Notes for Week 7 of *Confirmation*

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1 Some Properties of Confirmation Relations

Hempel (in his second installment) discusses various properties that confirmation relations might have. I will discuss a longer list of properties. Here are a bunch of properties that we'll discuss today. We will assume throughout most of our discussion that all of our E's, H's, and K's are logically contingent.

(M_E) If E confirms H relative to K, then $E \,\&\, E'$ confirms H relative to K (provided that E' does not contain any constant symbols not already contained in $\{H, E, K\}$).

(M_K) If E confirms H relative to K, then E confirms H relative to $K \,\&\, K'$ (provided that K' does not contain any constant symbols not already contained in $\{H, E, K\}$).

(NC) $\ulcorner \phi x \,\&\, \psi x \urcorner$ confirms $\ulcorner (\forall y)(\phi y \supset \psi y) \urcorner$ relative to (some/all/specific) K.

(SCC) If E confirms H relative to K and $H \vDash_K H'$, then E confirms H' relative to K.

(CCC) If E confirms H relative to K and $H' \vDash_K H$, then E confirms H' relative to K.

(CC) If E confirms H relative to K and E confirms H' relative to K, then $K \nvDash {\sim}(H \,\&\, H')$.

(CC′) If E confirms H relative to K and E confirms H' relative to K, then $K \nvDash {\sim}(H \equiv H')$.

(EC) If $E \vDash_K H$, then E confirms H relative to K.

(CEC) If $H \vDash_K E$, then E confirms H relative to K.

(EQC_E) If E confirms H relative to K and $K \vDash E \equiv E'$, then E' confirms H relative to K.

(EQC_H) If E confirms H relative to K and $K \vDash H \equiv H'$, then E confirms H' relative to K.

(EQC_K) If E confirms H relative to K and $K \Dashv\vDash K'$, then E confirms H relative to K'.

(NT) For some E, H, and K, E confirms H relative to K.
And, for every E/K, there exists an H such that E does *not* confirm H relative to K.

(ST) If E confirms H relative to K and E confirms H relative to ${\sim}K$, then E confirms H relative to $\top$.

As exercises, let's think about some subsets of this large set of conditions. Consider the following triples:

- (NT), (CEC), (SCC)
  - Inconsistent. Pick an E. Then, by (NT), E does not confirm (some) H relative to (some) K. But, by (CEC), E confirms $H \,\&\, E$, relative to K. Then, by (SCC), E confirms H, relative to K. Contradiction.
- (NT), (EC), (CCC)
  - Inconsistent. Pick an E. Then, by (NT), E does not confirm H relative to K. By (CCC), E does not confirm $H \vee E$ relative to K (if it did, then, by (CCC), it would also confirm the logically stronger H, contrary to our initial assumption). By (EC), E confirms $H \vee E$ relative to K. Contradiction.
- (NT), (CCC), (SCC)
  - Inconsistent. By (NT), (some) E confirms (some) H relative to (some) K. By (CCC), E confirms $H \,\&\, H'$ relative to K. By (SCC), E confirms H' relative to K. But, H' was arbitrary here. So, we have found an E/K such that, for all H', E confirms H' relative to K, which contradicts (NT).

As an exercise, it is useful to think about the consistency of other subsets of this large set of conditions.

2 Hempel's Theory of Confirmation

In order to understand Hempel's theory, we first need the concept of *the development of a (closed, first-order) statement H with respect to a set of individual constants I*, which I will write as $\text{dev}_I(H)$. This is defined as:

> $\text{dev}_I(H)$ is (*i*) the *conjunction* of the I-instances of H, if H is a *universal* ($\forall$) claim, (*ii*) the *disjunction* of the I-instances of H, if H is an *existential* ($\exists$) claim, and (iii) $\text{dev}_I(H) = H$, if H is quantifier-free.

