

CHAPTER II

PROBABILITY IN RELATION TO THE THEORY OF KNOWLEDGE

1. I DO not wish to become involved in questions of epistemology to which I do not know the answer; and I am anxious to reach as soon as possible the particular part of philosophy or logic which is the subject of this book. But some explanation is necessary if the reader is to be put in a position to understand the point of view from which the author sets out; I will, therefore, expand some part of what has been outlined or assumed in the first chapter.

2. There is, first of all, the distinction between that part of our belief which is rational and that part which is not. If a man believes something for a reason which is preposterous or for no reason at all, and what he believes turns out to be true for some reason not known to him, he cannot be said to believe it *rationally*, although he believes it and it is in fact true. On the other hand, a man may rationally believe a proposition to be *probable*, when it is in fact false. The distinction between rational belief and mere belief, therefore, is not the same as the distinction between true beliefs and false beliefs. The highest degree of rational belief, which is termed *certain* rational belief, corresponds to *knowledge*. We may be said to know a thing when we have a certain rational belief in it, and *vice versa*. For reasons which will appear from our account of probable degrees of rational belief in the following paragraph, it is preferable to regard *knowledge* as fundamental and to define *rational belief* by reference to it.

3. We come next to the distinction between that part of our rational belief which is certain and that part which is only probable. Belief, whether rational or not, is capable of degree. The highest degree of rational belief, or rational certainty of

belief, and its relation to knowledge have been introduced above. What, however, is the relation to knowledge of *probable* degrees of rational belief?

The proposition (*say, q*) that we *know* in this case is not the same as the proposition (*say, p*) in which we have a probable degree (*say, a*) of rational belief. If the evidence upon which we base our belief is *h*, then what we *know*, namely *q*, is that the proposition *p* bears the probability-relation of degree *a* to the set of propositions *h*; and this knowledge of ours justifies us in a rational belief of degree *a* in the proposition *p*. It will be convenient to call propositions such as *p*, which do not contain assertions about probability-relations, "primary propositions"; and propositions such as *q*, which assert the existence of a probability-relation, "secondary propositions."¹

4. Thus knowledge of a proposition always corresponds to certainty of rational belief in it and at the same time to actual truth in the proposition itself. We cannot know a proposition unless it is in fact true. A probable degree of rational belief in a proposition, on the other hand, arises out of knowledge of some corresponding secondary proposition. A man may rationally believe a proposition to be probable when it is in fact false, if the secondary proposition on which he depends is true and certain; while a man cannot rationally believe a proposition to be probable even when it is in fact true, if the secondary proposition on which he depends is not true. Thus rational belief of whatever degree can only arise out of knowledge, although the knowledge may be of a proposition secondary, in the above sense, to the proposition in which the rational degree of belief is entertained.

5. At this point it is desirable to colligate the three senses in which the term *probability* has been so far employed. In its most fundamental sense, I think, it refers to the logical relation between two sets of propositions, which in § 4 of Chapter I. I have termed the probability-relation. It is with this that I shall be mainly concerned in the greater part of this Treatise. Derivative from this sense, we have the sense in which, as above, the term *probable* is applied to the degrees of rational belief arising out of knowledge of secondary propositions which assert the

¹ This classification of "primary" and "secondary" propositions was suggested to me by Mr. W. E. Johnson.

existence of probability-relations in the fundamental logical sense. Further it is often convenient, and not necessarily misleading, to apply the term *probable* to the proposition which is the object of the probable degree of rational belief, and which bears the probability-relation in question to the propositions comprising the evidence.

6. I turn now to the distinction between direct and indirect knowledge—between that part of our rational belief which we know directly and that part which we know by argument.

We start from things, of various classes, with which we have, what I choose to call without reference to other uses of this term, *direct acquaintance*. Acquaintance with such things does not in itself constitute knowledge, although knowledge arises out of acquaintance with them. The most important classes of things with which we have direct acquaintance are our own sensations, which we may be said to *experience*, the ideas or meanings, about which we have thoughts and which we may be said to *understand*, and facts or characteristics or relations of sense-data or meanings, which we may be said to *perceive*;—experience, understanding, and perception being three forms of direct acquaintance.

The objects of knowledge and belief—as opposed to the objects of direct acquaintance which I term sensations, meanings, and perceptions—I shall term *propositions*.

