paradox (and other paradoxes of consistency) with the views recently expressed by a pair of philosophers who share our commitment to (†) — *i.e.*, to the claim that Preface cases (and other similar cases) show that deductive consistency is not a necessary requirement of ideal epistemic rationality.

4. Christensen and Kolodny on Coherence Requirements

We are not alone in our view that Prefaces (and other paradoxes of deductive consistency) suffice to establish (†).[25] For instance, David Christensen and Niko Kolodny agree with us about Prefaces and (†). But, Christensen and Kolodny react to the paradoxes of deductive consistency in a more radical way. They endorse

($\star$) There are no coherence requirements (in the relevant sense) for full belief.

That is to say, both Christensen and Kolodny endorse eliminativism regarding all (formal, synchronic, epistemic) coherence requirements for full belief. It is illuminating to compare and contrast the views of Christensen and Kolodny with our own views about paradoxes of consistency and proper responses to them.

Christensen (2004) accepts the following package of pertinent views.[26]

(C_1) Partial beliefs (*viz.*, credences) are subject to a formal, synchronic, epistemic coherence requirement (of ideal rationality): probabilism.

($\star$) Full beliefs are not subject to any formal, synchronic, epistemic coherence requirements (of ideal rationality).

(C_2) Epistemic phenomena that appear to be adequately explainable only by appeal to coherence requirements for full belief (and facts about an agent's full beliefs) can be adequately explained entirely by appeal to probabilism (and facts about an agent's credences).

We agree with Christensen about (C_1). In fact, our framework for grounding coherence requirements for full belief is inspired by analogous arguments for probabilism as a coherence requirement for partial belief. We will return to this important parallel below. Christensen's (C_2) is part of an error theory regarding epistemological explanations that appear to involve coherence requirements for full belief as (essential) explanans. Some such error theory is needed — given ($\star$) — since epistemologists often seem to make essential use of such coherence-explanans.

[25] Other authors besides Christensen (2004), Kolodny (2007), Foley (1992) and Klein (1985) have claimed that paradoxes of consistency place pressure on the claim that (EB) entails (CB). For instance, Kyburg (1970) maintains that the lottery paradox supports (†). We are focusing on preface cases here, since we think they are, ultimately, more compelling than lottery cases (see *fn.* 38).

[26] Strictly speaking, Christensen (2004) never explicitly endorses ($\star$) or (C_2) in their full generality. He focuses on deductive consistency as a coherence-explanans, and he argues that it can be "eliminated" from such explanations, in favor of appeals only to probabilism (and facts about the agents credences). So, our ($\star$) and (C_2) may be stronger than the principles Christensen actually accepts. In recent personal communication, Christensen has voiced some sympathy with the (existence and explanatory power of) the coherence requirements for full belief developed here. Having said that, our "straw man Christensen" allows for a more perspicuous contrast in the present context.

Kolodny (2007), on the other hand, accepts the following pair:

(K_1) No attitudes (full belief, partial belief, or otherwise) are subject to any formal, synchronic, epistemic coherence requirements (of ideal rationality).

(K_2) Epistemic phenomena that appear to be adequately explainable only by appeal to coherence requirements for full belief (together with facts about an agent's full beliefs) can be adequately explained entirely by appeal to the Evidential Norm (EB), together with facts about an agent's full beliefs.

Kolodny's (K_1) is far more radical than anything Christensen accepts. Of course, (K_1) entails ($\star$), but it also entails universal eliminativism about coherence requirements in epistemology. Kolodny doesn't think there are any (ineliminable) coherence requirements (or any ineliminable requirements of ideal rationality, for that matter), period. He doesn't even recognize probabilism as a coherence requirement for credences. As a result, Kolodny needs a different error theory to "explain away" the various epistemological explanations that seem to appeal essentially to coherence requirements for full belief. His error theory [(K_2)] uses the Evidential Norm for full belief (EB), along with facts about the agent's full beliefs, to explain away such appeals to "coherence requirements". So, Kolodny's error theory differs from Christensen's in a crucial respect: Kolodny appeals to local/narrow-scope norms for full belief to explain away apparent uses of coherence requirements for full belief; whereas, Christensen appeals to global/wide-scope requirements of partial belief to explain away apparent uses of coherence requirements for full belief. This is (partly) because Kolodny is committed to the following general claim:

(K_3) Full beliefs are an essential (and ineliminable) part of epistemology (*i.e.*, the full belief concept is ineliminable from some epistemological explanations).

We agree with Kolodny about (K_3). We, too, think that full belief is a crucial (and ineliminable) epistemological concept. (Indeed, this is one of the reasons we are offering a new framework for grounding coherence requirements for full belief!) Christensen, on the other hand (at least on our reading, see *fn.* 26), seems to be unsympathetic to (K_3).

One last epistemological principle will be useful for the purposes of comparing and contrasting our views with the views of Christensen and Kolodny.[27]

(‡) If there are any coherence requirements for full belief, then deductive consistency [(CB)] is one of them. [*i.e.*, If $\neg(\star)$, then (CB).]

Christensen and Kolodny both accept (‡), albeit in a trivial way. They both reject the antecedent of (‡) [*i.e.*, they both accept ($\star$)]. We, on the other hand, aim to provide a principled way of rejecting (‡). That is to say, we aim to ground new coherence requirements for full belief, which are distinct from deductive consistency. We think this is the proper response to the paradoxes of consistency [and (†)].

In the next section, we present our formal framework for grounding coherence requirements for (opinionated) full belief. But, first, we propose a desideratum for such coherence requirements, inspired by the considerations adduced so far.

($\mathcal{D}$) Coherence requirements for (opinionated) full belief should never come into conflict with either alethic or evidential norms for (opinionated) full

[27] The "If ..., then ..." in (‡) is a material conditional. That is, (‡) asserts: either ($\star$) or (CB).

belief. That is, coherence requirements for (opinionated) full belief should be entailed by both the Truth Norm (TB) and the Evidential Norm (EB).

In light of (†), deductive consistency [(CB)] violates desideratum ($\mathcal{D}$). If a coherence requirement satisfies desideratum ($\mathcal{D}$), we will say that it is conflict-proof. Next, we explain how to ground conflict-proof coherence requirements for (opinionated) full belief.

5. Our (Naïve) Framework and (Some of) its Coherence Requirements

As it happens, our preferred alternative(s) to (CB) were not initially motivated by thinking about paradoxes of consistency. They were inspired by some recent arguments for probabilism as a (synchronic, epistemic) coherence requirement for credences. James Joyce (1998; 2009) has offered arguments for probabilism that are rooted in considerations of accuracy (*i.e.*, in alethic considerations). We won't get into the details of Joyce's arguments here.[28] Instead, we present a general framework for grounding coherence requirements for sets of judgments of various types, including both credences and full beliefs. Our unified framework constitutes a generalization of Joyce's argument for probabilism. Moreover, when our approach is applied to full belief, it yields coherence requirements that are superior to (CB), in light of preface cases (and other similar paradoxes of consistency).

Applying our framework to judgment sets $\mathbf{J}$ of type $\mathfrak{J}$ only requires completing three steps. The three steps are as follows:

> **Step 1**. Say what it means for a set $\mathbf{J}$ of type $\mathfrak{J}$ to be perfectly accurate (at a possible world w). We use the term "vindicated" to describe the perfectly accurate set of judgments of type $\mathfrak{J}$, at w, and we use $\mathbf{J}_w$ to denote this vindicated set.[29]
>
> **Step 2**. Define a measure of distance between judgment sets, $d(\mathbf{J},\mathbf{J}')$. We use d to gauge a set $\mathbf{J}$'s distance from vindication at w [*viz.*, $d(\mathbf{J},\mathbf{J}_w)$].
>
> **Step 3**. Adopt a fundamental epistemic principle, which uses $d(\mathbf{J},\mathbf{J}_w)$ to ground a (synchronic, epistemic) coherence requirement for judgment sets $\mathbf{J}$ of type $\mathfrak{J}$.

This is all very abstract. To make things more concrete, let's look at the simplest application of our framework — to the case of (opinionated) full belief. Let:

$$B(p) \overset{\text{def}}{=} S \text{ believes that } p$$

$$D(p) \overset{\text{def}}{=} S \text{ disbelieves that } p.$$

Our agents will be forming (opinionated) judgments on some salient agenda $\mathcal{A}$, which is a (possibly proper) subset of some finite boolean algebra of propositions. That is, for each $p \in \mathcal{A}$, S either believes p or S disbelieves p, and not both.[30] In this way, an agent can be represented by her "belief set" $\mathbf{B}$, which is just the set of her beliefs (B) and disbeliefs (D) over some salient agenda $\mathcal{A}$. Similarly, we think of propositions as sets of (classical) possible worlds, so that a proposition is true at any world that it contains, and false at any world it doesn't contain.[31] With our (naïve) setup in place, we're ready for the three steps.

