```
<< PrSAT`
```

■ Computing the Measure on the State Descriptions

■ Finding Models in Maher's System

In[157]:=
```
x_ ⊃ y_ := ¬ x ⋁ y;
x_ ≡ y_ := (x ⊃ y) ⋀ (y ⊃ x);
map = {a16 → s1, a12 → s2, a13 → s5, a6 → s6, a14 → s3, a7 → s4, a8 → s7, a2 → s8,
  a15 → s9, a9 → s10, a10 → s13, a3 → s14, a11 → s11, a4 → s12, a5 → s15, a1 → s16};
MaherModel[f_, g_] := PrSAT[{Pr[Fa ⋀ Ga ⋀ Fb ⋀ Gb] == s1, Pr[Fa ⋀ Ga ⋀ Fb ⋀ ¬ Gb] == s2,
    Pr[Fa ⋀ Ga ⋀ ¬ Fb ⋀ Gb] == s3, Pr[Fa ⋀ Ga ⋀ ¬ Fb ⋀ ¬ Gb] == s4,
    Pr[Fa ⋀ ¬ Ga ⋀ Fb ⋀ Gb] == s5, Pr[Fa ⋀ ¬ Ga ⋀ Fb ⋀ ¬ Gb] == s6,
    Pr[Fa ⋀ ¬ Ga ⋀ ¬ Fb ⋀ Gb] == s7, Pr[Fa ⋀ ¬ Ga ⋀ ¬ Fb ⋀ ¬ Gb] == s8,
    Pr[¬ Fa ⋀ Ga ⋀ Fb ⋀ Gb] == s9, Pr[¬ Fa ⋀ Ga ⋀ Fb ⋀ ¬ Gb] == s10,
    Pr[¬ Fa ⋀ Ga ⋀ ¬ Fb ⋀ Gb] == s11, Pr[¬ Fa ⋀ Ga ⋀ ¬ Fb ⋀ ¬ Gb] == s12,
    Pr[¬ Fa ⋀ ¬ Ga ⋀ Fb ⋀ Gb] == s13, Pr[¬ Fa ⋀ ¬ Ga ⋀ Fb ⋀ ¬ Gb] == s14,
    Pr[¬ Fa ⋀ ¬ Ga ⋀ ¬ Fb ⋀ Gb] == s15, Pr[¬ Fa ⋀ ¬ Ga ⋀ ¬ Fb ⋀ ¬ Gb] == s16} //. {γF → f, γG → g}];
```

In[160]:=
```
MNC = MaherModel[1 / 1000, 1 / 10]
```

Out[160]=
$$\left\{\{Fa \to \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, Fb \to \{a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}\},\ Ga \to \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, Gb \to \{a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\},\ \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}\},\ \left\{a_1 \to \frac{16\,772\,211}{20\,000\,000}, a_2 \to \frac{10\,989}{20\,000\,000}, a_3 \to \frac{10\,989}{20\,000\,000}, a_4 \to \frac{1\,197\,801}{20\,000\,000}, a_5 \to \frac{1\,197\,801}{20\,000\,000}, a_6 \to \frac{5811}{20\,000\,000},\ a_7 \to \frac{999}{20\,000\,000}, a_8 \to \frac{999}{20\,000\,000}, a_9 \to \frac{999}{20\,000\,000}, a_{10} \to \frac{999}{20\,000\,000}, a_{11} \to \frac{798\,867}{20\,000\,000},\ a_{12} \to \frac{201}{20\,000\,000}, a_{13} \to \frac{201}{20\,000\,000}, a_{14} \to \frac{333}{20\,000\,000}, a_{15} \to \frac{333}{20\,000\,000}, a_{16} \to \frac{467}{20\,000\,000}\right\}\right\}$$

In[161]:=
```
EvaluateProbability[Pr[(Fa ⊃ Ga) ⋀ (Fb ⊃ Gb) | Fa ⋀ Ga], MNC] // N
```