Here are some examples to illustrate how $\text{dev}_I(H)$ is computed. Let $I = \{a, b\}$. Then, we have:

- $\text{dev}_I[(\forall x)Bx] = Ba \,\&\, Bb$.
- $\text{dev}_I[(\exists x)Rx] = Ra \vee Rb$.
- $\text{dev}_I[(\forall x)(Rx \supset Bx)] = (Ra \supset Ba) \,\&\, (Rb \supset Bb)$.
- $\text{dev}_I[(\forall x)(\exists y)Lxy] = \text{dev}_I[(\exists y)Lay \,\&\, (\exists y)Lby] = (Laa \vee Lab) \,\&\, (Lba \vee Lbb)$

It can be shown that logically equivalent hypotheses have logically equivalent developments. In fact, if $H \vDash H'$, then $\text{dev}_I(H) \vDash \text{dev}_I(H')$. Proving this is a useful exercise. We will just assume it in what follows.

With $\text{dev}_I(H)$ understood, we are now ready for Hempel's definition of confirmation:

> **Definition**. *E directly Hempel-confirms H* relative to K iff $E \vDash_K \text{dev}_{I(E\&K)}(H)$ with respect to the set $I(E \,\&\, K)$ of individual constants occurring in $E \,\&\, K$. *E Hempel-confirms H* relative to K iff E directly Hempel-confirms (relative to K) every member of some set S such that $S \vDash_K H$. [If E Hempel-confirms H relative to $\top$, then we just say E Hempel-confirms H, for short.]
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> – Caveat #1: An "observation report" E is a closed, quantifier-free sentence. We will assume the same about "background evidence" K. This makes Hempel's theory unable to handle certain sorts of evidence (*e.g.*, statistical evidence, or even other sorts of evidential statements—although, as Hempel notes, the definition could be extended to general sentences in first-order languages).
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> – Caveat #2: It is assumed that E contains all of its constant symbols *essentially*. For instance, while $E \stackrel{\text{def}}{=} Ra$ is logically equivalent to $E' \stackrel{\text{def}}{=} Ra \,\&(Qb \vee {\sim}Qb)$, they can partake in different confirmation relations, if we don't restrict $I(E')$ to the constant symbols E' contains essentially. So, strictly speaking, we need to think of $I(E \,\&\, K)$ as the smallest set of individual constants occurring in sentences E' that are logically equivalent to $E \,\&\, K$. This set can always be computed easily.

Here are some examples to illustrate the definition:

- $E \stackrel{\text{def}}{=} Ra$ and $E' \stackrel{\text{def}}{=} Ra \,\&\, (Qb \vee {\sim}Qb)$ both directly Hempel-confirm $(\forall x)Rx$. This is because of caveat #2. When we compute $I(E')$, we get $\{a\}$ and not $\{a, b\}$, since $E' \dashv\vDash E$. Thus, 'b' is *inessential* in E'.

- $Ra \,\&\, Ba$ does *not directly* Hempel-confirm $Rb \supset Bb$. This is because $Ra \,\&\, Ba \nvDash Rb \supset Bb$. But, $Ra \,\&\, Ba$ *does* Hempel-confirm $Rb \supset Bb$. Let $S \stackrel{\text{def}}{=} \{(\forall x)(Rx \supset Bx)\}$. Then, $Ra \,\&\, Ba$ directly Hempel-confirms the only member of S $[(\forall x)(Rx \supset Bx)]$, since $Ra \,\&\, Ba \vDash Ra \supset Ba = \text{dev}_{I(Ra\&Ba)}[(\forall x)(Rx \supset Bx)]$. $\therefore$ We have (indirect) Hempel-confirmation, since $S \vDash Rb \supset Bb$. Does $Ra \& Ba$ Hempel-confirm $Rb \& Bb$?

- Let $I \stackrel{\text{def}}{=} \{a, b\}$, $H \stackrel{\text{def}}{=} (\forall x)(\forall y)Rxy$, $E \stackrel{\text{def}}{=} Raa \,\&\, Rab \,\&\, Rbb \,\&\, Rba$, and $E' \stackrel{\text{def}}{=} Raa \,\&\, Rab \,\&\, Rbb$. E Hempel confirms H, but E' does not. Moreover, Raa Hempel confirms H. This example shows that Hempel-confirmation is not *unrestrictedly* monotonic in E. But, subject to the caveat that E_2 does not contain any constant symbols that do not occur in $\{E_1, H\}$, we have that $E_1 \& E_2$ confirms H, if E_1 does. In other words, Hempel's theory entails (M_E) and (M_K) in the restricted forms stated above.