Step 1 is straightforward. It is clear what it means for a set $\mathbf{B}$ of this type to be perfectly accurate/vindicated at a world w. The vindicated set $\mathbf{B}_w$ is given by:

$$\mathbf{B}_w \text{ contains } B(p)\ [D(p)] \text{ just in case } p \text{ is true [false] at } w.$$

This is clearly the best explication of $\mathbf{B}_w$, since $B(p)$ $[D(p)]$ is accurate just in case p is true [false]. Given the accuracy conditions for B/D, Step 1 is uncontroversial.

Step 2 is less straightforward, because there are a great many ways one could measure "distance between opinionated sets of beliefs/disbeliefs". For simplicity, we adopt perhaps the most naïve distance measure, which is given by:

$$d(\mathbf{B},\mathbf{B}') \overset{\text{def}}{=} \text{the number of judgments on which } \mathbf{B} \text{ and } \mathbf{B}' \text{ disagree.}^{32}$$

In particular, if you want to know how far your judgment set $\mathbf{B}$ is from vindication at w [*i.e.*, if you want to know the value of $d(\mathbf{B},\mathbf{B}_w)$] just count the number of mistakes you have made at w. To be sure, this is a very naïve measure of distance from vindication. As it turns out, however, we (ultimately) won't need to rely on such a strong (or naïve) assumption about $d(\mathbf{B},\mathbf{B}_w)$. In the end, we'll only need a much weaker assumption about $d(\mathbf{B},\mathbf{B}_w)$. But, for now, let's run with our naïve, "counting of mistakes at w" definition of $d(\mathbf{B},\mathbf{B}_w)$. We'll return to this issue later.

Step 3 is the philosophically most important step. Before we get to our favored fundamental epistemic principle(s), we will digress briefly to discuss a stronger fundamental epistemic principle that one might find (*prima facie*) plausible. Given our naïve setup, it turns out that there is *a* choice of fundamental epistemic principle that yields deductive consistency [(CB)] as a coherence requirement for opinionated full belief. Specifically, consider the following principle:

[28] There are some important disanalogies between Joyce's argument for probabilism and our analogous arguments regarding coherence requirements for full belief. Happily, the worry (articulated in Easwaran & Fitelson 2012) that Joyce's argument for probabilism may violate the credal analogue of ($\mathcal{D}$) does not apply to our present arguments (see *fn.* 42).

[29] As a heuristic, you can think of $\mathbf{J}_w$ as the set of judgments of type $\mathfrak{J}$ that an omniscient agent (*i.e.*, an agent who is omniscient about the facts at world w) would have.

[30] Our assumption of opinionation, relative to a salient agenda $\mathcal{A}$, results in no significant loss of generality for present purposes. As we have explained above, we do not think suspension of belief (on the Preface agenda — there are many propositions outside this agenda on which it may be reasonable to suspend) is an evidentially plausible way of responding to the Preface Paradox. Consequently, one of our present aims is to provide a response to paradoxes of consistency that allows for full opinionation (on the salient agendas). Moreover, there are other applications of the present framework for which opinionation is required. Briggs et al. (2014) show how to apply the present framework to the paradoxes of judgment aggregation, which presuppose opinionation on the salient agendas. Finally, we want to present the simplest and clearest version of our framework here. The naïve framework we present here can be generalized in various ways. Specifically, generalizing the present framework to allow for suspension of judgment (on the salient agendas) is, of course, desirable (Sturgeon 2008; Friedman 2013). See (Easwaran 2013) for a generalization of the present framework which allows for suspension of judgment on the salient agendas (see *fn.* 39). And, see (Fitelson 2014) for several other interesting generalizations of the present framework.

[31] It is implicit in our (highly idealized) framework that agents satisfy a weak sort of logical omniscience, in the sense that if two propositions are logically equivalent, then they may be treated as the same proposition in all models of the present framework. As such, we're assuming that agents cannot have distinct attitudes toward logically equivalent (classical, possible-worlds) propositions. We have already explained why such idealizations are okay in the present context (see *fn.* 9). However, it is important to note that we are not assuming agents satisfy a stronger sort of "omniscience" — an agent may believe some propositions while disbelieving some other proposition entailed by them (*i.e.*, our logical omniscience presupposition does not imply closure for rational belief). In other words, our agents are aware of all logical relations, but their judgment sets may not be closed under them.

[32] This is called the Hamming distance between the binary vectors $\mathbf{B}$ and $\mathbf{B}'$ (Deza and Deza 2009).

Possible Vindication (PV). There exists some possible world w at which all of the judgments in $\mathbf{B}$ are accurate. Or, to put this more formally, in terms of our distance measure d: $(\exists w)[d(\mathbf{B}, \mathbf{B}_w) = 0]$.

Given our naïve setup, it is easy to show that (PV) is equivalent to (CB).[33] As such, a defender of (CB) would presumably find (PV) attractive as a fundamental epistemic principle. However, as we have seen in previous sections, preface cases (and other paradoxes of consistency) have led many philosophers (including us) to reject (CB) as a rational requirement. This motivates the adoption of fundamental principles that are weaker than (PV). Interestingly, as we mentioned above, our rejection of (PV) was not (initially) motivated by Prefaces and the like. Rather, our adoption of fundamental principles weaker than (PV) was motivated (initially) by analogy with Joyce's argument(s) for probabilism as a coherence requirement for credences.

In the case of credences, the analogue of (PV) is clearly too strong. The vindicated set of credences (*i.e.*, the credences an omniscient agent would have) are such that they assign maximal confidence to all truths and minimal confidence to all falsehoods (Joyce, 1998). As a result, in the credal case, (PV) would require that all of one's credences be extremal. One doesn't need Preface cases (or any other subtle or paradoxical cases) to see that this would be an unreasonably strong (rational) requirement. It is for this reason that Joyce (and all others who argue in this way for probabilism) back away from the analogue of (PV) to strictly weaker epistemic principles — specifically, to accuracy-dominance avoidance principles, which are credal analogues of the following fundamental epistemic principle.

Weak Accuracy-Dominance Avoidance (WADA). $\mathbf{B}$ is not weakly[34] dominated in distance from vindication. Or, to put this more formally (in terms of d), there does not exist an alternative belief set $\mathbf{B}'$ such that:

(i) $(\forall w)[d(\mathbf{B}', \mathbf{B}_w) \leq d(\mathbf{B}, \mathbf{B}_w)]$, and

(ii) $(\exists w)[d(\mathbf{B}', \mathbf{B}_w) < d(\mathbf{B}, \mathbf{B}_w)]$.

(WADA) is a very natural principle to adopt, if one is not going to insist that — as a requirement of rationality — it must be possible for an agent to achieve perfect accuracy in her doxastic state. In the credal case, the analogous requirement was clearly too strong to count as a rational requirement. In the case of full belief, one needs to think about Preface cases (and the like) to see why (PV) is too strong. Retreating from (PV) to (WADA) is analogous to what one does in decision theory, when one backs off a principle of maximizing (actual) utility to some less demanding requirement of rationality (*e.g.,* dominance avoidance, maximization

[33]Here, we're assuming a slight generalization of the standard notion of consistency. Standardly, consistency applies only to beliefs (not disbeliefs), and it requires that there be a possible world in which all the agent's beliefs are true. More generally, we may define consistency as the existence of a possible world in which all the agent's judgments (both beliefs and disbeliefs) are accurate. Given this more general notion of consistency, (PV) and (CB) are equivalent in the present framework.

[34]Strictly speaking, Joyce *et al.* opt for the apparently weaker principle of avoiding strict dominance. However, in the credal case (assuming continuous, strictly proper scoring rules), there is no difference between weak and strict dominance (Schervish *et al.* 2009). In this sense, there is no serious disanalogy. Having said that, it is worth noting that, in the case of full belief, there is a significant difference between weak dominance and strict dominance. This difference will be discussed in some detail in §6 below. In the meantime, whenever we say "dominated" what we mean is weakly dominated in the sense of (WADA).

of expected utility, minimax, *etc.*).[35] Of course, there is a sense in which "the best action" is the one that maximizes actual utility; but, surely, maximization of actual utility is not a rational requirement. Similarly, there is clearly a sense in which "the best doxastic state" is the perfectly accurate [(TB)], or possibly perfectly accurate [(CB)/(PV)], doxastic state. But, in light of the paradoxes of consistency, (TB) and (CB) turn out not to be rational requirements either. One of the main problems with the existing literature on the paradoxes of consistency is that no principled alternative(s) to deductive consistency have been offered as coherence requirements for full belief. Such alternatives are just what our Joyce-style arguments provide.

If a belief set $\mathbf{B}$ satisfies (WADA), then we say $\mathbf{B}$ is non-dominated. This leads to the following, new coherence requirement for (opinionated) full belief:

(NDB) All (opinionated) agents S should (at any given time t) have sets of full beliefs (and disbeliefs) that are non-dominated.

