§ 87. Hempel's Analysis of the Concept of Confirming Evidence

Some interesting investigations by Hempel concerning the concept of confirming evidence are here discussed. Hempel shows correctly that two widespread conceptions are too narrow: Nicod's criterion (the law 'all swans are white' is confirmed by observations of white swans and only by these) and the prediction-criterion (a hypothesis is confirmed by given evidence if and only if one part of this evidence can be deduced from the other part with the help of the hypothesis). Hempel lays down some general conditions which, in his view, a concept must fulfil in order to be an adequate explicatum of the concept of confirming evidence. It is shown that some of these conditions are not valid, that is to say, no adequate explicatum can fulfil them.

In this and the next sections we shall discuss investigations made by Carl G. Hempel concerning confirmation in general and especially the classificatory concept. The following discussion is chiefly based on an article of his published in two parts in *Mind* 1945 ([Studies]; references in the following are to this article); some of his technical results had been published previously ([Syntactical], 1943). The first-mentioned article gives a clear and illuminating exposition of the whole problem situation concerning confirmation and the distinction between the classificatory, the comparative, and the quantitative concepts of confirmation. A number of points in this problem complex are here clarified for the first time. For instance, Hempel's distinction between the pragmatical concept of the confirmation of a hypothesis by an observer and the logical (semantical) concept of the confirmation of a hypothesis on the basis of an evidence sentence is important; likewise his distinction of the three phases in the procedure of testing a given hypothesis (*op. cit.*, p. 114): making observations, confronting the hypothesis with the observation report, accepting or rejecting the hypothesis. These distinctions are valuable tools for clarifying the situation for many discussions and controversies at the present time concerning confirmation, the foundations of empiricism, verifiability, and related problems.

The main part of Hempel's article concerns the problem of an explication for the classificatory concept of confirmation. We shall now discuss his views in detail. His explicandum is as follows: a sentence (or a class of sentences, or perhaps an individual) represents confirming (corroborating, favorable) evidence or constitutes a confirming instance for a given hypothesis. In his general discussion and in the examples, no reference is made to any *prior* evidence. Thus Hempel's explicandum corresponds to our dyadic relation $\mathfrak{C}_0(h,i)$ ('h is confirmed by i') rather than to the triadic relation $\mathfrak{C}(h,i;e)$ ('h is confirmed by i on the basis of the prior evidence e'). Therefore we shall in the following compare the explicata discussed by Hempel with $\mathfrak{C}_0$.

Hempel starts with a critical discussion of an explicatum which seems widely accepted (*op. cit.*, pp. 9 ff.); he quotes the following passages by Jean *Nicod* as a clear formulation for it: "Consider the formula or the law: *A entails B*. How can a particular proposition, or more briefly, a fact, affect its probability? If this fact consists of the presence of B in a case of A, it is favourable to the law '*A entails B*'; on the contrary, if it consists of the absence of B in a case of A, it is unfavourable to this law. It is conceivable that we have here the only two direct modes in which a fact can influence the probability of a law. . . . Thus, the entire influence of particular truths or facts on the probability of universal propositions or laws would operate by means of these two elementary relations which we shall call *confirmation* and *invalidation*" ([Induction], p. 219). Hempel refers here also to R. M. Eaton's discussion on "Confirmation and Infirmation" ([Logic], chap. iii), which is based on Nicod's conception. Thus, according to Nicod's criterion, the fact that the individual b is both M and M', or the sentence '$Mb \,.\, M'b$' describing this fact, is confirming evidence for the law '$(x)(Mx \supset M'x)$'. Hempel discusses this criterion in detail, and I agree entirely with his views. As he points out, the criterion is applicable only to a quite special, though important, form of hypothesis. But even if restricted to this form, the criterion does not constitute a necessary condition; in other words, it is clearly too narrow (in the sense of § 86). Hempel shows that it is not in accord with the Equivalence Condition for Hypotheses (see below, H8.22). For instance, '$Mb \,.\, M'b$' is confirming evidence, according to Nicod's criterion, for the law stated above, but not for the L-equivalent law '$(x)(\sim M'x \supset \sim Mx)$'. This is an instance of what Hempel calls the *paradox of confirmation.* He discusses this paradox in detail and reveals its main sources (*op. cit.*, pp. 13–21). [We have briefly indicated this paradox earlier (§ 46) and we shall discuss it later (in Vol. II) in connection with the universal inductive inference; we shall try to throw some light on the problem from the point of view of our inductive logic; our results will essentially be in agreement with Hempel's views.] Nicod's criterion may be taken as a sufficient condition for the concept of confirming evidence if it is restricted to laws of the form mentioned with only *one* variable. That in the case of laws with several variables it is not even sufficient is shown by Hempel with the help of the following counterexample (*op. cit.*, p. 13 n.), which is interesting and quite surprising. Let the hypothesis be the law '$(x)(y)[\sim(Rxy \,.\, Ryx) \supset (Rxy \,.\, \sim Ryx)]$'. [Incidentally, by an unfortunate misprint in the footnote mentioned, the second conjunctive component in the antecedent was omitted.] Now the fact described by '$Rab \,.\, \sim Rba$' fulfils both the antecedent and the consequent in the law; hence this fact should be taken as a confirming case according to Nicod's criterion. However, since the law stated is L-equivalent to '$(x)(y)Rxy$', the fact mentioned is actually disconfirming.