- *No consistent* E can confirm the following hypothesis (H), which is *true* on $\mathbb{N}$:

  $$(H) \qquad (\forall x)(\exists y)(x < y) \,\&\, (\forall x)(x \nless x) \,\&\, (\forall x)(\forall y)(\forall z)[(x < y \,\&\, y < z) \supset x < z]$$

  since $\text{dev}_I(H)$ *is inconsistent, for any finite* I. That is an interesting consequence of Hempel's definition.

- Let $H \stackrel{\text{def}}{=} (\forall x)(Rx \supset Bx)$. Which of the following six propositions Hempel-confirm H?

E_1: $Ra \,\&\, Ba$	E_2: $\sim Ra$	E_3: Ba
E_4: $\sim Ra \,\&\, \sim Ba$	E_5: $\sim Ra \,\&\, Ba$	E_6: $Ra \,\&\, \sim Ba$

  Answer: *All but* E_6 *Hempel-confirm* H. Indeed, all but E_6 *directly* Hempel-confirm H.

- Consider the following argument (given by Hempel, and endorsed also by Goodman):

  > If E consists *only* of one nonraven [$\sim Ra$], then E confirms that all objects are nonravens [$(\forall x)\sim Rx$], and *a fortiori* E supports the weaker assertion that all nonblack objects are nonravens [$(\forall x)(\sim Bx \supset \sim Rx)$]. [Therefore, "one nonblack nonraven" $\sim Ba \,\&\, \sim Ra$ also confirms that all nonblack objects are nonravens, and hence that all ravens are black.]

(i)	$\sim Ra$ confirms $(\forall x)\sim Rx$.	(NC)
(ii)	$(\forall x)\sim Rx \vDash (\forall x)(\sim Bx \supset \sim Rx)$	(Logic)
(iii)	$\sim Ra$ confirms $(\forall x)(\sim Bx \supset \sim Rx)$	(i), (ii), (SCC)
(iv)	$(\forall x)(\sim Bx \supset \sim Rx) \;\Dashv\vDash\; (\forall x)(Rx \supset Bx)$	(Logic)
(v)	$\sim Ra$ confirms $(\forall x)(Rx \supset Bx)$	(iii), (iv), (EQC$_H$)
(PC)	$\sim Ba \,\&\, \sim Ra$ confirms $(\forall x)(Rx \supset Bx)$	(v), (M$_E$)

This "independent argument" for (PC) depends essentially on (NC), (SCC), *and* (M$_E$). Hempel and Goodman don't even *state* the final conclusion! This shows how subtle (M$_E$) is. They make this error because they talk about the *object* "one nonraven" doing the confirming. And, "one nonblack nonraven" *is* "one nonraven"!

3 Hypothetico–Deductive Confirmation ("The Prediction Criterion")

The hypothetico-deductive (HD) account of confirmation is typically not defended by anyone who is serious about confirmation theory.[1] But, it is often used as a foil in contemporary discussions. Here it is (Hempel calls this "the prediction criterion" — his definition is a bit more complicated, but I'll ignore that):

> **Definition**. E *HD-confirms* H relative to K iff $H \vDash_K E$.

While Hempel and HD confirmation are both defined in terms of $\vDash_K$, it's a pretty significant difference that Hempel has E on the left hand side of $\vDash_K$, whereas HD has E on the right-hand side of $\vDash_K$.

4 Two Probabilistic Confirmation Concepts

As we'll see in two weeks, Carnap discusses two distinct probabilistic confirmation concepts.

4.1 Confirmation as Firmness ("Absolute" Confirmation)

Confirmation as firmness is a conditional-probability-threshold concept:

> **Definition**. E *confirms*$_f$ H relative to K iff $\Pr(H \mid E \,\&\, K) > t$, for some threshold value t.

Here, one is usually also offered an *interpretation* of $\Pr(\cdot \mid \cdot)$. But, I will ignore this for now. We'll return to that issue later in the seemster (I will argue that logical confirmation concepts do not require any interpretation of Pr, although their epistemic correlates will involve some interpretation of Pr).

[1] Quine (in his "Natural Kinds" paper, which we'll discuss when we get to "grue") occasionally talks about (HD) confirmation, but he seems to be confused about the distinction between (HD) and Hempelian confirmation. In fact, they differ radically.