Interestingly, (NDB) is strictly weaker than (CB). Moreover, (NDB) is weaker than (CB) in an appropriate way, in light of our Preface Paradoxes (and other similar paradoxes of consistency). Our first two theorems (each with an accompanying definition) help to explain why.

The first theorem states a necessary and sufficient condition for (*i.e.*, a characterization of) non-dominance: we call it *Negative* because it identifies certain objects, the non-existence of which is necessary and sufficient for non-dominance. The second theorem states a sufficient condition for non-dominance: we call it *Positive* because it states that in order to show that a certain belief set $\mathbf{B}$ is non-dominated, it's enough to construct a certain type of object.

Definition 1 (Witnessing Sets). $\mathbf{S}$ *is a* ***witnessing set*** *iff (a) at every world* w*, at least half of the judgments*[36] *in* $\mathbf{S}$ *are inaccurate; and, (b) at some world more than half of the judgments in* $\mathbf{S}$ *are inaccurate.*

Theorem 1 (Negative). $\mathbf{B}$ *is non-dominated iff* $\mathbf{B}$ *contains no witnessing set.*
[*We will use "(NWS)" to abbreviate the claim that "no subset of* $\mathbf{B}$ *is a witnessing set." Thus, Theorem 1 can be stated equivalently as:* $\mathbf{B}$ *is non-dominated iff (NWS).*]

It is an immediate corollary of this first theorem that if $\mathbf{B}$ is deductively consistent [*i.e*, if $\mathbf{B}$ satisfies (PV)], then $\mathbf{B}$ is non-dominated. After all, if $\mathbf{B}$ is deductively consistent, then there is a world w such that no judgments in $\mathbf{B}$ are inaccurate at w (*fn.* 33). However, while deductive consistency guarantees non-dominance, the converse is not the case, *i.e.*, non-dominance does not ensure deductive consistency. This will be most perspicuous as a consequence of our second theorem.

[35]The analogy to decision theory could be made even tighter. We could say that being accuracy-dominated reveals that you are in a position to recognize *a priori* that another option is guaranteed to do better at achieving the "epistemic aim" of getting as close to the truth as possible. This decision-theoretic stance dovetails nicely with the sentiments expressed by Foley (*op. cit.*). See §8 for further discussion of (and elaboration on) this epistemic decision-theoretic stance.

[36]Here, we rely on naïve counting. This is unproblematic, since all of our agendas are finite. The coherence norm we'll propose in the end (see §7) will not be based on counting and (as a result) will be applicable to both finite and infinite agendas. All Theorems are proved in the Appendix.

Definition 2. *A probability function* Pr **represents** *a belief set* **B** *iff for every* $p \in \mathcal{A}$

(i) **B** *contains* $B(p)$ *iff* $\Pr(p) > 1/2$*, and*

(ii) **B** *contains* $D(p)$ *iff* $\Pr(p) < 1/2$.

Theorem 2 (Positive)**.** **B** *is non-dominated if there exists a probability function* Pr *that represents* **B**.[37]

To appreciate the significance of Theorem 2, it helps to think about a standard lottery case.[38] Consider a fair lottery with $n \geq 3$ tickets, exactly one of which is the winner. For each $j \leqslant n$, let p_j be the proposition that the j^{th} ticket is not the winning ticket; let q be the proposition that some ticket is the winner; and, let these $n + 1$ propositions exhaust the agenda $\mathcal{A}$. (Note that the agenda leaves out conjunctions of these propositions.) Finally, let Lottery be the following opinionated belief set on $\mathcal{A}$:

$$\left\{B(p_j) \mid 1 \leqslant j \leqslant n\right\} \cup \{B(q)\}.$$

In light of Theorem 2, Lottery is non-dominated. The probability function that assigns each ticket equal probability of winning represents Lottery. However, Lottery is not deductively consistent. Hence, (NDB) is strictly weaker than (CB).[39]

Not only is (NDB) weaker than (CB), it is weaker than (CB) in a desirable way. More precisely, in accordance with desideratum ($\mathcal{D}$), we will now demonstrate that (NDB) is entailed by both alethic considerations [(TB)/(CB)] and evidential considerations [(EB)]. While there is considerable disagreement about the precise content of the Evidential Norm for full belief (EB), there is widespread agreement (at least, among evidentialists) that the following is a necessary condition for satisfying (EB).

> **Necessary Condition for Satisfying (EB).** **B** satisfies (EB), *i.e.*, all judgments in **B** are supported by the total evidence, ***only if***:
>
> ($\mathcal{R}$) There exists some probability function that probabilifies (*i.e.*, assigns probability greater than 1/2 to) each belief in **B** and dis-probabilifies (*i.e.*, assigns probability less than 1/2 to) each disbelief in **B**.

Most evidentialists agree that probabilification — relative to some probability function — is a minimal necessary condition for justification. Admittedly, there is plenty of disagreement about which probability function is implicated in ($\mathcal{R}$).[40]

[37] The question: "Does a belief set **B** have a representing probability function?" is decidable (Fitelson 2008). So is the question "Does a belief set **B** have a witnessing set?". This suffices to ensure that coherence properties (in our sense) of (finite) belief sets are formal in the intended sense of §1.

[38] We are not endorsing the belief set Lottery in this example as epistemically rational. Indeed, we think that the lottery paradox is not as compelling — as a counterexample to (EB) ⇒ (CB) — as the preface paradox is. On this score, we agree with Pollock (1990) and Nelkin (2000). We are just using this lottery example to make a formal point about the logical relationship between (CB) and (NDB).

[39] It is worth noting that belief sets like Lottery can remain non-dominated even if we allow alternative judgment sets to suspend judgment on some or all of the propositions in the salient agenda. This is yet another reason why our restriction to opinionated judgment sets (over salient agendas) results in no real loss of generality (see *fn.* 30). See (Easwaran, 2013) for further discussion.

[40] Internalists like Fumerton (1995) require that the function $\Pr(\cdot)$ which undergirds (EB) should be "internally accessible" to the agent (in various ways). Externalists like Williamson (2000) allow for "inaccessible" evidential probabilities. And, subjective Bayesians like Joyce (2005) say that $\Pr(\cdot)$ should reflect the agent's subjective degrees of belief (*viz.*, credences). Despite this disagreement, most evidentialists agree that (EB) entails ($\mathcal{R}$), which is all we need for present purposes.

But, because our Theorem 2 only requires the existence of *some* probability function that probabilifies S's beliefs and dis-probabilifies S's disbeliefs, it is sufficient to ensure (on most evidentialist views) that (EB) entails ($\mathcal{R}$). Assuming we're correct in our assessment that Prefaces (and other similar paradoxes of consistency) imply (†), this is precisely the entailment that fails for (CB), and the reason why (CB) fails to satisfy desideratum ($\mathcal{D}$) [*i.e.*, why (CB) fails to be conflict-proof], while (NDB) does satisfy it.[41] Thus, by grounding coherence for full beliefs in the same way Joyce grounds probabilism for credences, we are naturally led to coherence requirements for (opinionated) full belief that are plausible alternatives to (CB).[42] This gives us a principled way to accept (†) while rejecting (‡), and it paves the way for a novel and compelling response to the Preface (and other similar paradoxes of consistency). Figure 1 depicts the logical relations between the epistemic requirements and norms we have discussed so far.

$$\begin{tikzcd}
\text{(TB)} \arrow[d, Rightarrow] & & \text{(EB)} \arrow[ddl, Rightarrow] \\
\text{(CB)/(PV)} \arrow[dr, Rightarrow] & & \\
& (\mathcal{R}) \arrow[d, Rightarrow] & \\
\text{(NWS)} \arrow[r, Leftrightarrow] & \text{(NDB)/(WADA)} &
\end{tikzcd}$$

Figure 1. Logical relations between requirements and norms (so far)

In the next two sections, we will elaborate on the family of coherence requirements generated by our framework.

6. A Family of New Coherence Requirements for Full Belief

The analysis above revealed two coherence requirements that are strictly weaker than deductive consistency: ($\mathcal{R}$) and (NDB). There is, in fact, a large family of such requirements. This family includes requirements that are even weaker than (NDB), as well as requirements that are stronger than ($\mathcal{R}$). Regarding the former, the most interesting requirement that is weaker than (NDB) is generated by replacing weak-accuracy-dominance avoidance with strict-accuracy-dominance avoidance, *i.e.*, by adopting (SADA), rather than (WADA), as the fundamental epistemic principle.

[41] Another way to see why there can't be preface-style counterexamples to (NDB) is to recognize that such cases would have to involve not only the (reasonable) belief that *some* of one's beliefs are false, but the much stronger (and unreasonable/irrational) belief that *most* of one's beliefs are false.