Hempel proposes (p. 22) to take the concept of confirming evidence not, like Nicod, as a relation between an object or fact and a sentence, but as a semantical relation—or, alternatively, a syntactical (i.e., purely formal) relation—between two sentences, as we do with $\mathfrak{C}_0$ (and c). A language system L is presupposed. The primitive predicates in L designate directly observable properties or relations. An observation sentence is a basic sentence (atomic sentence or negation, D16-6b) in L. An observation report in the narrower sense is a class or conjunction of a finite number of observation sentences (*op. cit.*, p. 23); an observation report in the wider sense is any nongeneral sentence. We shall henceforth use the term in the wider sense. [Hempel uses the wider sense in the more technical paper [Syntactical], p. 126. In the text of [Studies] he uses the narrower sense, but he mentions the wider sense in footnotes (pp. 108, 111) and declares that the narrower sense was used in the text only for greater convenience of exposition and that all results, definitions, and theorems remain applicable if the wider sense is adopted. Thus our use of the wider sense is justified; it will facilitate the construction of some examples.] Hempel admits also contradictory sentences as observation reports (p. 103, footnote 1); however, we shall exclude them, in accord with our general requirement that the evidence referred to by any confirmation concept be non-L-false. (This requirement was later accepted by Hempel [Degree], p. 102; our exclusion here will not affect the results of the subsequent discussion of Hempel's views.) Hempel restricts the evidence e referred to by the concept of confirmation to observation reports, but the hypothesis h may be any sentence of the language L. The structure of L is similar to that of our systems $\mathfrak{L}$ except that L does not contain a sign of identity.

Hempel makes (*op. cit.*, pp. 97 ff.) a critical examination of another explicatum of the concept of confirming evidence, which is often used at least implicitly and which at first glance appears as quite plausible. This explicatum, which Hempel calls the *prediction-criterion* of confirmation, is based on the consideration that it is customary to regard a hypothesis as confirmed if a prediction made with its help is borne out by the facts. This consideration suggests the following definition: An observation report $\mathfrak{K}_i$ confirms the hypothesis $h =_{Df}$ $\mathfrak{K}_i$ can be divided into two mutually exclusive subclasses $\mathfrak{K}_{i1}$ and $\mathfrak{K}_{i2}$ such that $\mathfrak{K}_{i2}$ is not empty, and every sentence of $\mathfrak{K}_{i2}$ can be logically deduced from (i.e., is L-implied by) $\mathfrak{K}_{i1}$ together with h but not from $\mathfrak{K}_{i1}$ alone. Hempel shows that this concept is indeed a sufficient condition for the explicatum sought, but not a necessary condition; in other words, it is not too wide, but it is clearly too narrow. The chief reason is the obvious fact that most scientific hypotheses do not simply express a conditional connection between observable properties but have a more general and often more complex form. This is illustrated by the simple example of the sentence '$(x)[(y)R_1xy \supset (\exists z)R_2xz]$' in an infinite universe, where R_1 and R_2 are observable relations. If we take any instance of this universal sentence, say with 'b' for 'x', then we see that the antecedent (i.e., '$(y)R_1by$') is not L-implied by any finite class of observation sentences, and that the consequent (i.e., '$(\exists z)R_2bz$') does not L-imply any observation sentence. This shows that it is "a considerable over-simplification to say that scientific hypotheses and theories enable us to derive predictions of future experiences from descriptions of past ones" (p. 100). The logical connection which a scientific hypothesis establishes between observation reports is in general not merely of a deductive kind; it is rather a combination of deductive and nondeductive steps. The latter are inductive in one wide sense of this word; Hempel calls them 'quasi-inductive'.

After these discussions of Nicod's criterion and the prediction-criterion resulting in the rejection of both explicata as too narrow, Hempel proceeds to the positive part of his discussion. He states a number of general conditions for the adequacy of any explicatum for the concept of confirming evidence (pp. 102 ff.); we shall discuss them in the present section. Then he defines his own explicatum and shows that it fulfils the conditions of adequacy; this will be discussed in the next section. Hempel's *conditions of adequacy* are as follows ('H' is here attached to his numbers); the evidence e is always an observation report as explained earlier, while the hypothesis h may be any sentence of the language L.

(H8.1) *Entailment Condition:* If h is entailed by e (i.e., $\vdash e \supset h$), then e confirms h.

(H8.2) *Consequence Condition:* If e confirms every sentence of the class $\mathfrak{K}_i$ and h is a consequence of (i.e., L-implied by) $\mathfrak{K}_i$, then e confirms h.

The following two more special conditions follow from H8.2.

(H8.21) *Special Consequence Condition:* If e confirms h, then it also confirms every consequence of h (i.e., sentence L-implied by h).

(H8.22) *Equivalence Condition for Hypotheses:* If h and h' are L-equivalent and e confirms h then e confirms h'.

(H8.3) *Consistency Condition:* The class whose elements are e and all the hypotheses confirmed by e is consistent (i.e., not L-false).

The following two more special conditions follow from H8.3.

(H8.31) If e and h are incompatible (i.e., L-exclusive, $e \cdot h$ is L-false), then e does not confirm h.

(H8.32) If h and h' are incompatible (i.e., L-exclusive), then e does not confirm both h and h'.

(H8.4) *Equivalence Condition for Observation Reports* (*op. cit.*, p. 110 n.): If e and e' are L-equivalent and e confirms h, then e' confirms h.