Out[161]= 0.8995

In[162]:=
```
EvaluateProbability[Pr[(Fa ⊃ Ga) ⋀ (Fb ⊃ Gb)], MNC] // N
```

Out[162]= 0.998491

In[163]:=
```
cons = And @@ AlgebraicForm[{Pr[Fa ⋀ Ga ⋀ Fb ⋀ Gb] == s1, Pr[Fa ⋀ Ga ⋀ Fb ⋀ ¬ Gb] == s2,
      Pr[Fa ⋀ Ga ⋀ ¬ Fb ⋀ Gb] == s3, Pr[Fa ⋀ Ga ⋀ ¬ Fb ⋀ ¬ Gb] == s4,
      Pr[Fa ⋀ ¬ Ga ⋀ Fb ⋀ Gb] == s5, Pr[Fa ⋀ ¬ Ga ⋀ Fb ⋀ ¬ Gb] == s6,
      Pr[Fa ⋀ ¬ Ga ⋀ ¬ Fb ⋀ Gb] == s7, Pr[Fa ⋀ ¬ Ga ⋀ ¬ Fb ⋀ ¬ Gb] == s8,
      Pr[¬ Fa ⋀ Ga ⋀ Fb ⋀ Gb] == s9, Pr[¬ Fa ⋀ Ga ⋀ Fb ⋀ ¬ Gb] == s10,
      Pr[¬ Fa ⋀ Ga ⋀ ¬ Fb ⋀ Gb] == s11, Pr[¬ Fa ⋀ Ga ⋀ ¬ Fb ⋀ ¬ Gb] == s12,
      Pr[¬ Fa ⋀ ¬ Ga ⋀ Fb ⋀ Gb] == s13, Pr[¬ Fa ⋀ ¬ Ga ⋀ Fb ⋀ ¬ Gb] == s14,
      Pr[¬ Fa ⋀ ¬ Ga ⋀ ¬ Fb ⋀ Gb] == s15, Pr[¬ Fa ⋀ ¬ Ga ⋀ ¬ Fb ⋀ ¬ Gb] == s16}, {Fa, Fb, Ga, Gb}];
FindMaherModel[props_] := FindInstance[
   0 < γF < 1 && 0 < γG < 1 && AlgebraicForm[(props && cons), {Fa, Fb, Ga, Gb}] //. map, {γF, γG}];
```

In[165]:=
```
FindMaherModel[Pr[(Fa ⊃ Ga) ⋀ (Fb ⊃ Gb) | Fa ⋀ Ga] < Pr[(Fa ⊃ Ga) ⋀ (Fb ⊃ Gb)]]
```

Out[165]=
$$\left\{\left\{\gamma_F \to \frac{1}{16}, \gamma_G \to \frac{1}{2}\right\}\right\}$$

In[166]:=
```
FindMaherModel[Pr[Fb ≡ Gb | Fa ⋀ Ga] < Pr[Fb ≡ Gb]]
```

Out[166]=
$$\left\{\left\{\gamma_F \to \frac{1}{8}, \gamma_G \to \frac{1}{4}\right\}\right\}$$

In[167]:= **FindMaherModel[Pr[Gb | Fa ⋀ Ga] < Pr[Gb]]**

Out[167]= {}

In[168]:= **FindMaherModel[Pr[Fb ≡ Gb | Fa ≡ Ga] < Pr[Fb ≡ Gb]]**

Out[168]= {}

In[169]:= **FindMaherModel[Pr[Gb | Fa ≡ Ga] < Pr[Gb]]**

Out[169]= $\left\{\left\{\gamma_F \to \frac{1}{4}, \gamma_G \to \frac{1}{4}\right\}\right\}$

In[170]:= **FindMaherModel[Pr[Gb | (Fa ≡ Ga) ⋀ Fb] < Pr[Gb | Fb]]**

Out[170]= {}

In[171]:= **FindMaherModel[Pr[Fb ≡ Gb | Fa ⋀ Ga ⋀ Fb] < Pr[Fb ≡ Gb | Fb]]**

Out[171]= {}

In[172]:= **FindMaherModel[Pr[Gb | (Fa ≡ Ga) ⋀ ¬ Fb] < Pr[Gb | ¬ Fb]]**

Out[172]= {}

In[173]:= **FindMaherModel[Pr[Gb | (Fa ⋀ Ga) ⋀ ¬ Fb] < Pr[Gb | ¬ Fb]]**

Out[173]= {}

In[174]:= **FindMaherModel[Pr[Gb | Ga] > Pr[Gb | Ga ⋀ (Fa ⋀ ¬ Fb)] > Pr[Gb]]**

Out[174]= $\left\{\left\{\gamma_F \to \frac{1}{2}, \gamma_G \to \frac{1}{2}\right\}\right\}$

In[175]:= **FindMaherModel[Not[Pr[Gb | Ga] > Pr[Gb | Ga ⋀ (Fa ⋀ ¬ Fb)] > Pr[Gb]]]**

Out[175]= {}

■ Computing the Measure on the State Descriptions (full, 4 parameter case)