[42] We have given a general, theoretical argument to the effect that the Evidential Norm for full belief [(EB)] entails our coherence requirement(s) for full belief. We know of no analogous general argument for credences. In (Easwaran & Fitelson 2012), we raised the possibility of counterexamples to the analogous theoretical claim: ($\mathcal{E}$) the evidential norm for credences (independently) entails probabilism. Joyce (2013) and Pettigrew (2013a) take steps toward general arguments for ($\mathcal{E}$).

Strict Accuracy-Dominance Avoidance (SADA). **B** is not strictly dominated in distance from vindication. Or, to put this more formally (in terms of d), there does not exist an alternative belief set $\mathbf{B}'$ such that:

$$(\forall w)[d(\mathbf{B}', \mathbf{B}_w) < d(\mathbf{B}, \mathbf{B}_w)].$$

It is obvious that (WADA) entails (SADA). That the converse entailment does not hold can be shown by producing an example of a doxastic state that is weakly, but not strictly, dominated in d-distance from vindication. We present such an example in the APPENDIX. What about requirements "in between" (CB) and ($\mathcal{R}$)?

One way to bring out requirements that are weaker than (CB) but stronger than ($\mathcal{R}$) is to think of (CB) and ($\mathcal{R}$) as "limiting cases" of the following parametric family.

Parametric Family of Probabilistic Requirements Between ($\mathcal{R}$) and (CB)

($\mathcal{R}_r$) There exists a probability function Pr such that, for every $p \in \mathcal{A}$:

(i) **B** contains $B(p)$ iff $\Pr(p) > r$, and

(ii) **B** contains $D(p)$ iff $\Pr(p) < 1 - r$,

where $r \in [1/2, 1)$.

What we have been calling ($\mathcal{R}$) is (obviously) equivalent to member ($\mathcal{R}_{1/2}$) of the above family. And, as the value of r approaches 1, the corresponding requirement ($\mathcal{R}_r$) approaches (CB) in logical strength. This gives rise to a continuum of coherence requirements that are "in between" (CB) and ($\mathcal{R}$) in terms of their logical strength. (CB) is equivalent to the following, extremal probabilistic requirement.

Extremal Probabilistic Equivalent to (CB)

(CB_{Pr}) There exists a probability function Pr such that, for every $p \in \mathcal{A}$:

(i) **B** contains $B(p)$ iff $\Pr(p) = 1$, and

(ii) **B** contains $D(p)$ iff $\Pr(p) = 0$.

To see that (CB_{Pr}) is equivalent to (CB), note that a belief set **B** is consistent (*i.e.*, possibly perfectly accurate) just in case there is a truth-value assignment function that assigns $\top$ to all p such that $B(p) \in \mathbf{B}$ and $\bot$ to all p such that $D(p) \in \mathbf{B}$. But, this is equivalent to the existence of an indicator function that assigns 1 to all the believed propositions in **B** and 0 to all the disbelieved propositions in **B**. And, such indicator functions just are probability functions of the sort required by (CB_{Pr}).

In the next section, we'll look more closely at our family of new coherence requirements, with an eye toward narrowing the field.

7. A Closer Look at Our Family of New Coherence Requirements

First, we note that there is a clear sense in which (NDB) seems to be too weak. (NDB) doesn't even rule out belief sets that contain contradictory pairs of beliefs. For instance, the belief set $\{B(P), B(\neg P)\}$ on the simple agenda $\{P, \neg P\}$ is not weakly dominated in d-distance from vindication. This can be seen in TABLE 1. In TABLE 1, a "+" denotes an accurate judgment (at a world) and a "−" denotes an inaccurate judgment (at a world). As you can see, the belief set $\mathbf{B} \overset{\text{def}}{=} \{B(P), B(\neg P)\}$ contains one accurate judgment and one inaccurate judgment in each of the two salient possible worlds, *i.e.*, $d(\mathbf{B}, \mathbf{B}_{w_1}) = 1$ and $d(\mathbf{B}, \mathbf{B}_{w_2}) = 1$. None of the other

			$\mathbf{B}$		$\mathbf{B}'$		$\mathbf{B}''$		$\mathbf{B}'''$	
	P	$\neg P$	$B(P)$	$B(\neg P)$	$B(P)$	$D(\neg P)$	$D(P)$	$B(\neg P)$	$D(P)$	$D(\neg P)$
w_1	**F**	**T**	−	+	−	−	+	+	+	−
w_2	**T**	**F**	+	−	+	+	−	−	−	+

TABLE 1. Why (NDB) seems too weak

three possible (opinionated) belief sets on $\mathcal{A}$ weakly accuracy-dominates **B**. Specifically, let $\mathbf{B}' \overset{\text{def}}{=} \{B(P), D(\neg P)\}$, $\mathbf{B}'' \overset{\text{def}}{=} \{D(P), B(\neg P)\}$ and $\mathbf{B}''' \overset{\text{def}}{=} \{D(P), D(\neg P)\}$. Then:

$$1 = d(\mathbf{B}, \mathbf{B}_{w_1}) < d(\mathbf{B}', \mathbf{B}_{w_1}) = 2,$$
$$1 = d(\mathbf{B}, \mathbf{B}_{w_2}) < d(\mathbf{B}'', \mathbf{B}_{w_2}) = 2,$$
$$1 = d(\mathbf{B}, \mathbf{B}_{w_1}) = d(\mathbf{B}''', \mathbf{B}_{w_1}), \text{ and}$$
$$1 = d(\mathbf{B}, \mathbf{B}_{w_2}) = d(\mathbf{B}''', \mathbf{B}_{w_2}).$$

Therefore, none of $\mathbf{B}'$, $\mathbf{B}''$ or $\mathbf{B}'''$ weakly accuracy-dominates **B**, which implies that **B** satisfies (NDB). But, intuitively, **B** should count as incoherent. After all, **B** violates ($\mathcal{R}$), which implies that **B** cannot be supported by the total evidence — whatever the total evidence is. This suggests that (NDB) is too weak to serve as "the" (strongest, universally binding) coherence requirement for (opinionated) full belief. Indeed, we think a similar argument could be given to show that no requirement that is (strictly) weaker than ($\mathcal{R}$) can be "the" coherence requirement for full belief. Dominance requirements like (NDB) have other shortcomings, besides being too weak.

Dominance avoidance conditions like (WADA) and (SADA) are defined in terms of the naïve "mistake-counting" measure of distance from vindication $d(\mathbf{B}, \mathbf{B}_w)$. Such simple counting measures work fine for finite belief sets, but there seems to be no clear way to apply such naïve distance measures to infinite belief sets. On the other hand, probabilistic requirements like ($\mathcal{R}$) can be applied (in a uniform way) to both finite and infinite belief sets.

There is another problem (NDB) inherits from (WADA)'s reliance on the naïve, mistake-counting measure of distance from vindication $d(\mathbf{B}, \mathbf{B}_w)$. This measure seems to require that each proposition in the agenda receive "equal weight" in the calculation of **B**'s distance from vindication. One might (for various reasons) want to be able to assign different "weights" to different propositions when calculating the overall distance from vindication of a doxastic state.[43] An examination of the proof of Theorem 2 (see the APPENDIX) reveals that if a belief set **B** satisfies ($\mathcal{R}$), then **B** will minimize expected distance from vindication, relative to its representing probability function Pr. The proof of this result requires only that the measure of distance from vindication be additive — *i.e.*, that each judgment in **B** receive an "inaccuracy score" and that these "inaccuracy scores" are added up across the members of **B**. In other words, if we adopt ($\mathcal{R}$) as our (ultimate) coherence requirement, then — so long as our measure of distance from vindication is additive — coherent belief sets will be guaranteed to be non-dominated. So, another advantage of adopting ($\mathcal{R}$) — as opposed to (NDB) — as "the" coherence requirement

[43]Joyce's (1998; 2009) argument(s) for probabilism allow(s) for different weights to be assigned to different propositions in the calculation of distance from vindication (of a credence function).

for full belief is that it allows us to use any additive distance measure we like, while preserving the (Joycean) connection between coherence and the avoidance of accuracy-dominance.