Now we shall examine these conditions of adequacy stated by Hempel. We interpret these conditions as referring to the concept of initial confirming evidence as explicandum; we shall soon come back to the question whether Hempel has not sometimes a different explicandum in mind. Thus we shall apply the conditions to $\mathfrak{C}_o$; but when we accept one of them, we shall state not only a condition (b) for $\mathfrak{C}_o$, but first a more general condition (a) for $\mathfrak{C}$; (b) is then a special case of (a) with 't' for 'e'. It is presupposed for (a) that $e \cdot i$ is non-L-false, because otherwise $\mathfrak{c}(h, e \cdot i)$ would have no value and hence the subsequent condition (2) could not be applied; and it is presupposed for (b) that e is not L-false. Our statements of conditions will have the same numbers as Hempel's but with 'C' instead of 'H'. For this discussion we remember that we found that $\mathfrak{C}$ is the same as positive relevance and $\mathfrak{C}_o$ the same as initial positive relevance; therefore we shall make use of the results concerning relevance concepts stated in the preceding chapter. Our examination will be based on the view that any adequate explicatum for the classificatory concept of confirmation must be in accord with at least one adequate explicatum for the quantitative concept of confirmation; in other words, a relation $\mathfrak{C}_o$ proposed as explicatum cannot be accepted as adequate unless there is at least one c-function $\mathfrak{c}$, which is an adequate explicatum for probability$_1$, such that, if $\mathfrak{C}_o(h,i)$ then

(1) $$\mathfrak{c}(h,i) > \mathfrak{c}(h,t)\ .$$

Analogously, it is necessary for the adequacy of a proposed explicatum $\mathfrak{C}$ that there is at least one adequate $\mathfrak{c}$ such that, if $\mathfrak{C}(h,i,e)$, then

(2) $$\mathfrak{c}(h, e \cdot i) > \mathfrak{c}(h,e)\ .$$

In examining Hempel's statements of conditions of adequacy or our subsequent statements, we shall regard such a statement as valid if there is at least one explicatum $\mathfrak{C}_o$ (or $\mathfrak{C}$) which is adequate in the sense just

explained, i.e., in accord with an adequate c-function, and which satisfies the statement generally, i.e., for any sentences as arguments.

The *entailment condition* H8.1 may appear at first glance as quite plausible. And it is indeed valid in ordinary cases. However, it does not hold in some special cases as we shall see by the subsequent counterexamples. Therefore we restate it in the following qualified form.

(C8.1) *Entailment Condition.* Let h be either a sentence in a finite system or a nongeneral sentence in the infinite system.

a. If $\vdash e \cdot i \supset h$ and not $\vdash e \supset h$, then $\mathfrak{C}(h,i,e)$.

b. If $\vdash i \supset h$ and h is not L-true, then $\mathfrak{C}_o(h,i)$.

The following theorem shows that the entailment condition in the modified form C8.1 is valid.

T87-1.

a. Any instance of the relation $\mathfrak{C}$ which is required by C8.1a is in accord with every regular c-function.

Proof. Let $\vdash e \cdot i \supset h$ and not $\vdash e \supset h$. It was presupposed that $e \cdot i$ is not L-false. Therefore, for every regular $\mathfrak{c}$, $\mathfrak{c}(h, e \cdot i) = 1$ (T59-1b) and $\mathfrak{c}(h,e) < 1$ (T59-5a). Thus this instance of $\mathfrak{C}$ is in accord with $\mathfrak{c}$ ((3) in § 86).

b. Any instance of $\mathfrak{C}_o$ required by C8.1b is in accord with every regular c-function. (From (a), with 't' for e.)

In C8.1a, we have excluded the case that $\vdash e \supset h$. This restriction is necessary, because in this case $\mathfrak{c}(h,e) = 1 = \mathfrak{c}(h, e \cdot i)$; hence $\mathfrak{c}$ is not increased. For the same reason, the case that h is L-true must be excluded in C8.1b.

For the sake of simplicity, we have stated C8.1 only for the case that h is a sentence in a finite system or a nongeneral sentence in the infinite system. However, C8.1 is valid also if h is a general sentence in the infinite system except in the case where h is almost L-implied by e (D58-1c) with respect to any of the c-functions on which $\mathfrak{C}$ is based. In the latter case, $\mathfrak{c}(h,e) = 1$ although not $\vdash e \supset h$; thus here again $\mathfrak{c}$ is not increased and hence i is not positively relevant. (This holds if positive relevance is defined by D65-1a; see, however, the subsequent remark concerning the alternative definition D'.)

We considered in § 65 the following *example* in the infinite system: h is '$(\exists x)Px$', i is 'Pb', 't' is taken as e. We mentioned that, for certain c-functions, e.g., $\mathfrak{c}^*$, h is almost L-true. Although in every finite system the $\mathfrak{c}$ of h on 't' is increased by the addition of i, in the infinite system $\mathfrak{c}(h,t)$ is already 1 and hence is not increased by the addition of i. Therefore i is here irrelevant to h; i is not confirming evidence for h.

Cases of the kind of this example suggested the alternative definition (D') for positive relevance indicated in § 65. If this alternative definition is chosen, then in cases like the above example i is called positive to h and hence is regarded as confirming evidence for h. Then the restriction of h to nongeneral sentences in the infinite system in C8.1 can be omitted. (But the restricting conditions in (a) that not $\vdash e \supset h$ and in (b) that h is not L-true remain.)

The *equivalence conditions* for hypotheses (H8.22) and for observation reports (H8.4) are obviously valid, because the corresponding principles hold for all regular c-functions (T59-1i and h). For $\mathfrak{C}$, the former condition can be generalized; the hypotheses h and h' need only be L-equivalent with respect to e, i.e., $\vdash e \supset (h \equiv h')$ (cf. T59-2j).