■ Finding Models in Maher's System (full, 4 parameter case)

```
x_ ⊃ y_ := ¬ x ⋁ y;
x_ ≡ y_ := (x ⊃ y) ⋀ (y ⊃ x);
map = {a16 → s1, a12 → s2, a13 → s5, a6 → s6, a14 → s3, a7 → s4, a8 → s7, a2 → s8,
  a15 → s9, a9 → s10, a10 → s13, a3 → s14, a11 → s11, a4 → s12, a5 → s15, a1 → s16};
MaherModel[f_, g_, l_, i_] := PrSAT[{Pr[Fa ⋀ Ga ⋀ Fb ⋀ Gb] == s1, Pr[Fa ⋀ Ga ⋀ Fb ⋀ ¬ Gb] == s2,
    Pr[Fa ⋀ Ga ⋀ ¬ Fb ⋀ Gb] == s3, Pr[Fa ⋀ Ga ⋀ ¬ Fb ⋀ ¬ Gb] == s4,
    Pr[Fa ⋀ ¬ Ga ⋀ Fb ⋀ Gb] == s5, Pr[Fa ⋀ ¬ Ga ⋀ Fb ⋀ ¬ Gb] == s6,
    Pr[Fa ⋀ ¬ Ga ⋀ ¬ Fb ⋀ Gb] == s7, Pr[Fa ⋀ ¬ Ga ⋀ ¬ Fb ⋀ ¬ Gb] == s8,
    Pr[¬ Fa ⋀ Ga ⋀ Fb ⋀ Gb] == s9, Pr[¬ Fa ⋀ Ga ⋀ Fb ⋀ ¬ Gb] == s10,
    Pr[¬ Fa ⋀ Ga ⋀ ¬ Fb ⋀ Gb] == s11, Pr[¬ Fa ⋀ Ga ⋀ ¬ Fb ⋀ ¬ Gb] == s12,
    Pr[¬ Fa ⋀ ¬ Ga ⋀ Fb ⋀ Gb] == s13, Pr[¬ Fa ⋀ ¬ Ga ⋀ Fb ⋀ ¬ Gb] == s14,
    Pr[¬ Fa ⋀ ¬ Ga ⋀ ¬ Fb ⋀ Gb] == s15, Pr[¬ Fa ⋀ ¬ Ga ⋀ ¬ Fb ⋀ ¬ Gb] == s16} //.
   {γF → f, γG → g, λ → l, ι → i}];
```

```
MNC = MaherModel[1 / 1000, 1 / 10, 2, 1 / 2]
```