What about requirements stronger than ($\mathcal{R}$)? For instance, could some requirement ($\mathcal{R}_r$) — with $r > 1/2$ — be a better candidate than ($\mathcal{R}$) for "the" coherence requirement for (opinionated) full belief? We suspect this will depend on the context in which a doxastic state is being evaluated. Recall that the key premise in our argument that ($\mathcal{R}$) satisfies desideratum ($\mathcal{D}$) [*i.e.*, that ($\mathcal{R}$) is conflict-proof] was the assumption that the Evidential Norm (EB) entails ($\mathcal{R}$). It is uncontroversial (among probabilist-evidentialists) that this entailment holds in all contexts. What happens to this entailment when we replace ($\mathcal{R}$) with ($\mathcal{R}_r$) and $r > 1/2$? When $r > 1/2$, it will no longer be uncontroversial — even among probabilist-evidentialists — that (EB) entails ($\mathcal{R}_r$) in all contexts. For instance, consider a Lockean, who thinks that one ought to believe P just in case $\Pr(P) > r$, where the value of the threshold r (and perhaps the evidential probability function Pr) depends on context. If $r > 1/2$, then such a probabilist-evidentialist will only accept the entailment (EB) $\Rightarrow$ ($\mathcal{R}_r$) in some contexts. As a result, when $r > 1/2$, the entailment (EB) $\Rightarrow$ ($\mathcal{R}_r$) will not hold uncontroversially, in a context-independent way. However, $r = 1/2$ seems too low to generate a strong enough coherence requirement in most (if not all) contexts, *i.e.*, the requirement ($\mathcal{R}$) = ($\mathcal{R}_{1/2}$) seems too weak in most (if not all) contexts.

To see this, consider the class of minimal inconsistent sets of propositions of size n: $\mathbb{B}_n$. That is, each member of $\mathbb{B}_n$ is an inconsistent set of size n containing no inconsistent proper subset. For instance, each member of $\mathbb{B}_2$ will consist of a contradictory pair of propositions. We've already seen that believing both elements of a member of $\mathbb{B}_2$ is ruled out as incoherent by ($\mathcal{R}$) = ($\mathcal{R}_{1/2}$), but not by (NDB). However, believing each element of a member of $\mathbb{B}_3$ (*e.g.*, believing each of the propositions in $\{P, Q, \neg(P \,\&\, Q)\}$) will *not* be ruled out by ($\mathcal{R}_{1/2}$). In order to rule out inconsistent belief sets of size three, we would need to raise the threshold r to $2/3$. In other words, ($\mathcal{R}_{2/3}$) is the weakest requirement in the ($\mathcal{R}_r$) family that rules out believing each member of a three-element minimal inconsistent set (*i.e.*, each member of some set in $\mathbb{B}_3$). In general, we have the following theorem.[44]

Theorem 3. *For all $n \geq 2$ and for each set of propositions $\mathbf{P} \in \mathbb{B}_n$, if $r \geq \frac{n-1}{n}$ then ($\mathcal{R}_r$) rules out believing every member of $\mathbf{P}$, while if $r < \frac{n-1}{n}$, then ($\mathcal{R}_r$) doesn't rule out believing every member of $\mathbf{P}$.*

In preface (or lottery) cases, n is typically quite large. As a result, in order to rule out such large inconsistent belief sets as incoherent, ($\mathcal{R}_r$) would require a large threshold $r = \frac{n-1}{n}$. For example, ruling out inconsistent belief sets of size 5 *via* ($\mathcal{R}_r$) requires a threshold of $r = 0.8$, and ruling out inconsistent belief sets of size 10 *via* ($\mathcal{R}_r$) requires a threshold of $r = 0.9$. We think this goes some way toward explaining why smaller inconsistent belief sets seem "less coherent" than

[44]Hawthorne & Bovens (1999) prove some very similar formal results; and Christensen (2004) and Sturgeon (2008) appeal to such formal results in their discussions of the paradoxes of consistency. However, all of these authors presuppose that the probabilities involved in their arguments are the credences of the agent in question. Our probability functions need not be credence functions (see *fn.* 40). Indeed, we need not even assume here that agents have degrees of belief. This is important for us, since we do not want to presuppose any sort of reduction (or elimination) of full belief to (or in favor of) partial belief (or any other quantitative concept). See (Fitelson 2014) for further discussion.

larger inconsistent belief sets.[45] Having said that, we don't think there is a precise "universal threshold" r such that ($\mathcal{R}_r$) is "the" (strongest universally binding) coherence requirement. There are precise values of r that yield clear-cut cases of universally binding coherence requirements ($\mathcal{R}_r$), *e.g.*, $r = 1/2$. And, there are precise values of r which yield coherence requirements ($\mathcal{R}_r$) that we take to be clearly not universally binding, *e.g.*, $r = 1 - \epsilon$, for some minuscule ϵ.[46] What, exactly, happens in between? We'll have to leave that question for a future investigation.[47]

In the next section, we'll elaborate briefly on an illuminating decision-theoretic analogy that we mentioned in passing (*e.g.*, *fn.* 35). Then, we'll tie up a few remaining theoretical loose ends. Finally, we'll consider a troublesome example for our framework, inspired by Caie's (2013) analogous recent counterexample to Joyce's argument for probabilism.

8. A Decision-Theoretic Analogy

If we think of closeness to vindication as a kind of epistemic utility (Pettigrew, 2013b; Greaves, 2013), then we may think of ($\mathcal{R}$) as an expected epistemic utility maximization principle. On this reading, ($\mathcal{R}$) is tantamount to the requirement that an agent's belief set should maximize expected epistemic utility, relative to some evidential probability function. Expected utility maximization principles are stronger (*i.e.*, less permissive) than dominance principles, which (again) explains why ($\mathcal{R}$) is stronger than (NDB).

We could push this decision-theoretic analogy even further. We could think of the decision-theoretic analogue of (TB) as a principle of *actually* maximizing utility (AMU), *i.e.*, choose an act that maximizes utility in the *actual* world. We could think of the decision-theoretic analogue of (CB) as a principle of *possibly* maximizing utility (PMU), *i.e.*, choose an act that maximizes utility in some *possible* world. And, as we have already discussed (see *fn.* 35), (NDB) would be analogous to a (weak) dominance principle in epistemic decision theory. The general correspondence

[45]More generally, it seems that the ($\mathcal{R}_r$) can do a lot of explanatory work. For instance, we mentioned above (*fn.* 7) that even Kolodny — a coherence eliminativist — should be able to benefit from our framework and analysis. We think Kolodny can achieve a more compelling explanatory error theory by taking, *e.g.*, ($\mathcal{R}$), rather than (CB), as his target. There is a much tighter conceptual connection between (EB) — which is Kolodny's central explanatory epistemic principle — and ($\mathcal{R}$). For this reason, we believe that a shift from (CB) to ($\mathcal{R}$) would make the explanatory aims of Kolodny's error theory easier to achieve. Finally, we suspect that debates about the existence of coherence requirements would become more interesting if we stopped arguing about whether (CB) is a coherence requirement and started arguing about whether ($\mathcal{R}$) or (NDB) [or the other ($\mathcal{R}_r$)] are coherence requirements.

[46]When we say that $r = 1 - \epsilon$, for some minuscule ϵ, leads to a coherence requirement ($\mathcal{R}_r$) that is not universally binding, we are not making a claim about any specific probability function (see *fn.* 44). For instance, we're not assuming a Lockean thesis with a "universal" threshold of $r = 1 - \epsilon$. It is important to remember that ($\mathcal{R}_r$) asserts only the existence of some probability function that assigns a value greater than r to all beliefs in $\mathbf{B}$ and less than $1 - r$ to all disbeliefs in $\mathbf{B}$. This is why ($\mathcal{R}_{1-\epsilon}$) is strictly logically weaker than (CB), and (therefore) no more controversial than (CB).

[47]Fully answering this question would require (among other things) a more substantive analysis of the relationship between the Evidential Norm (EB) and the requirements ($\mathcal{R}_r$), in cases where $r > 1/2$. That sort of analysis is beyond the scope of this paper, but will be taken up in (Fitelson 2014).

between the epistemic norms and requirements discussed above and the analogous decision-theoretic principles is summarized in TABLE 2.[48]

Epistemic Principle	Analogous Decision-Theoretic Principle
(TB)	(AMU) Do ϕ only if ϕ maximizes utility in the *actual* world.
(CB)	(PMU) Do ϕ only if ϕ maximizes u in *some possible* world.
($\mathcal{R}$)	(MEU) Do ϕ only if ϕ maximizes *expected* utility.[49]
(WADA)	(WDOM) Do ϕ only if ϕ is *not weakly dominated* in utility.
(SADA)	(SDOM) Do ϕ only if ϕ is *not strictly dominated* in utility.

TABLE 2. A Decision-Theoretic Analogy

Like (CB), the principle of "possible maximization of utility" (PMU) is not a requirement of rationality. And, as in the case of (CB), in order to see the failure of (PMU), one needs to consider "paradoxical" cases. See (Parfit 1988; Kolodny & MacFarlane 2010) for discussions of a "paradoxical" decision problem (the so-called "Miner Puzzle") in which the rational act is one that does not maximize utility in any possible world. From this decision-theoretic perspective, it is not surprising that deductive consistency (CB) turns out to be too demanding to be a universally binding rational requirement (Foley, 1992).[50]

9. TYING UP A FEW THEORETICAL LOOSE ENDS

As we have seen, the accuracy-dominance avoidance requirement (NDB) is equivalent to a purely combinatorial condition (NWS) defined in terms of witnessing sets. Similar purely combinatorial conditions exist for some of our other coherence requirements as well. Consider these variations on the concept of a witnessing set.