The *consequence condition* H8.2 and the special consequence condition H8.21 are not valid, as we shall see. In his discussion of H8.21, Hempel refers (p. 105, n. 1) to William Barrett ([Dewey], p. 312), whose view that "not every observation which confirms a sentence need also confirm all its consequences" is obviously in contradiction to the consequence condition. Barrett supports his view by pointing to "the simplest case: the sentence 'C' is an abbreviation of '$A \cdot B$', and the observation O confirms 'A', and so 'C', but is irrelevant to 'B', which is a consequence of 'C' ". This situation can indeed occur, as we shall see; thus Barrett is right in rejecting the consequence condition. Now Hempel points out that Barrett, in the phrase "and so 'C' " just quoted, seems to presuppose tacitly the *converse consequence condition:* if e confirms h, then it confirms also any sentence of which h is a consequence. Hempel shows correctly that a simultaneous requirement of both the consequence condition and the converse consequence condition would immediately lead to the absurd result that any observation report e confirms any hypothesis h (because e confirms e, hence $e \cdot h$, hence h). Since he accepts the consequence condition, he rejects the converse consequence condition. On the other hand, Barrett, accepting the latter, rejects the former. Each of the two incompatible conditions has a certain superficial plausibility. Which of them is valid? The answer is, neither.

In our investigation of the possible relevance situations for two hypotheses (§§ 70, 71) we found the following results, which hold for all regular c-functions. It is possible that, on the same evidence e, which may be factual or tautological, i is positive to h but negative to $h \vee k$, although the latter is L-implied by the former. This is possible not only if i is negative to k but also if i is irrelevant or even positive to k (§ 71, case 4a). We have indicated there a general procedure for constructing cases of this kind, and given a numerical example (§ 71, example for 4a). This shows that *the consequence condition is not valid,* that is, not in accord with any regular c-function. We have further found that it is possible that i is positive to h but negative to $h \cdot k$, although the latter L-implies the former. This is possible even if i is positive to k (§ 71, case 3a). Here likewise a general construction procedure has been indicated and a numerical example given (§ 71, example for 3a). This shows that *the converse consequence condition is not valid.*

A remark made by Hempel in his discussion of Barrett is interesting because it throws some light on the reasoning which led Hempel to the consequence condition. Hempel quotes Barrett's statement that "the degree of confirmation for the consequence of a sentence cannot be less than that of the sentence itself". This statement is correct; it does indeed hold for every regular c-function (T59-2d). Hempel agrees with this principle but regards it as incompatible with a renunciation of the special consequence condition, "since the latter may be considered simply as the correlate, for the non-gradated [i.e., classificatory] relation of confirmation, of the former principle which is adapted to the concept of degree of confirmation". This seems to show that here Hempel has in mind as explicandum the following relation: 'the degree of confirmation of h on i is greater than r', where r is a fixed value, perhaps 0 or 1/2. This interpretation seems indicated also by another remark which Hempel makes in support of the consequence condition: "An observation report which confirms certain hypotheses would invariably be qualified as confirming any consequence of those hypotheses. Indeed: any such consequence is but an assertion of all or part of the combined content of the original hypotheses and has therefore to be regarded as confirmed by any evidence which confirms the original hypotheses" (p. 103). This reasoning may appear at first glance quite plausible; but this is due, I think, only to the inadvertent transition to the explicandum mentioned above. This relation, however, is not the same as our original explicandum, the classificatory concept of confirmation as used, for instance, by a scientist when he says something like this: 'The result of the experiment just made supplies confirming evidence for my hypothesis'. Hempel's general discussions give the impression that he too is originally thinking of this explicandum, when he refers to favorable and unfavorable data, both of which are regarded as relevant and distinguished from irrelevant data, and when he speaks of given evidence as strengthening or weakening a given hypothesis. The difference between the two explicanda is easily seen as follows. Let r be a fixed value. The result that the degree of confirmation of h after the observation i is $q > r$ does not by itself show that i furnishes a positive contribution to the confirmation of h; for it may be that the prior degree of confirmation of h (i.e., before the observation i) was already q, in which case i is irrelevant; or it may have been even greater than q, in which case i is negative. [Example. Let h be '$P_1b \vee P_2b$', and i 'P_3a'. Take $r = 1/2$. For many $\mathfrak{c}$-functions $\mathfrak{c}(h,i) = \mathfrak{c}(h,t) = 3/4$. Therefore i is (initially) irrelevant to h, although $\mathfrak{c}(h,i) > 1/2$.] And, the other way round, the result that the posterior degree of confirmation of h is higher than the prior one does not necessarily make it higher than r (unless $r = 0$). Thus we see that the essential criterion for the concept of confirming evidence must take into account not simply the posterior degree of confirmation but rather a comparison between this and the prior one.

The consistency condition H8.3 is not valid; it seems to me not even plausible. The special condition H8.31, requiring compatibility of the hypothesis with the evidence, is certainly valid. We restate it here in the general form as Compatibility Condition:

(C8.31) *Compatibility Condition.*

a. If i and h are L-exclusive with respect to e, that is, if $e \cdot i \cdot h$ is L-false, then not $\mathfrak{C}(h,i,e)$.

b. If i and h are L-exclusive, that is, if $i \cdot h$ is L-false, then not $\mathfrak{C}_0(h,i)$.

The following theorem shows that C8.31 is valid, no matter on which $\mathfrak{c}$-function or class of $\mathfrak{c}$-functions $\mathfrak{C}$ is based.

T87-2.

a. If a relation $\mathfrak{C}$ holds in any instance excluded by C8.31a, then it is not in accord with any regular $\mathfrak{c}$-function.

Proof. Let $e \cdot i \cdot h$ be L-false. Then, for every regular $\mathfrak{c}$, $\mathfrak{c}(h,e \cdot i) = 0$ (T59-1e), hence not $> \mathfrak{c}((h,e)$.

b. If a relation $\mathfrak{C}_0$ holds in any instance excluded by C8.31b, then it is not in accord with any regular $\mathfrak{c}$-function. (From (a), with 't' for e.)