Now our knowledge of propositions seems to be obtained in two ways: directly, as the result of contemplating the objects of acquaintance; and indirectly, *by argument*, through perceiving the probability-relation of the proposition, about which we seek knowledge, to other propositions. In the second case, at any rate at first, what we know is not the proposition itself but a secondary proposition involving it. When we know a secondary proposition involving the proposition *p* as subject, we may be said to have indirect knowledge *about p*.

Indirect knowledge about *p* may in suitable conditions lead to rational belief in *p* of an appropriate degree. If this degree is that of certainty, then we have not merely indirect knowledge *about p*, but indirect knowledge *of p*.

7. Let us take examples of direct knowledge. From acquaintance with a sensation of yellow I can pass directly to a knowledge of the proposition “I have a sensation of yellow.” From acquaintance with a sensation of yellow and with the

meanings of “yellow,” “colour,” “existence,” I may be able to pass to a direct knowledge of the propositions “I understand the meaning of yellow,” “my sensation of yellow exists,” “yellow is a colour.” Thus, by some mental process of which it is difficult to give an account, we are able to pass from direct acquaintance with things to a knowledge of propositions about the things of which we have sensations or understand the meaning.

Next, by the contemplation of propositions of which we have direct knowledge, we are able to pass indirectly to knowledge of or about other propositions. The mental process by which we pass from direct knowledge to indirect knowledge is in some cases and in some degree capable of analysis. We pass from a knowledge of the proposition *a* to a knowledge about the proposition *b* by perceiving a logical relation between them. With this logical relation we have direct acquaintance. The logic of knowledge is mainly occupied with a study of the logical relations, direct acquaintance with which permits direct knowledge of the secondary proposition asserting the probability-relation, and so to indirect knowledge about, and in some cases of, the primary proposition.

It is not always possible, however, to analyse the mental process in the case of indirect knowledge, or to say by the perception of *what* logical relation we have passed from the knowledge of one proposition to knowledge about another. But although in some cases we *seem* to pass directly from one proposition to another, I am inclined to believe that in all legitimate transitions of this kind some logical relation of the proper kind must exist between the propositions, even when we are not explicitly aware of it. In any case, whenever we pass to knowledge about one proposition by the contemplation of it in relation to another proposition of which we have knowledge—even when the process is unanalysable—I call it an *argument*. The knowledge, such as we have in ordinary thought by passing from one proposition to another without being able to say what logical relations, if any, we have perceived between them, may be termed uncompleted knowledge. And knowledge, which results from a distinct apprehension of the relevant logical relations, may be termed knowledge proper.

8. In this way, therefore, I distinguish between direct and

indirect knowledge, between that part of our rational belief which is based on direct knowledge and that part which is based on argument. About what *kinds* of things we are capable of knowing propositions directly, it is not easy to say. About our own existence, our own sense-data, some logical ideas, and some logical relations, it is usually agreed that we have direct knowledge. Of the law of gravity, of the appearance of the other side of the moon, of the cure for phthisis, of the contents of Bradshaw, it is usually agreed that we do *not* have direct knowledge. But many questions are in doubt. Of *which* logical ideas and relations we have direct acquaintance, as to whether we can ever know directly the existence of *other people*, and as to when we are knowing propositions about sense-data directly and when we are interpreting them—it is not possible to give a clear answer. Moreover, there is another and peculiar kind of derivative knowledge—by memory.

At a given moment there is a great deal of our knowledge which we know neither directly nor by argument—we remember it. We may remember it as knowledge, but forget how we originally knew it. What we once knew and now consciously remember, can fairly be called knowledge. But it is not easy to draw the line between conscious memory, unconscious memory or habit, and pure instinct or irrational associations of ideas (acquired or inherited)—the last of which cannot fairly be called knowledge, for unlike the first two it did not even arise (in us at least) out of knowledge. Especially in such a case as that of what our eyes tell us, it is difficult to distinguish between the different ways in which our beliefs have arisen. We cannot always tell, therefore, what is remembered knowledge and what is not knowledge at all; and when knowledge is remembered, we do not always remember at the same time whether, originally, it was direct or indirect.

Although it is with knowledge by argument that I shall be mainly concerned in this book there is one kind of direct knowledge, namely of secondary propositions, with which I cannot help but be involved. In the case of every argument, it is only directly that we can know the secondary proposition which makes the argument itself valid and rational. When we know something by argument this must be through direct acquaintance with some logical relation between the conclusion and the premiss.