Definition 3 (Witnessing$_1$ Sets)**. S** *is a **witnessing$_1$ set** iff at every world w, more than half of the judgments in* **S** *are inaccurate.*

Definition 4 (Witnessing$_2$ Sets)**. S** *is a **witnessing$_2$ set** iff at every world w, at least half of the judgments in* **S** *are inaccurate.*

Corresponding to each of these types of witnessing sets is a requirement stating that no subset of **B** should be a witnessing set of that type. To wit:

[48]The double line between (CB) and ($\mathcal{R}$) in Table 2 is intended to separate rational requirements like ($\mathcal{R}$) from principles like (CB) that are too demanding to be (universally binding) rational requirements. We're not sure exactly how to draw this line, but we think that reflection on how analogous lines are drawn on the decision-theoretic side may shed light on this question (Fitelson, 2014).

[49]Strictly speaking, in order to make the analogy tight, we need to precisify (MEU), as follows:

(MEU) Do ϕ only if ϕ maximizes expected utility, relative to *some* probability function.

This is weaker than the standard interpretation of (MEU), which involves maximizing expected utility relative to a *specific* probability function (*viz.*, the agent's credence function in the decision situation).

[50]For a nice survey of some of the recent fruits of this (epistemic) decision-theoretic stance, see (Pettigrew 2013b). And, see (Greaves, 2013; Berker, 2013) for some meta-epistemological worries about taking this sort of epistemic decision-theoretic stance. We don't think the worries expressed by Greaves and Berker are ultimately problematic for our present approach, but we don't have the space here to properly allay them.

(NW$_1$S) No subset of **B** is a witnessing$_1$ set.

(NW$_2$S) No subset of **B** is a witnessing$_2$ set.

It turns out that (NW$_1$S) and (NW$_2$S) are intimately connected with (SADA) and ($\mathcal{R}$), respectively. These connections are established by the following two theorems:

Theorem 4. **B** *is non-strictly-dominated iff* **B** *contains no witnessing$_1$ set.* [*In other words, the following equivalence holds: (SADA) $\iff$ (NW$_1$S) .*]

Theorem 5. **B** *is probabilistically representable (in the sense of Definition 2) only if*[51] **B** *contains no witnessing$_2$ set* [*i.e., ($\mathcal{R}$) $\Longrightarrow$ (NW$_2$S)*].

This brings us to our final Theorem. Consider the following condition, which requires that there be no contradictory pairs of judgments in a belief set:

(NCP) **B** does not contain any contradictory pairs of judgments. That is, there is no proposition p such that either $\{B(p), B(\neg p)\} \subseteq \mathbf{B}$ or $\{D(p), D(\neg p)\} \subseteq \mathbf{B}$.

Theorem 6. **B** *is probabilistically representable (in the sense of Definition 2) only if*[52] **B** *satisfies both (NDB) and (NCP)* [*i.e., ($\mathcal{R}$) $\Longrightarrow$ (NDB & NCP)*].

That ties up the remaining theoretical loose ends. FIGURE 2 depicts the known (*fn.* 51) logical relationships between all the requirements and norms discussed above. In the next section, we discuss a worrisome example inspired by Caie's (2013) recent counterexample to Joyce's argument for probabilism.

10. A WORRISOME EXAMPLE FOR OUR FRAMEWORK

Michael Caie (2013) has recently given a problematic example for (perhaps even a counterexample to) Joyce's argument for probabilism. We don't have space here to fully analyze Caie's example. But, there is an obvious analogue of Caie's example in our framework for full belief. Consider the following (self-referential) claim:

(P) S does not believe that P.

That is, P says of itself that it is not believed by S. Consider the agenda $\mathcal{A} \stackrel{\text{def}}{=} \{P, \neg P\}$. There seems to be a sound argument for the (worrisome) claim that there are no coherent opinionated belief sets for S on $\mathcal{A}$. This can be seen *via* TABLE 3.

The "$\times$"s in TABLE 3 indicate that these entries in the table are ruled out as logically impossible (given the definition of P). As such it appears that **B** and **B**$'''$ strictly accuracy-dominate their (live) alternatives (*i.e.*, it appears that **B** strictly dominates **B**$'$ and **B**$'''$ strictly dominates **B**$''$). As a result, all of the consistent opinionated belief sets on $\mathcal{A}$ would seem to be ruled out by (SADA). As for **B** and **B**$'''$, these belief sets consist of contradictory pairs of propositions. We argued earlier that ($\mathcal{R}$) entails (SADA) and rules out any belief set containing contradictory pairs,

[51]It is an open question whether the converse of Theorem 5 is true [*i.e.*, it is an open question whether ($\mathcal{R}$) $\Longleftarrow$ (NW$_2$S)]. There are no small counterexamples to ($\mathcal{R}$) $\Longleftarrow$ (NW$_2$S). If this implication could be established, then it would show that the naïve counting measure of distance from vindication $d(\mathbf{B}, \mathbf{B}_w)$ is canonical (with respect to the additive measures of distance from vindication). That is, if (NW$_2$S) is equivalent to ($\mathcal{R}$), then a purely combinatorial (naïve, counting) condition is (by Theorem 2) sufficient to ensure non-dominance for any additive measure of distance from vindication.

[52]The converse is false [*i.e.*, ($\mathcal{R}$) $\not\Leftarrow$ (NDB & NCP)]. See the APPENDIX for a counterexample.

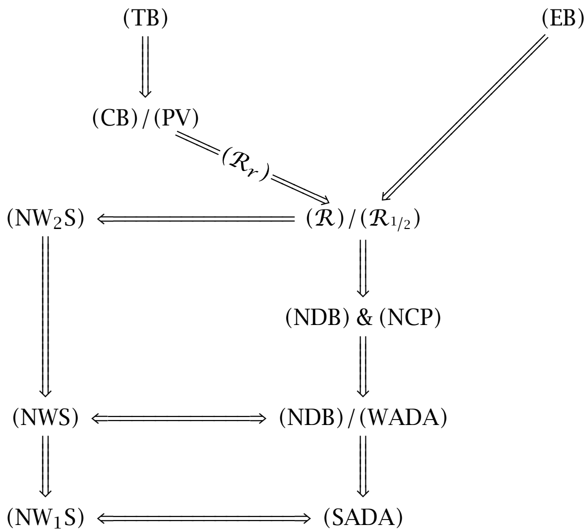

FIGURE 2. Logical relations between (all) requirements and norms

			B		**B**$'$		**B**$''$		**B**$'''$	
	P	$\neg P$	$B(P)$	$B(\neg P)$	$B(P)$	$D(\neg P)$	$D(P)$	$B(\neg P)$	$D(P)$	$D(\neg P)$
w_1	**F**	**T**	$-$	$+$	$-$	$-$	$\times$	$\times$	$\times$	$\times$
w_2	**T**	**F**	$\times$	$\times$	$\times$	$\times$	$-$	$-$	$-$	$+$

TABLE 3. Caie-style example in our framework

which seems to mean that ($\mathcal{R}$) rules out all (opinionated) belief sets on $\mathcal{A}$. However, one or both of these entailments might fail in the present case. Our earlier arguments assumed (as standard in probability theory) that none of the propositions exhibit dependence between doxastic states and the truth. Thinking about how to apply ($\mathcal{R}$) in this case requires reconsidering the relationship between a belief set and a probability function.

A probability function assigns numbers to various worlds. When evaluating a belief set with regards to various probability functions, we assumed that worlds specify the truth values of the propositions an agent might believe or disbelieve, while facts about what an agent believes in each world are specified by the belief set, and that every world and every belief set could go together. But with these propositions, there are problems in assessing each belief set with respect to a probability function that assigns positive probability to both worlds, since some belief sets and worlds make opposite specifications of the truth value of a single proposition. For instance, the proposition (P) is specified as false by **B** and **B**$'$, but as true by w_2, which is why those cells of the table are ruled out.

There are two alternatives that one might pursue. On the first alternative, one considers "restricted worlds" that only specify truth values of propositions that

don't concern the beliefs of the agent. On this alternative, there is only one restricted world, and thus only one relevant probability function (which assigns that restricted world probability 1). But this probability function doesn't say anything about the probability of (P), since (P) is a proposition that concerns the beliefs of the agent, and thus is specified by the belief set, and not the restricted worlds and the probability function. So ($\mathcal{R}$) doesn't even apply. But on this alternative, (WADA) and (SADA) still apply, and they rule out **B**$'$ and **B**$''$, while allowing **B** and **B**$'''$, even though they involve contradictory pairs.