On the other hand, the second special condition H8.32 seems to me invalid. Hempel himself shows that a set of physical measurements may confirm several quantitative hypotheses which are incompatible with each other (p. 106). This seems to me a clear refutation of H8.32. Hempel discusses possibilities of weakening or omitting this requirement, but he decides at the end to maintain it unchanged, without saying how he intends to overcome the difficulty which he has pointed out himself. Perhaps he thinks that he may leave aside this difficulty because the results of physical measurements cannot be formulated in the simple language L to which his analysis applies. However, it seems to me that there are similar but simpler counterexamples which can be formulated in our systems $\mathfrak{L}$ and in Hempel's system L. For instance, let i describe the frequency of

a property M in a finite population, and h and h' state two distinct values m and m' for the frequency of M in a sample of s individuals belonging to the population, such that the relative frequencies m/s and m'/s are both near to the relative frequency of M in the population as stated in i. Then i confirms both h and h', although they are incompatible with each other.

Example. Let i be a statistical distribution (D26-6c) for M and non-M with respect to 10,000 individuals with the cardinal number 8,000 for M. Let h be a statistical distribution with respect to 100 of these individuals with the cardinal number 80 for M, and similarly h' with respect to the same individuals and with the cardinal number 79. Note that a statistical distribution for a finite class has the form of a disjunction of conjunctions and does not contain variables or the sign of identity; therefore it occurs also in L and it is an observation report (in the wider sense). Let e be either the tautology 't' or a factual sentence irrelevant to h and to h' (on 't' and on i). Then for many $\mathfrak{c}$-functions (presumably including all adequate ones) $\mathfrak{c}(h,e \cdot i) > \mathfrak{c}(h,e)$ and $\mathfrak{c}(h',e \cdot i) > \mathfrak{c}(h',e)$. (These are cases of the direct inductive inference, see § 94.) Thus i is positively relevant and hence constitutes confirming evidence for both h and h'.

Hempel mentions in this context still another condition, which might be called the *Conjunction Condition:* if e confirms each of two hypotheses, then it also confirms their conjunction (p. 106). Hempel seems to accept this condition; he regards any violation of it as "intuitively rather awkward". However, this condition is not valid for our explicandum; we have found earlier that i may be positive both to h and to k but negative to $h \cdot k$ (see § 71, case 3a and the example for it; this was mentioned above as a refutation of the converse consequence condition). And it is not valid for the second explicandum either, no matter which value we choose for r.

This is seen as follows. Let r be any real number such that $0 \leqq r < 1$. Let q be $(1 - r)/2$; hence $q > 0$. Let i say that in a given finite population the relative frequencies are as follows: for '$P_1 \cdot P_2$', r; for '$P_1 \cdot \sim P_2$', q; for '$P_2 \cdot \sim P_1$', q; hence for '$\sim P_1 \cdot \sim P_2$', 0; for 'P_1', $r + q$; for 'P_2', $r + q$. Let h be 'P_1b' and h' 'P_2b', where b belongs to the population. Then (as we shall see later, T94-1e) for every symmetrical $\mathfrak{c}$-function and hence for every adequate one, the following holds. $\mathfrak{c}(h,i) = \mathfrak{c}(h',i) = r + q > r$; on the other hand, $\mathfrak{c}(h \cdot h',i) = r$.

What may be the reasons which have led Hempel to the consistency conditions H8.32 and H8.3? He regards it as a great advantage of any explicatum satisfying H8.3 "that it sets a limit, so to speak, to the strength of the hypotheses which can be confirmed by given evidence", as was pointed out to him by Nelson Goodman. This argument does not seem to have any plausibility for *our* explicandum, because a weak additional evidence can cause an increase, though a small one, in the confirmation even of a very strong hypothesis. But it is plausible for the second explicandum mentioned earlier: the degree of confirmation exceeding a fixed value r. Therefore we may perhaps assume that Hempel's acceptance of the conmay have been even greater than q, in which case i is negative. [Example. Let h be '$P_1b \vee P_2b$', and i 'P_3a'. Take $r = 1/2$. For many c-functions $c(h,i) = c(h,t) = 3/4$. Therefore i is (initially) irrelevant to h, although $c(h,i) > 1/2$.] And, the other way round, the result that the posterior degree of confirmation of h is higher than the prior one does not necessarily make it higher than r (unless $r = 0$). Thus we see that the essential criterion for the concept of confirming evidence must take into account not simply the posterior degree of confirmation but rather a comparison between this and the prior one.

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The following theorem shows that C8.31 is valid, no matter on which c-function or class of c-functions $\mathfrak{C}$ is based.

T87-2.

a. If a relation $\mathfrak{C}$ holds in any instance excluded by C8.31a, then it is not in accord with any regular c-function.

Proof. Let $e \cdot i \cdot h$ be L-false. Then, for every regular c, $c(h,e \cdot i) = 0$ (T59-1e), hence not $> c((h,e)$.

b. If a relation $\mathfrak{C}_0$ holds in any instance excluded by C8.31b, then it is not in accord with any regular c-function. (From (a), with 't' for e.)