$$\left\{\{\text{Fa} \to \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, \text{Fb} \to \{a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}\},\right.$$
$$\text{Ga} \to \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, \text{Gb} \to \{a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\},$$
$$\Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}\},$$
$$\left\{a_1 \to \frac{16\,772\,211}{20\,000\,000}, a_2 \to \frac{10\,989}{20\,000\,000}, a_3 \to \frac{10\,989}{20\,000\,000}, a_4 \to \frac{1\,197\,801}{20\,000\,000}, a_5 \to \frac{1\,197\,801}{20\,000\,000},\right.$$
$$a_6 \to \frac{5811}{20\,000\,000}, a_7 \to \frac{999}{20\,000\,000}, a_8 \to \frac{999}{20\,000\,000}, a_9 \to \frac{999}{20\,000\,000}, a_{10} \to \frac{999}{20\,000\,000}, a_{11} \to \frac{798\,867}{20\,000\,000},$$
$$\left.\left.a_{12} \to \frac{201}{20\,000\,000}, a_{13} \to \frac{201}{20\,000\,000}, a_{14} \to \frac{333}{20\,000\,000}, a_{15} \to \frac{333}{20\,000\,000}, a_{16} \to \frac{467}{20\,000\,000}\right\}\right\}$$

```
EvaluateProbability[Pr[(Fa ⊃ Ga) ⋀ (Fb ⊃ Gb) | Fa ⋀ Ga], MNC] // N
```

```
0.8995
```

```
EvaluateProbability[Pr[(Fa ⊃ Ga) ⋀ (Fb ⊃ Gb)], MNC] // N
```

```
0.998491
```

```
cons = And @@ AlgebraicForm[{Pr[Fa ⋀ Ga ⋀ Fb ⋀ Gb] == s1, Pr[Fa ⋀ Ga ⋀ Fb ⋀ ¬ Gb] == s2,
     Pr[Fa ⋀ Ga ⋀ ¬ Fb ⋀ Gb] == s3, Pr[Fa ⋀ Ga ⋀ ¬ Fb ⋀ ¬ Gb] == s4,
     Pr[Fa ⋀ ¬ Ga ⋀ Fb ⋀ Gb] == s5, Pr[Fa ⋀ ¬ Ga ⋀ Fb ⋀ ¬ Gb] == s6,
     Pr[Fa ⋀ ¬ Ga ⋀ ¬ Fb ⋀ Gb] == s7, Pr[Fa ⋀ ¬ Ga ⋀ ¬ Fb ⋀ ¬ Gb] == s8,
     Pr[¬ Fa ⋀ Ga ⋀ Fb ⋀ Gb] == s9, Pr[¬ Fa ⋀ Ga ⋀ Fb ⋀ ¬ Gb] == s10,
     Pr[¬ Fa ⋀ Ga ⋀ ¬ Fb ⋀ Gb] == s11, Pr[¬ Fa ⋀ Ga ⋀ ¬ Fb ⋀ ¬ Gb] == s12,
     Pr[¬ Fa ⋀ ¬ Ga ⋀ Fb ⋀ Gb] == s13, Pr[¬ Fa ⋀ ¬ Ga ⋀ Fb ⋀ ¬ Gb] == s14,
     Pr[¬ Fa ⋀ ¬ Ga ⋀ ¬ Fb ⋀ Gb] == s15, Pr[¬ Fa ⋀ ¬ Ga ⋀ ¬ Fb ⋀ ¬ Gb] == s16}, {Fa, Fb, Ga, Gb}];
FindMaherModel[props_] := FindInstance[0 < γF < 1 && 0 < γG < 1 && λ > 0 && 0 < ι < 1 &&
     AlgebraicForm[(props && cons), {Fa, Fb, Ga, Gb}] //. map, {γF, γG, λ, ι}];
```

```
FindMaherModel[Pr[(Fa ⊃ Ga) ⋀ (Fb ⊃ Gb) | Fa ⋀ Ga] < Pr[(Fa ⊃ Ga) ⋀ (Fb ⊃ Gb)]]
```

$$\left\{\left\{\gamma_F \to \frac{1}{8}, \gamma_G \to \frac{1}{4}, \lambda \to 2, \iota \to \frac{5}{8}\right\}\right\}$$

```
FindMaherModel[Pr[Fb ≡ Gb | Fa ⋀ Ga] < Pr[Fb ≡ Gb]]
```

$$\left\{\left\{\gamma_F \to \frac{1}{8}, \gamma_G \to \frac{1}{4}, \lambda \to 2, \iota \to \frac{1}{2}\right\}\right\}$$

```
FindMaherModel[Pr[Gb | Fa ⋀ Ga] < Pr[Gb]]
```

```
{}
```

```
FindMaherModel[Pr[Fb ≡ Gb | Fa ≡ Ga] < Pr[Fb ≡ Gb]]
```

```
{}
```

```
FindMaherModel[Pr[Gb | Fa ≡ Ga] < Pr[Gb]]
```

$$\left\{\left\{\gamma_F \to \frac{1}{4}, \gamma_G \to \frac{1}{4}, \lambda \to 2, \iota \to \frac{1}{2}\right\}\right\}$$

This one result changes. That is, conditional upon **Fb**, we can have **Fa ≡ Ga** negatively relevant to **Gb.**

```
FindMaherModel[Pr[Gb | (Fa ≡ Ga) ⋀ Fb] < Pr[Gb | Fb]]
```

$$\left\{\left\{\gamma_F \to \frac{1}{16}, \gamma_G \to \frac{1}{2}, \lambda \to 2, \iota \to \frac{13}{16}\right\}\right\}$$

```
FindMaherModel[Pr[Fb ≡ Gb | Fa ⋀ Ga ⋀ Fb] < Pr[Fb ≡ Gb | Fb]]
```

```
{}
```

Interestingly, conditional upon ***NOT*** Fb, both results *DO* hold. *This* is the salient point Goodman *should have* made. Now, the argument hinges completely on (RTE).

```
FindMaherModel[Pr[Gb | (Fa ≡ Ga) ⋀ ¬ Fb] < Pr[Gb | ¬ Fb]]
```

```
{}
```

```
FindMaherModel[Pr[Gb | Fa ⋀ Ga ⋀ ¬ Fb] < Pr[Gb | ¬ Fb]]
```

```
{}
```