On the second alternative, one considers full worlds that specify truth values for all propositions, and evaluates each belief set with respect to every world, even though some combinations of belief set and world can't both be actual. On this alternative, TABLE 3 should be replaced by TABLE 1 (*p.* 18). In this new table (which includes impossible pairs), (WADA) and (SADA) no longer rule out **B**$'$ and **B**$''$. However ($\mathcal{R}$) does rule out **B** and **B**$'''$. Thus, if one accepts all three of these principles whenever they apply, this second alternative gives the opposite coherence requirement to the first alternative.

This second alternative is endorsed by Briggs (2009, *pp.* 78-83) for talking about actual coherence requirements. She says that an agent with a belief set that is ruled out by the first alternative is "guaranteed to be wrong about something, even though his or her beliefs are perfectly coherent" (*op. cit.*, *p.* 79). The "guarantee" of wrongness only comes about because we allow the specification of the belief set to also specify parts of the world, instead of allowing the two to vary independently.

One might worry that the present context is sufficiently different from the types of cases that Briggs considered that one should go for the first alternative instead. But determining the right interpretation of ($\mathcal{R}$) in the current case is beyond the scope of this paper. Caie's examples show that some of our general results may need to be modified in cases where some of the relevant propositions either refer to or depend on the agent's beliefs.[53]

11. CONCLUSION

We have sketched a general framework for constructing coherence requirements for various types of judgment, by starting with a notion of vindication and a distance relation among judgment sets, and supplementing them with a fundamental epistemic principle. Many philosophers have perhaps implicitly assumed Possible Vindication [(PV)] as a fundamental epistemic principle, and thus derived deductive consistency [(CB)] as a coherence norm for full belief. We think that (PV) is too strong, and — following Joyce (1998; 2009) — we retreat to weak accuracy dominance avoidance (WADA). The resulting coherence requirement (NDB) is thus derived from considerations of accuracy, and therefore can't come into conflict with them. Moreover, on plausible understandings of evidential norms for belief [which require at least ($\mathcal{R}$)], this coherence requirement can't come in conflict with them either [depending on how we understand what is going on in examples involving state-act dependence (Caie 2012; 2013)]. As a result, our proposed coherence

[53]Caie's self-referential example is a special case of a more general class of examples which involve dependencies (causal *or* semantic) between an agent's beliefs and the truth-values of the objects of their beliefs. For further discussion of this broader class of examples involving state-act dependencies, see (Carr, 2013; Greaves, 2013).

requirement meets desideratum ($\mathcal{D}$) — it doesn't conflict with either alethic or evidential norms. Consequently, our new requirement allows (some) preface cases (with appropriate evidential structure) to conform to requirements of ideal epistemic rationality.

Thus, we think (NDB) is a coherence requirement that should replace (CB) in the relevant discussions. However, we suspect that something stronger, like ($\mathcal{R}$) [or perhaps even one of the ($\mathcal{R}_r$)], could provide a more robust coherence requirement. One question for future research is whether any such requirement can be derived within our framework, either by changing the notion of vindication, the distance measure, or the fundamental epistemic principle. Investigating these alternatives in the framework will be interesting in other ways as well. Is there some coherence requirement in this framework that deals well with infinite sets of judgments? What happens when we start with a notion of vindication that is evidential in nature, rather than alethic? Are there epistemic norms other than the alethic and evidential ones, and if so, do our coherence requirements conflict with them?

One final point: in accordance with desideratum ($\mathcal{D}$), our proposed coherence requirements are entailed by both evidential and alethic norms for full belief (given appropriate fundamental epistemic principles). If one accepts both the Jamesian and Cliffordian claims that there are alethic and evidential norms on belief, then one might suspect that this is in fact the role of coherence requirements. They may be the most general (rational, global/wide-scope) requirements on belief that are entailed by all of the basic epistemic norms.

Appendix

Proof of Theorem 1.
[**B** is non-dominated iff (⇔) **B** contains no witnessing set.]

(⇒) We prove the contrapositive. Suppose that $\mathbf{S} \subseteq \mathbf{B}$ is a witnessing set. Let $\mathbf{B}'$ agree with **B** on all judgments outside **S** and disagree with **B** on all judgments in **S**. By the definition of a witnessing set, $\mathbf{B}'$ must weakly dominate **B** in distance from vindication [$d(\mathbf{B}, \mathbf{B}_w)$]. Thus, **B** is dominated.

(⇐) We prove the contrapositive. Suppose that **B** is dominated, *i.e.*, that there is some $\mathbf{B}'$ that weakly dominates **B** in distance from vindication [$d(\mathbf{B}, \mathbf{B}_w)$]. Let $\mathbf{S} \subseteq \mathbf{B}$ be the set of judgments on which **B** and $\mathbf{B}'$ disagree. Then, **S** is a witnessing set.

Proof of Theorem 2.
[**B** is non-dominated if (⇐) there is a probability function Pr that represents **B**.]

Let Pr be a probability function that represents **B** in sense of Definition 2. Consider the expected distance, as calculated by Pr, of a belief set **B** — the sum over all worlds w of $\Pr(w) \cdot d(\mathbf{B}, \mathbf{B}_w)$. Since $d(\mathbf{B}, \mathbf{B}_w)$ is a sum of components for each proposition (1 if **B** disagrees with w on the proposition and 0 if they agree), and since expectations are linear, the expected distance from vindication is the sum of the expectation of these components. The expectation of the component for disbelieving p is $\Pr(p)$ while the expectation of the component for believing p is $1 - \Pr(p)$. Thus, if $\Pr(p) > 1/2$ then believing p is the attitude that uniquely minimizes the expectation, while if $\Pr(p) < 1/2$ then disbelieving p is the attitude that uniquely minimizes the expectation. Thus, since Pr represents **B**, this means that **B** has strictly lower expected distance from vindication than any other belief set with

respect to Pr (*i.e.*, that **B** uniquely minimizes expected distance from vindication, or maximizes expected closeness to vindication, relative to Pr). Suppose, for *reductio*, that some $\mathbf{B}'$ (weakly) dominates **B**. Then, $\mathbf{B}'$ must be no farther from vindication than **B** in any world, and thus $\mathbf{B}'$ must have expected distance from vindication no greater than that of **B**. But **B** has strictly lower expected distance from vindication than any other belief set. Contradiction. Therefore, no $\mathbf{B}'$ can dominate **B**, and so **B** must be non-dominated.

Proofs that (SADA) ⇏ (WADA) and (NDB) ⇏ ($\mathcal{R}$).

Consider a sentential language $\mathcal{L}$ with two atomic sentences X and Y. The Boolean algebra $\mathcal{B}$ generated by $\mathcal{L}$ contains 16 propositions (corresponding to the subsets of the set of four state descriptions of $\mathcal{L}$, *i.e.*, the set of four salient possible worlds). Table 4 depicts $\mathcal{B}$, and two opinionated belief sets ($\mathbf{B}_1$ and $\mathbf{B}_2$) on $\mathcal{B}$.

$\mathcal{B}$	$\mathbf{B}_1$	$\mathbf{B}_2$
$\neg X \,\&\, \neg Y$	D	D
$X \,\&\, \neg Y$	D	D
$X \,\&\, Y$	D	D
$\neg X \,\&\, Y$	D	D
$\neg Y$	D	D
$X \equiv Y$	D	D
$\neg X$	B	B
X	B	D
$\neg(X \equiv Y)$	D	D
Y	D	D
$X \vee \neg Y$	B	B
$\neg X \vee \neg Y$	B	B
$\neg X \vee Y$	B	B
$X \vee Y$	D	B
$X \vee \neg X$	B	B
$X \,\&\, \neg X$	D	D

Table 4. Examples showing (SADA) ⇏ (WADA) and (NDB) ⇏ ($\mathcal{R}$)

We have the following four salient facts regarding $\mathbf{B}_1$ and $\mathbf{B}_2$:

(1) $\mathbf{B}_1$ is weakly dominated (in distance from vindication) by belief set $\mathbf{B}_2$. [This can be verified *via* simple counting.] Thus, $\mathbf{B}_1$ violates (NDB)/(WADA).

(2) $\mathbf{B}_1$ is not strictly dominated (in distance from vindication) by any belief set over $\mathcal{B}$. [This can be verified by performing an exhaustive search on the set of all possible belief sets over $\mathcal{B}$.[54]] Thus, $\mathbf{B}_1$ satisfies (SADA).

[54] We have created a *Mathematica* notebook which verifies claims (1)–(4) of this Appendix. This notebook can be downloaded from `http://fitelson.org/ace.nb`.

(3) $\mathbf{B}_2$ is not weakly dominated (in distance from vindication) by any belief set over $\mathcal{B}$. [This can be verified by performing an exhaustive search on the set of all possible belief sets over $\mathcal{B}$ (see *fn.* 54).] Thus, $\mathbf{B}_2$ satisfies (WADA).