On the other hand, the second special condition H8.32 seems to me invalid. Hempel himself shows that a set of physical measurements may confirm several quantitative hypotheses which are incompatible with each other (p. 106). This seems to me a clear refutation of H8.32. Hempel discusses possibilities of weakening or omitting this requirement, but he decides at the end to maintain it unchanged, without saying how he intends to overcome the difficulty which he has pointed out himself. Perhaps he thinks that he may leave aside this difficulty because the results of physical measurements cannot be formulated in the simple language L to which his analysis applies. However, it seems to me that there are similar but simpler counterexamples which can be formulated in our systems $\mathfrak{L}$ and in Hempel's system L. For instance, let i describe the frequency of

a property M in a finite population, and h and h' state two distinct values m and m' for the frequency of M in a sample of s individuals belonging to the population, such that the relative frequencies m/s and m'/s are both near to the relative frequency of M in the population as stated in i. Then i confirms both h and h', although they are incompatible with each other.

Example. Let i be a statistical distribution (D26-6c) for M and non-M with respect to 10,000 individuals with the cardinal number 8,000 for M. Let h be a statistical distribution with respect to 100 of these individuals with the cardinal number 80 for M, and similarly h' with respect to the same individuals and with the cardinal number 79. Note that a statistical distribution for a finite class has the form of a disjunction of conjunctions and does not contain variables or the sign of identity; therefore it occurs also in L and it is an observation report (in the wider sense). Let e be either the tautology 't' or a factual sentence irrelevant to h and to h' (on 't' and on i). Then for many c-functions (presumably including all adequate ones) $c(h,e \cdot i) > c(h,e)$ and $c(h',e \cdot i) > c(h',e)$. (These are cases of the direct inductive inference, see § 94.) Thus i is positively relevant and hence constitutes confirming evidence for both h and h'.

Hempel mentions in this context still another condition, which might be called the *Conjunction Condition:* if e confirms each of two hypotheses, then it also confirms their conjunction (p. 106). Hempel seems to accept this condition; he regards any violation of it as "intuitively rather awkward". However, this condition is not valid for our explicandum; we have found earlier that i may be positive both to h and to k but negative to $h \cdot k$ (see § 71, case 3a and the example for it; this was mentioned above as a refutation of the converse consequence condition). And it is not valid for the second explicandum either, no matter which value we choose for r.

This is seen as follows. Let r be any real number such that $0 \leqq r < 1$. Let q be $(1 - r)/2$; hence $q > 0$. Let i say that in a given finite population the relative frequencies are as follows: for '$P_1 \cdot P_2$', r; for '$P_1 \cdot \sim P_2$', q; for '$P_2 \cdot \sim P_1$', q; hence for '$\sim P_1 \cdot \sim P_2$', 0; for 'P_1', $r + q$; for 'P_2', $r + q$. Let h be 'P_1b' and h' 'P_2b', where b belongs to the population. Then (as we shall see later, T94-1e) for every symmetrical c-function and hence for every adequate one, the following holds. $c(h,i) = c(h',i) = r + q > r$; on the other hand, $c(h \cdot h',i) = r$.

What may be the reasons which have led Hempel to the consistency conditions H8.32 and H8.3? He regards it as a great advantage of any explicatum satisfying H8.3 "that it sets a limit, so to speak, to the strength of the hypotheses which can be confirmed by given evidence", as was pointed out to him by Nelson Goodman. This argument does not seem to have any plausibility for *our* explicandum, because a weak additional evidence can cause an increase, though a small one, in the confirmation even of a very strong hypothesis. But it is plausible for the second explicandum mentioned earlier: the degree of confirmation exceeding a fixed value r. Therefore we may perhaps assume that Hempel's acceptance of the consistency condition is due again to an inadvertent shift to the second explicandum. This assumption seems corroborated by the following result. Although H8.32 is not valid for our explicandum, it is valid for the second explicandum if we take for r 1/2 or any greater value (<1). For if h and h' are L-exclusive, then it is impossible that $\mathfrak{c}(h,i)$ and $\mathfrak{c}(h',i)$ both exceed 1/2, because the sum of those two $\mathfrak{c}$-values is $\mathfrak{c}(h \vee h',i)$ (according to the special addition theorem, T59-1l), and hence cannot exceed 1.

§ 88. Hempel's Definition of Confirming Evidence

Hempel defines a concept *Cf* as an explicatum for confirming evidence, and he shows that *Cf* fulfils his conditions of adequacy, which we discussed in the preceding section. It is found that *Cf* is too narrow as an explicatum for the general concept of confirming evidence, but it seems adequate as an explicatum for the special case where the evidence shows that *all* observed individuals have the property referred to in the hypothesis.

This concludes the discussion of the concept of confirming evidence.

On the basis of his analysis of the problem of an explication of the concept of confirming evidence, Hempel proceeds to construct the definition of a dyadic relation *Cf* between sentences, which he proposes as an explicatum. (His construction is given in technical details in [Syntactical], pp. 130–42, and briefly outlined in [Studies], p. 109.) We shall briefly state the series of definitions, using our terminology and notation and omitting minor details not relevant for our discussion. We add again 'H' to the numbers in the latter article and call the first definition 'H9.0'. e is any molecular sentence, h any sentence of Hempel's language system L earlier indicated (similar to $\mathfrak{L}$ but without a sign of identity).

(H9.0) The *development* of h for a finite class C of individual constants $=_{\mathrm{Df}}$ the sentence formed from h by the following transformations: (1) every universal matrix $(\mathrm{i}_k)(\mathfrak{M}_k)$ is replaced by the conjunction of the substitution instances of its scope $\mathfrak{M}_k$ for all in in C; (2) every existential matrix $(\exists \mathrm{i}_k)(\mathfrak{M}_k)$ is replaced by the disjunction of the substitution instances of its scope $\mathfrak{M}_k$ for all in in C. (If h contains no variables, then its development is h itself.)