4.2 Confirmation as Increase in Firmness ("Incremental" Confirmation)

Confirmation as increase in firmness is a probabilistic relevance concept:

> **Definition**. E *confirms*$_i$ H relative to K iff $\Pr(E \mid H \,\&\, K) > \Pr(E \mid {\sim}H \,\&\, K)$.

As above, I will ignore the interpretive questions about Pr for now.

5 A Summary of the Properties of our Four Confirmation Concepts

So far, we have seen four confirmation concepts: Hempelian, HD, Firmness, and Increase in Firmness. These four concepts differ in important ways with respect to various properties, including our list of properties above. The following table summarizes these properties, for each of the four concepts.

	Does Concept Satisfy Condition?										
Concept	$\mathrm{EQC}_{E/H/K}$	EC	CC	NT	SCC	CCC	CEC	$\mathrm{M}_{E/K}$	NC	CC′	ST
Hempelian	Yes	Yes	Yes[1]	Yes	Yes	No	No	Yes	Yes	Yes[1]	Yes
HD	Yes	No	No	Yes	No	Yes	Yes	No	No	Yes[5]	Yes
Confirms$_f$	Yes	Yes[2]	No	Yes	Yes	No	No	No	No	Yes[4]	Yes
Confirms$_i$	Yes	Yes[3]	No	Yes	No	No	Yes[3]	No	No	Yes	No

It is a useful exercise to go through and convince yourself of all these answers. I won't dwell on these here, but I will briefly discuss one important subset of properties of Hempel-confirmation: (SCC), (NC), (M_E). These three properties are involved in both paradoxes of confirmation. We discussed the raven paradox last week [and again above in connection with the "independent argument" for (PC)]. When we get to "grue" in a few weeks, we'll see these properties playing a crucial role again. Here's an example. Let $Ox \overset{\text{def}}{=} x$ is examined (for the first time) prior to t (for some t in the distant future). Let any statement $\mathcal{E}$ of the form $\ulcorner Oa \,\&\, \psi a \urcorner$ be called an "observation statement". Now, in Hempel's theory of confirmation, we can reason as follows:

(a) Oa confirms $(\forall x)Ox$. (NC)

(b) $\ulcorner Oa \,\&\, \psi a \urcorner$ confirms $(\forall x)Ox$, for any ψ. (a), (M_E)

(c) $\ulcorner Oa \,\&\, \psi a \urcorner$ confirms $\ulcorner (\forall x)({\sim}Ox \supset \phi x) \urcorner$, for any ψ and any ϕ. (b), (SCC)

$\therefore$ *Any observation statement $\mathcal{E}$ confirms any universal generalization H about unexamined objects.*

As we'll see when we get to Goodman, this is one of the main "odd consequences" of Hempel's theory that Goodman complains about. Goodman does lots of fancy syntactic fiddling with antecedent and consequent predicate expressions of universal claims (*e.g.*, "emerose"). But, all that fancy footwork unnecessary. All one needs is to recognize that Hempel's theory entails (SCC), (NC), and (M_E). Apparently, Hempel and Goodman were unaware that Hempel's theory implies (M_E). I discussed this last week (see also my "ravens" paper).

6 Confirmation, Disconfirmation, Verification, and Falsification

On Hempel's theory, as well as the probabilistic theories, disconfirmation can be defined as follows:

> **Definition**. E *disconfirms* H relative to K iff E confirms ${\sim}H$ relative to K.

[1] Assuming that $E \,\&\, K$ is not self-contradictory.
[2] Assuming that $\Pr(E \mid K) \neq 0$.
[3] Assuming that $\Pr(H \mid K) \in (0,1)$, and $\Pr(E \mid K) \in (0,1)$.
[4] Assuming that $t \geq \frac{1}{2}$.
[5] Assuming that $K \nvDash E$.

And "confirmational neutrality" can be defined as *neither confirming nor disconfirming*. On the HD account, however, disconfirmation cannot be defined in this way. HD-disconfirmation is typically defined as $H \vDash_K \sim E$. This is different than saying E HD-confirms $\sim H$ relative to K, since that would be $\sim H \vDash_K E$.