(4) $\mathbf{B}_2$ is not represented (in the sense of Definition 2) by any probability function on $\mathcal{B}$. [This can be verified *via* Fitelson's (2008) decision procedure for probability calculus `PrSAT` (see *fn.* 54).[55]] Thus, $\mathbf{B}_2$ violates $(\mathcal{R})$.

Proof of Theorem 3.
[For all $n \geq 2$ and for each set of propositions $\mathbf{P} \in \mathbb{B}_n$, if $r \geq \frac{n-1}{n}$ then $(\mathcal{R}_r)$ rules out believing every member of $\mathbf{P}$, while if $r < \frac{n-1}{n}$, then $(\mathcal{R}_r)$ doesn't rule out believing every member of $\mathbf{P}$.]

Let $\mathbf{P}$ be a member of $\mathbb{B}_n$, *i.e.*, $\mathbf{P}$ consists of n propositions, there is no world in which all of these n propositions are true, but for each proper subset $\mathbf{P}' \subset \mathbf{P}$ there is a world in which all members of $\mathbf{P}'$ are true.

Let $\phi_1, \ldots, \phi_n$ be the n propositions in $\mathbf{P}$. Let each w_i be a world in which ϕ_i is false, but all other members of $\mathbf{P}$ are true. Let Pr be the probability distribution that assigns probability $1/n$ to each world w_i and 0 to all other worlds. If $r < \frac{n-1}{n}$, then Pr shows that the belief set $\mathbf{B_P} := \{B(\phi_1), \ldots, B(\phi_n)\}$, which includes the belief that ϕ_i for each $\phi_i \in \mathbf{P}$, satisfies $(\mathcal{R}_r)$. This establishes the second half of the theorem.

For the first half, we will proceed by contradiction. Thus, assume that $\mathbf{P}$ is a member of $\mathbb{B}_n$ such that the belief set $\mathbf{B_P} := \{B(\phi_1), \ldots, B(\phi_n)\}$, which includes the belief that ϕ_i for each $\phi_i \in \mathbf{P}$, is *not* ruled out by $\left(\mathcal{R}_{\frac{n-1}{n}}\right)$. Then there must be some Pr such that for each i, $\Pr(\phi_i) > \frac{n-1}{n}$. This means that for each i, $\Pr(\neg\phi_i) < 1/n$. Since the disjunction of finitely many propositions is at most as probable as the sum of their individual probabilities, this means that $\Pr(\neg\phi_1 \vee \ldots \vee \neg\phi_n) < 1$. But since $\mathbf{P}$ is inconsistent, $\neg\phi_1 \vee \ldots \vee \neg\phi_n$ is a tautology, and therefore must have probability 1. This is a contradiction, so $\mathbf{B_P}$ must be ruled out by $\left(\mathcal{R}_{\frac{n-1}{n}}\right)$.

Proof of Theorem 4.
[$\mathbf{B}$ is not strictly dominated iff ($\Leftrightarrow$) $\mathbf{B}$ contains no witnessing$_1$ set.
That is: (SADA) $\Leftrightarrow$ (NW$_1$S)]

($\Rightarrow$) We'll prove the contrapositive. Suppose that $\mathbf{S} \subseteq \mathbf{B}$ is a witnessing$_1$ set. Let $\mathbf{B}'$ agree with $\mathbf{B}$ on all judgments outside $\mathbf{S}$ and disagree with $\mathbf{B}$ on all judgments in $\mathbf{S}$. By the definition of a witnessing$_1$ set, $\mathbf{B}'$ must strictly dominate $\mathbf{B}$ in distance from vindication $[d(\mathbf{B}, \mathbf{B}_w)]$. Thus, $\mathbf{B}$ is strictly dominated.

($\Leftarrow$) We prove the contrapositive. Suppose $\mathbf{B}$ is strictly dominated, *i.e.*, that there is some $\mathbf{B}'$ that strictly dominates $\mathbf{B}$ in distance from vindication $[d(\mathbf{B}, \mathbf{B}_w)]$. Let $\mathbf{S} \subseteq \mathbf{B}$ be the set of judgments on which $\mathbf{B}$ and $\mathbf{B}'$ disagree. Then, $\mathbf{S}$ is a witnessing$_1$ set.

Proof of Theorem 5.
[$\mathbf{B}$ is probabilistically representable (in the sense of Definition 2) only if $\mathbf{B}$ contains no witnessing$_2$ set. That is, $(\mathcal{R}) \Longrightarrow$ (NW$_2$S).]

[55] In fact, this can be verified easily by hand, since the set $\mathbf{B}_2$ contains two contradictory pairs of judgments: $\{D(Y), D(\neg Y)\}$ and $\{D(X \equiv Y), D(\neg(X \equiv Y))\}$. Moreover, we've already seen an example of this phenomenon in §7, above, when we showed (NDB) does not rule out contradictory pairs.

In our proof of Theorem 2, we established that if Pr represents $\mathbf{B}$, then $\mathbf{B}$ has strictly lower expected distance from vindication than any other belief set with respect to Pr. Assume, for *reductio*, that $\mathbf{S} \subseteq \mathbf{B}$ is a witnessing$_2$ set for $\mathbf{B}$. Let $\mathbf{B}'$ agree with $\mathbf{B}$ on all judgments outside $\mathbf{S}$ and disagree with $\mathbf{B}$ on all judgments in $\mathbf{S}$. Then by the definition of a witnessing$_2$ set, $\mathbf{B}'$ must be no farther from vindication than $\mathbf{B}$ in any world. But this contradicts the fact that $\mathbf{B}$ has strictly lower expected distance from vindication than $\mathbf{B}'$ with respect to Pr. So the witnessing$_2$ set must not exist.

Proof of Theorem 6.
[$\mathbf{B}$ is probabilistically representable (in the sense of Definition 2) only if $\mathbf{B}$ satisfies both (NDB) and (NCP). That is, $(\mathcal{R}) \Longrightarrow$ (NDB & NCP).]

Theorem 2 implies $(\mathcal{R}) \Longrightarrow$ (NDB). And, it is obvious that $(\mathcal{R}) \Longrightarrow$ (NCP), since no probability function can probabilify both members of a contradictory pair and no probability function can dis-probabilify both members of a contradictory pair.

Counterexample to the Converse of Theorem 6.
[$(\mathcal{R}) \not\Leftarrow$ (NDB & NCP).]

Let there be six possible worlds, $w_1, w_2, w_3, w_4, w_5, w_6$. Consider the agenda $\mathcal{A}$ consisting of the following four propositions (*i.e.*, $\mathcal{A} \stackrel{\text{def}}{=} \{p_1, p_2, p_3, p_4\}$).

$$\begin{aligned}
p_1 &= \{w_1, w_2, w_3\} \\
p_2 &= \{w_1, w_4, w_5\} \\
p_3 &= \{w_2, w_4, w_6\} \\
p_4 &= \{w_3, w_5, w_6\}
\end{aligned}$$

Let $\mathbf{B} \stackrel{\text{def}}{=} \{B(p_1), B(p_2), B(p_3), B(p_4)\}$. $\mathbf{B}$ is itself a witnessing$_2$ set, since, in every possible world, exactly two beliefs (*i.e.*, exactly half of the beliefs) in $\mathbf{B}$ are accurate. So by Theorem 5, $\mathbf{B}$ is not probabilistically representable. However, $\mathbf{B}$ satisfies (NDB). To see this, note that every belief set on $\mathcal{A}$ has an expected distance from vindication of 2, relative to the uniform probability distribution. This implies that no belief set on $\mathcal{A}$ dominates any other belief set on $\mathcal{A}$. Finally, $\mathbf{B}$ satisfies (NCP), since every pair of beliefs in $\mathbf{B}$ is consistent.[56]

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[56] We can extend this counterexample to an example that isn't restricted to $\mathcal{A}$, but in fact is opinionated across the whole algebra $\mathcal{B}$ of propositions constructible out of the six possible worlds. Here's how to extend $\mathbf{B}$ to a belief set $\mathbf{B}'$ over $\mathcal{B}$. For the propositions p that are true in four or more worlds, $\mathbf{B}'$ contains $B(p)$. For the propositions q that are true in two or fewer worlds, $\mathbf{B}'$ contains $D(q)$. For each pair of complementary propositions that are true in exactly three worlds, $\mathbf{B}'$ includes belief in one and disbelief in the other (*i.e.*, $\mathbf{B}'$ may include any consistent pair of attitudes toward each pair of complementary propositions that are true in exactly three worlds). By construction, this set $\mathbf{B}'$ satisfies (NCP). Additionally, the belief set $\mathbf{B}'$ is tied for minimal expected distance from vindication on $\mathcal{B}$, relative to the uniform probability distribution on $\mathcal{B}$. Therefore, $\mathbf{B}'$ satisfies (NDB).

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