(H9.1) $Cfd(e,h)$, *e directly confirms h* $=_{\mathrm{Df}}$ e L-implies the development of h for the class of those in which occur essentially in e (i.e., which occur in every sentence L-equivalent to e).

(H9.2) $Cf(e,h)$, *e confirms h* $=_{\mathrm{Df}}$ h is L-implied by a class of sentences each of which is directly confirmed by e.

Example. Let e be '$Pa_1 \cdot Pa_2 \cdot \ldots \cdot Pa_{10}$', l '$(x)Px$', and h 'Pa_{12}'. Then $Cfd(e,l)$; and, since $\vdash l \supset h$, $Cf(e,h)$; but not $Cfd(e,h)$.

(H9.3) *e disconfirms h* $=_{\mathrm{Df}}$ e confirms non-h.

(H9.4) e is *neutral* with respect to h $=_{\mathrm{Df}}$ e neither confirms nor disconfirms h.

Now let us see whether the concept *Cf* defined by H9.2 seems adequate as an explicatum for our explicandum, the concept of confirming evidence. Hempel shows that *Cf* satisfies all his conditions of adequacy earlier stated. While he takes this fact as an indication of adequacy, it will make us doubtful, since we found that some of the requirements are invalid.

It follows from our refutation of the special consequence condition H8.21 and the special consistency condition H8.32 that no R can possibly fulfil all of the following four conditions:

(i) R is not clearly too wide (in the sense of § 86),
(ii) R is not clearly too narrow,
(iii) R satisfies H8.21,
(iv) R satisfies H8.32.

For if (ii) and (iii) are fulfilled, then our counterexamples to H8.21 lead to cases where R holds but the explicandum does clearly not hold; hence (i) is not fulfilled. And if (i) and (iv) are fulfilled, then our counterexamples to H8.32 lead to cases which are excluded by H8.32 but in which the explicandum clearly holds; hence (ii) is not fulfilled.

Since Hempel has shown that his explicatum *Cf* satisfies all his requirements, among them H8.21 and H8.32, *Cf* must be either clearly too wide or clearly too narrow or both. I am not aware of any cases in which *Cf* holds but the explicandum does clearly not hold. Thus we may assume, unless and until somebody finds counterinstances, that *Cf* is not clearly too wide. However, it is clearly too narrow; we shall see, indeed, that *Cf* is limited to some quite special kinds of cases of the explicandum. The result that a proposed explicatum is found too narrow constitutes a much less serious objection than the result that it is too wide. In the former case the proposed concept may still be useful; it may be an adequate explicatum for a subkind of the explicandum within a limited field. It seems that this is the case with *Cf*.

We shall now consider the four most important kinds of inductive reasoning as explained earlier (§ 44B) and examine, for each of them, under what conditions *Cf* holds. In the following discussion the population is assumed to be finite. Individuals not referred to in the evidence e are called new individuals. 'rf' means relative frequency. (In (1) and (2) we restrict the present discussion, for the sake of simplicity, to a hypothesis h concerning one individual.)

1. *Direct inference.* *e* is a statistical distribution (D26-6c) to the effect that the rf of a property in the population, say, the primitive property *P*, has the value *r*; *h* is '*Pb*', where *b* belongs to the population.

1a. Let *r* be 1; that is, all individuals in the population are known to be *P*. Then *Cf* holds, but this case is trivial because *e* L-implies *h*.

1b. Let $0 < r < 1$. *Cf* does *not* hold. However, if *r* is close to 1, most people would regard *e* as confirming evidence for *h*. This holds even for both explicanda: (i) c is increased by adding *e* to '*t*'; (ii) c(*h*,*e*) exceeds the fixed value *q*, say 1/2. (We shall see later (T94-1e) that for every symmetrical c, and hence for every adequate c, c(*h*,*e*) = *r*.)

2. *Predictive inference.* *e* is a statistical distribution to the effect that the rf of a property, say, *P*, in a given sample is *r*; *h* is the singular prediction '*Pd*', where *d* is a new individual.

2a. Let *r* be 1; that is, all individuals in the observed sample have been found to be *P*. Then *Cf* holds (see the above example following H9.2).

2b. Let $0 < r < 1$. *Cf* does *not* hold. However, if *r* is close to 1, most people would regard *e* as confirming evidence for *h*, in the sense of either explicandum (as in 1b). (For any adequate c-function, in the case of a sufficiently large sample c(*h*,*e*) is close or equal to *r*.)

Example. Let *e* and *h* be as in the earlier example following (H9.2) and *i* '$\sim Pa_{11}$'. (*i* is negative to *h* on *e*.) Then not *Cf*(*e* . *i*,*h*).

2c. Let the evidence contain, in addition to *e* with $r = 1$, irrelevant data on additional individuals. Then *Cf* does *not* hold.

Example. Let *e* and *h* be as above, and *i'* be 'P_2a_{11}'. (*i'* is irrelevant to *h* on *e*.) Then not *Cf*(*e* . *i'*,*h*). However, for every adequate c, c(*h*,*e* . *i'*) = c(*h*,*e*). Therefore, since *e* is regarded as confirming evidence for *h*, *e* . *i'* will usually be regarded so too.

3. *Inverse inference.* *e* is a statistical distribution to the effect that the rf of *P* in a given sample of a population is *r*; *h* is a statistical distribution saying that the rf of *P* in the population is *r'*.

3a. Let *r* and *r'* be 1, that is, all individuals in the sample and in the population are stated to be *P*. Here *Cf*(*e*,*h*) holds, and even *Cfd*(*e*,*h*) (see *Cfd*(*e*,*l*) in the example following H9.2).