In *all* knowledge, therefore, there is some direct element; and logic can never be made purely mechanical. All it can do is so to arrange the reasoning that the logical relations, which have to be perceived directly, are made explicit and are of a simple kind.

9. It must be added that the term *certainty* is sometimes used in a merely psychological sense to describe a state of mind without reference to the logical grounds of the belief. With this sense I am not concerned. It is also used to describe the highest degree of rational belief; and this is the sense relevant to our present purpose. The peculiarity of certainty is that knowledge of a secondary proposition involving certainty, together with knowledge of what stands in this secondary proposition in the position of evidence, leads to *knowledge of*, and not merely *about*, the corresponding primary proposition. Knowledge, on the other hand, of a secondary proposition involving a degree of probability lower than certainty, together with knowledge of the premiss of the secondary proposition, leads only to a *rational belief of the appropriate degree* in the primary proposition. The knowledge present in this latter case I have called knowledge *about* the primary proposition or conclusion of the argument, as distinct from knowledge *of* it.

Of probability we can say no more than that it is a lower degree of rational belief than certainty; and we may say, if we like, that it deals with degrees of certainty.¹ Or we may make probability the more fundamental of the two and regard certainty as a special case of probability, as being, in fact, the *maximum probability*. Speaking somewhat loosely we may say that, if our premisses make the conclusion certain, then it *follows* from the premisses; and if they make it very probable, then it very nearly follows from them.

It is sometimes useful to use the term “impossibility” as the negative correlative of “certainty,” although the former sometimes has a different set of associations. If *a* is certain, then the contradictory of *a* is impossible. If a knowledge of *a* makes *b* certain, then a knowledge of *a* makes the contradictory

¹ This view has often been taken, *e.g.*, by Bernoulli and, incidentally, by Laplace; also by Fries (see Czuber, *Entwicklung*, p. 12). The view, occasionally held, that probability is concerned with degrees of truth, arises out of a confusion between certainty and truth. Perhaps the Aristotelian doctrine that future events are neither true nor false arose in this way.

of b impossible. Thus a proposition is impossible with respect to a given premiss, if it is disproved by the premiss; and the relation of impossibility is the relation of minimum probability.¹

10. We have distinguished between rational belief and irrational belief and also between rational beliefs which are certain in degree and those which are only probable. Knowledge has been distinguished according as it is direct or indirect, according as it is of primary or secondary propositions, and according as it is of or merely *about* its object.

In order that we may have a rational belief in a proposition p of the degree of certainty, it is necessary that one of two conditions should be fulfilled—(i.) that we know p directly; or (ii.) that we know a set of propositions h , and also know some secondary proposition q asserting a certainty-relation between p and h . In the latter case h may include secondary as well as primary propositions, but it is a necessary condition that all the propositions h should be *known*. In order that we may have rational belief in p of a lower degree of probability than certainty, it is necessary that we know a set of propositions h , and also know some secondary proposition q asserting a probability-relation between p and h .

In the above account one possibility has been ruled out. It is assumed that we cannot have a rational belief in p of a degree less than certainty except through knowing a secondary proposition of the prescribed type. Such belief can only arise, that is to say, by means of the perception of some probability-relation. To employ a common use of terms (though one inconsistent with the use adopted above), I have assumed that all direct knowledge is certain. All knowledge, that is to say, which is obtained in a manner strictly direct by contemplation of the objects of acquaintance and without any admixture whatever of argument and the contemplation of the logical bearing of any other knowledge on this, corresponds to *certain* rational belief and not to a merely probable degree of rational belief. It is true that there do *seem* to be degrees of knowledge and rational belief, when the source of

¹ Necessity and Impossibility, in the senses in which these terms are used in the theory of Modality, seem to correspond to the relations of Certainty and Impossibility in the theory of probability, the other modals, which comprise the intermediate degrees of possibility, corresponding to the intermediate degrees of probability. Almost up to the end of the seventeenth century the traditional treatment of modals is, in fact, a primitive attempt to bring the relations of probability within the scope of formal logic.

the belief is solely in acquaintance, as there are when its source is in argument. But I think that this appearance arises partly out of the difficulty of distinguishing direct from indirect knowledge, and partly out of a confusion between *probable* knowledge and *vague* knowledge. I cannot attempt here to analyse the meaning of vague knowledge. It is certainly not the same thing as knowledge proper, whether certain or probable, and it does not seem likely that it is susceptible of strict logical treatment. At any rate I do not know how to deal with it, and in spite of its importance I will not complicate a difficult subject by endeavouring to treat adequately the theory of vague knowledge.