Hempel has an interesting discussion of the cases in which E *verifies* or *falsifies* H (relative to $\top$), according to his theory of confirmation (and his "logical" understanding of verification and falsification). According to Hempel, E *verifies* (or *conclusively confirms*) H if $E \vDash H$, and E *falsifies* (or *conclusively disconfirms*) H if $E \vDash \sim H$. As a result, Hempel says that "Whether a hypothesis is verifiable, or falsifiable, in this sense depends exclusively on its logical form." Hempel then discusses the following four kinds of hypotheses:

1. Closed, Quantifier-Free Hypotheses. According to Hempel, such hypotheses are both verifiable and falsifiable, because, for any such H, there exist "observation reports" E (*i.e.*, other closed, quantifier-free first-order sentences) that entail H, and there also exist E's that are incompatible with H.

2. Purely Existential Hypotheses. According to Hempel, such hypotheses are verifiable but not falsifiable, because, for any such H, there will exist E's (*i.e.*, closed, quantifier-free first-order sentences) that entail H, but there will not exist any E's that are incompatible with H.

3. Purely Universal Hypotheses. According to Hempel, such hypotheses are falsifiable but not verifiable, because, for any such H, there will exist E's (*i.e.*, closed, quantifier-free first-order sentences) that are incompatible with H, but there will not exist any E's that entail H.

4. Mixed Quantifier Hypotheses. In general, such hypotheses will tend to be neither verifiable nor falsifiable. But, there are some such hypotheses that are verifiable [$(\forall x)(\exists y)(Px \vee Qy)$ is verified by Qa], and some that are falsifiable [$(\forall x)(\exists y)(Px \,\&\, Qy)$ is falsified by $\sim Pa$].

Of course, Hempel realizes that this logical taxonomy only captures one sense of verification/falsification (which he calls "relative verification/falsification"). He contrasts this with "absolute verification/falsification", which he says "does not belong to formal logic, but rather to pragmatics". In this broader setting, Hempel has a very interesting discussion of "three phases of the scientific test of a given hypothesis":

> The first phase consists in the performance of suitable experiments or observations and the ensuing acceptance of observation sentences, or of observation reports, stating the results obtained; the next phase consists in confronting the given hypothesis with the accepted observation reports, i.e., in ascertaining whether the latter constitute confirming, disconfirming or irrelevant evidence with respect to the hypothesis; the final phase consists either in accepting or rejecting the hypothesis on the strength of the confirming or disconfirming evidence constituted by the accepted observation reports, or in suspending judgment, awaiting the establishment of further relevant evidence.

Hempel says that the current study is concerned primarily with "phase 2", which he takes to be *purely logical*. What about the other two "phases", which belong to "pragmatics"? Hempel tells us that:

> The first phase, on the other hand, is of a pragmatic character; it involves no logical confrontation of sentences with other sentences. It consists in performing certain experiments or systematic observations and noting the results. The latter are expressed in sentences which have the form of observation reports, and their acceptance by the scientist is connected (by causal, not by logical relations) with experiences occurring in those tests.
>
> The third phase, too, can be construed as pragmatic, namely as consisting in a decision on the part of the scientist or a group of scientists to accept (or reject, or leave in suspense, as the case may be) a given hypothesis after ascertaining what amount of confirming or of disconfirming evidence for the hypothesis is contained in the totality of the accepted observation sentences. However, it may well be attempted to give a reconstruction of this phase in purely logical terms. This would require the establishment of general "rules of acceptance"; roughly speaking, these rules would state how well a given hypothesis has to be confirmed by the accepted observation reports to be scientifically acceptable itself; i.e., the rules would formulate criteria for the acceptance or rejection of a hypothesis by reference to the kind and amount of confirming or disconfirming evidence for it embodied in the totality of accepted observation reports; possibly, these criteria would also refer to such additional factors as the "simplicity" of the hypothesis in question, the manner in which it fits into the system of previously accepted theories, etc.

For Hempel, this reflects a change in view. Previously, Hempel had argued that

> ...the only possible interpretation of the phrase "Sentence S is true" is "S is highly confirmed by accepted observation reports". I should now reject this view. As the work of A. Tarski, R. Carnap, and others has shown, it is possible to define a semantical concept of truth which is not synonymous with that of strong confirmation, and which corresponds much more closely to what has customarily been referred to truth, especially in logic, but also in other contexts. ...In the light of these considerations, it seems advisable to me to reserve the term "truth" for the semantical concept; I should now phrase the statements in the *Analysis* articles as dealing with confirmation.