3b. Let $0 < r < 1$. Then *Cf* holds for *no* value of *r'*. However, for *r'* equal or near to *r*, many people, though not all, would regard *e* as confirming evidence for *h*.

4. *Universal inductive inference.* Let *h* be a universal sentence, say '$(x)Mx$', and *e* be a conjunction of sentences concerning the individuals of a given sample not containing negative instances. (If '*b*' occurs essentially in *e*, it is called a positive instance for *h* if *e* L-implies '*Mb*', a negative instance if *e* L-implies '$\sim Mb$', and a neutral instance if it is neither a positive nor a negative instance.)

4a. Let *e* contain only positive instances. Then *Cf* and even *Cfd* hold. (This case is the same as 3a.)

4b. Let *e* contain both positive and neutral instances. Then *Cf* does *not* hold.

Example. Let *l*, *e*, and *i'* be as previously. Then not *Cf*(*e* . *i'*,*l*). However, many will regard *i'* as irrelevant to *l* on *e*, that is, c(*l*,*e* . *i'*) = c(*l*,*e*). Since now *e* is regarded as confirming *l*, that is, c(*h*,*e*) > c(*h*,*t*), c(*h*,*e* . *i'*) is likewise > c(*h*,*t*). Hence *e* . *i'* will be regarded as confirming *h*.

Thus we see that in each of the kinds of inductive inference just discussed *Cf* holds only in the special case where the evidence ascribes to *all* individuals essentially occurring in it the property in question. Although this case is of great importance, it is very limited. In the great majority of the cases in which scientists speak of confirming evidence, the rf in *e* is not 1 or 0 but has an intermediate value. These cases are not covered by *Cf*. However, *Cf* can presumably be regarded as an adequate explicatum for the concept of confirming evidence in the special case described.

Hempel's investigations of the problem of confirming evidence supplied the first thoroughgoing and clear analysis of the whole problem complex. As such they remain valuable independently of his attempted solution of the particular problem of finding a nonquantitative explicatum for the concept of confirming evidence. The latter problem is today no longer as important as it was at the time Hempel made his investigations. He himself has defined, in the meantime, in collaboration with others, an interesting concept *dc*, proposed as an explicatum for degree of confirmation (see Hempel and Oppenheim [Degree], and Helmer and Oppenheim [Degree]); this will be discussed in a later chapter (in Vol. II). Some years ago those who worked on these problems expected that, if and when a definition of degree of confirmation were to be constructed, it would be based on a definition of a nonquantitative concept of confirming evidence. However, today it is seen that this is not the case either for Hempel's definition of *dc* nor for my definition of c*, and it is not regarded as probable that it will be the case for other definitions which will be proposed. It appears at present more promising to proceed in the opposite direction, that is, to define a quantitative form of the concept of confirming evidence on the basis of an explicatum for degree of confirmation, for instance, $\mathfrak{C}^*$ (or $\mathfrak{C}^*_0$) based on c* (see (5) and (6) in § 86) or analogous concepts based on Hempel's *dc* or on other explicata for degree of confirmation.

This concludes the discussion of the classificatory concept of confirming evidence. We have not found an adequate explicatum defined in nonquantitative terms. The concepts which were considered as possible explicata were found to be too narrow. However, we have a theory of confirming evidence in quantitative terms. The general part of this theory, which refers to all regular c-functions, was constructed in the preceding chapter as the theory of relevance. Later we shall find specific results concerning relevance with respect to the function c*.

CHAPTER VIII

THE SYMMETRICAL c-FUNCTIONS

In this chapter we return to quantitative inductive logic. A special kind of regular c-functions is introduced, called symmetrical c-functions. The definition is as follows. An m-function is called symmetrical (D90-1) if it has the same value for any state-descriptions which are isomorphic (D26-3a), i.e., such that one is constructed from the other by replacing individual constants with others. Then a c-function is called symmetrical (D91-1) if it is based upon a symmetrical m-function. It is shown (T91-2) that any symmetrical c-function fulfils the requirement of invariance, that is to say, its value for two sentences is not changed if the individual constants occurring in the sentences are replaced with other ones. It seems generally, though tacitly, agreed that any adequate explicatum for probability$_1$, i.e., degree of confirmation, must fulfil this requirement and hence be symmetrical. Theorems concerning symmetrical c-functions are developed (§§ 92–96), among them theorems concerning the direct inductive inference, that is, the inference from the frequency of a property in a population to its frequency in a sample (§ 94). (The other inductive inferences will be dealt with only in later chapters, because they presuppose the choice of a particular c-function.) The classical formulas of the binomial law (§ 95) and of Bernoulli's theorem (§ 96) are here construed as approximations for special cases of the direct inference.

This chapter presupposes §§ 25–27 of the earlier chapter on deductive logic.

§ 90. Symmetrical m-Functions

It seems plausible to require that an adequate concept of degree of confirmation should treat all individuals on a par. Those c-functions which fulfil this requirement will later (§ 91) be called symmetrical. As a preliminary step toward this concept we define here (D1) symmetrical m-functions as those regular m-functions which ascribe to any two isomorphic (D26-3a) state-descriptions the same value.

In the preceding chapter we have discussed the two nonquantitative concepts of confirmation, viz., the comparative concept 𝔐𝔆 and the classificatory concept ℭ. Now we return to the investigation of the quantitative concept, the concept of degree of confirmation. This investigation was begun in chapter v. There we introduced the general concept of regular c-functions and stated theorems which hold indiscriminately for *all* c-functions, no matter whether or not they are adequate explicata for our explicandum, the quantitative concept of probability$_1$ or degree of confirmation. In the present chapter we strengthen the assumptions underlying our system of inductive logic. Our final aim will be to choose one