I assume then that only true propositions can be known, that the term "probable knowledge" ought to be replaced by the term "probable degree of rational belief," and that a probable degree of rational belief cannot arise directly but only as the result of an argument, out of the knowledge, that is to say, of a secondary proposition asserting some logical probability-relation in which the object of the belief stands to some known proposition. With arguments, if they exist, the *ultimate* premisses of which are known in some other manner than that described above, such as might be called "probable knowledge," my theory is not adequate to deal without modification.¹

For the objects of certain belief which is based on direct knowledge, as opposed to certain belief arising indirectly, there is a well-established expression; propositions, in which our rational belief is both certain and direct, are said to be *self-evident*.

11. In conclusion, the relativity of knowledge to the individual may be briefly touched on. Some part of knowledge—knowledge of our own existence or of our own sensations—is clearly relative to individual experience. We cannot speak of knowledge absolutely—only of the knowledge of a particular person. Other parts of knowledge—knowledge of the axioms of logic, for example—may seem more objective. But we must admit, I think, that this too is relative to the constitution of the human mind, and that the constitution of the human mind may vary in some degree from man to man. What is self-evident to me and what

¹ I do not mean to imply, however, at any rate at present, that the ultimate premisses of an argument need always be *primary* propositions.

I really know, may be only a probable belief to you, or may form no part of your rational beliefs at all. And this may be true not only of such things as *my* existence, but of some logical axioms also. Some men—indeed it is obviously the case—may have a greater power of logical intuition than others. Further, the difference between some kinds of propositions over which human intuition seems to have power, and some over which it has none, may depend wholly upon the constitution of our minds and have no significance for a perfectly objective logic. We can no more assume that all true secondary propositions are or ought to be universally known than that all true primary propositions are known. The perceptions of some relations of probability may be outside the powers of some or all of us.

What we know and what probability we can attribute to our rational beliefs is, therefore, subjective in the sense of being relative to the individual. But given the body of premisses which our subjective powers and circumstances supply to us, and given the kinds of logical relations, upon which arguments can be based and which we have the capacity to perceive, the conclusions, which it is rational for us to draw, stand to these premisses in an objective and wholly logical relation. Our logic is concerned with drawing conclusions by a series of steps of certain specified kinds from a *limited* body of premisses.

With these brief indications as to the relation of Probability, as I understand it, to the Theory of Knowledge, I pass from problems of ultimate analysis and definition, which are not the primary subject matter of this book, to the logical theory and superstructure, which occupies an intermediate position between the ultimate problems and the applications of the theory, whether such applications take a generalised mathematical form or a concrete and particular one. For this purpose it would only encumber the exposition, without adding to its clearness or its accuracy, if I were to employ the perfectly exact terminology and minute refinements of language, which are necessary for the avoidance of error in very fundamental enquiries. While taking pains, therefore, to avoid any divergence between the substance of this chapter and of those which succeed it, and to employ only such periphrases as could be translated, *if desired*, into perfectly exact language, I shall not cut myself off from the convenient, but looser, expressions, which have been habitually employed

by previous writers and have the advantage of being, in a general way at least, immediately intelligible to the reader.¹

¹ This question, which faces all contemporary writers on logical and philosophical subjects, is in my opinion much more a question of *style*—and therefore to be settled on the same sort of considerations as other such questions—than is generally supposed. There are occasions for very exact methods of statement, such as are employed in Mr. Russell's *Principia Mathematica*. But there are advantages also in writing the English of Hume. Mr. Moore has developed in *Principia Ethica* an intermediate style which in his hands has force and beauty. But those writers, who strain after exaggerated precision without going the whole hog with Mr. Russell, are sometimes merely pedantic. They lose the reader's attention, and the repetitious complication of their phrases eludes his comprehension, without their really attaining, to compensate, a complete precision. Confusion of thought is not always best avoided by technical and unaccustomed expressions, to which the mind has no immediate reaction of understanding; it is possible, under cover of a careful formalism, to make statements, which, if expressed in plain language, the mind would immediately repudiate. There is much to be said, therefore, in favour of understanding the substance of what you are saying *all the time*, and of never reducing the substantives of your argument to the mental status of an *x* or *y*.