Thus, Hempel has moved from a rather strong form of verificationism to a weaker form. Nonetheless, the view Hempel outlines here does presuppose a rather strong connection between logic and epistemology (or "pragmatics"). According to Hempel, it is a pragmatic question as to "the *kind and amount* of confirming or disconfirming evidence for" H that will be required for acceptance of H. However, he seems to insist that *confirming/disconfirming* (accepted) evidence reports will play *some* role in the epistemological story. One might wonder *why*. Here, one needn't look beyond the cases of *conclusive* confirmation/disconfirmation to see non-trivial problems. The conclusive relations are just the *logical* relations of entailment and refutation. There are two immediate epistemological issues that arise here. First, there is the fact that we are not *logically omniscient*, and so there will be many cases in which we simply *do not know that* the conclusive confirmation/disconfirmation relations obtain in the first place. Second, even in cases where we *do* know that these relations obtain, it is not clear precisely what *epistemological significance* this should have with respect to H. For instance, if we come to accept E (which, as Hempel is right to remind us, might itself be because of how "well confirmed" E is by what we already accept), and then we realize that E conclusively confirms H, must we then accept H as a result? Perhaps not. Perhaps we are already convinced (perhaps by some other route) that H is a crazy hypothesis (indeed, we might even have discovered an inconsistency here in what we currently accept). In that case, maybe it would be better to give up our acceptance of E instead. Unfortunately, this sort of issue is not often discussed in this strand of the historical literature. This is one of the issues I'd like to focus on in coming weeks (especially, when we get to Goodman).

Note that various other presuppositions about the relationship between logic and epistemology have already been lurking in Hempel's discussion. When he discusses the raven paradox, he seems to be appealing to an *epistemic* intuition: if we *already know* $\sim Ra$, then *observing* a's color won't give us any *evidence* about the color of ravens. He uses this (plausible) intuition to "train" our intuitions about *confirmation*. But, why should epistemic intuitions be relevant to confirmation theory at all? After all, "phase 2" is supposed to be purely logical. And, "phase 3" is pragmatic (or epistemic). But, he seems to be using (epistemic) "phase 3" intuitions to hone his (logical) theory of "phase 2". This same methodology will be used by Goodman in his argument *against* Hempel's theory of confirmation. While this may seem unfair to Hempel (who does recognize significant differences between logic and epistemology), it does seem to cohere with what Hempel himself *does* in his own methodology. Keep your eyes open for this sort of methodological maneuver.

In closing, it might be useful to consider a few "bridge principles" that one might think (or hope) are in force when it comes to the relationship between deductive entailment and inference/belief:

① If an agent S's belief set (or "acceptance set") B entails p (and S *knows* $B \vDash p$), then it would be reasonable for S to infer/believe p.

② If S knows that $B \vDash p$, then S should *not* be such that *both*: S believes $\bigwedge B$, *and* S does not believe p.

③ If S knows that $B \vDash p$, then S should *not* be such that *both*: S believes each of the $B_i \in B$, *and* S does not believe p.

Principle ① is quite strong, and it is clearly false. There are various cases in which the right thing for S to do is to reject some $B_i \in B$, rather than accepting/inferring/believing p. Principle ② seems more plausible (it is "wide scope", rather than "narrow scope"), but it doesn't yield very interesting constraints on S (especially, regarding p), since S is probably never in a position where it would be reasonable for them to believe $\bigwedge B$ (indeed, they may not even be able to *grasp* the conjunction of everything they now believe/accept). Principle ③ may also seem more plausible than ①, but it seems less plausible than ②. Preface paradox cases seem to be counterexamples to ③ (especially, if we take "global" preface paradox cases in which $p = \bigwedge B$).

As an exercise, you might try to think up some more general "bridge principles" for Hempel-confirmation (not only for the conclusive cases of entailment/refutation). I think the prospects are rather dim. Later on, we'll discuss various "bridge principles" for our probabilistic confirmation concepts (and some appropriate "epistemic correlates"). Meanwhile, I recommend the first two chapters of Harman's *Change in View*.