mentally different meanings of 'probability'. Another distinction has been made between subjective and objective probability. However, I believe that practically all authors really have an objective concept of probability in mind and that the appearance of subjectivist conceptions is in most cases caused only by occasional unfortunate formulations; this will be discussed soon (§ 12).

Other distinctions which have been made are more or less similar to our distinction between probability$_1$ and probability$_2$. For instance, F. P. Ramsey ([Foundations] (1926), p. 157) says: "... the general difference of opinion between statisticians who for the most part adopt the frequency theory of probability and logicians who mostly reject it renders it likely that the two schools are really discussing different things, and that the word 'probability' is used by logicians in one sense and by statisticians in another".

It seems to me that practically all authors on probability have meant either probability$_1$ or probability$_2$ as their explicandum, despite the fact that their various explanations appear to refer to a number of quite different concepts.

For one group of authors, the question of their explicandum is easily answered. In the case of all those who support a frequency theory of probability, i.e., who define their explicata in terms of relative frequency (e.g. as a limit or in some other way), there can be no doubt that their explicandum is probability$_2$. Their formulations are, in general, presented in clear and unambiguous terms. Often they state explicitly that their explicandum is relative frequency. And even in the cases where this is not done, the discussion of their explicata leaves no doubt as to what is meant as explicandum.

This, however, covers only one of the various conceptions, i.e., explicata proposed, and only one of the many different explanations of explicanda which have been given and of which some examples were mentioned earlier. It seems clear that the other explanations do not refer to the statistical, empirical concept of relative frequency, and I believe that practically all of them, in spite of their apparent dissimilarity, are intended to refer to probability$_1$. Unfortunately, many of the phrases used are more misleading than helpful in our efforts to find out what their authors actually meant as explicandum. There is, in particular, one point on which many authors in discussions on probability$_1$, or on logical problems in general, commit a certain typical confusion or adopt incautiously other authors' formulations which are infected by this confusion. I am referring to what is sometimes called psychologism in logic. This will be discussed in the next two sections.

§ 11. Psychologism in Deductive Logic

Logical relations, e.g., logical consequence, are (i) logical, i.e., nonfactual, based merely upon meanings, (ii) objective, i.e., not dependent upon anybody's thinking about them. Most logicians treat them within their systems as objective relations, but, in spite of this, many characterize them in their general preliminary remarks in subjectivistic terms, e.g., with reference to actual thinking or believing. We call this discrepancy primitive psychologism in (deductive) logic. A qualified psychologism refers, not to actual, but to correct or rational thinking. This is usually meant in an objectivistic sense; in this case, the reference to thinking is gratuitous.

Those who work in the history of science or the methodology of science are familiar with the fact that there is frequently a discrepancy between what an author actually does and what he says he does; in particular, between the sense in which he actually uses a term or a sentence and the sense which he explicitly attributes to it. This holds especially for abstract terms and general principles. Consequently, in order to find out which sense a certain term has for the author, it is often not sufficient to look at his explicit explanations. We should also examine how he uses the term and, especially, how he argues pro or con statements in which the term occurs. And if these two tests are not in good agreement, the latter is more reliable than the first; it gives a better indication of the actual sense of the term for the author, that is, his general habit of using it. Suppose, for instance, we wish to know what a certain historian or political scientist means by 'democracy'. The best way is to observe under what conditions he applies this term and, still more important, what reasons he gives for these applications; we can accelerate the procedure by asking him questions as to whether and why he would apply the term to a country whose form of government was such and such. Of course, the direct way of asking: "What do you mean by 'democracy'?" is much simpler and quicker, and in many cases it will do. But there is always the danger that, instead of defining his actual meaning, he will give a definition which he has read in a theoretical book by a political scientist or even by a philosopher.

The discrepancy here discussed is likewise found in exact fields. Frege has repeatedly shown (especially in his *Über die Zahlen des Herrn H. Schubert* [Jena, 1899]) that the definitions of 'number' given by some mathematicians are deplorably inadequate and would lead to absurd and never intended applications, while the actual use of the term in the construction of a theory of numbers is quite correct.

The discrepancy discussed takes a special form in the case of logic. Before we approach the logical concept of probability$_1$, which is one of the fundamental concepts of inductive logic, let us look at the older and more familiar field of deductive logic, logic in the narrower sense. The task of logic (in this sense) has been the same for Aristotle as for modern (symbolic) logic, although the form of the systems constructed for the solution of this task has undergone considerable change in the course of the development. The task is the establishment of certain relations between sentences (or the propositions expressed by the sentences) usually called logical relations, among them, as one of the fundamental concepts of logic, the relation of logical consequence or deducibility. We cannot give here a full and exact characterization of these relations but will only indicate some of their characteristics. (i) They are independent of the contingency of the facts of nature, hence formal (in the traditional, not the syntactical, sense; see [Semantics], p. 232, meaning II); consequently, for ascertaining one of these relations in a concrete case, we need only know the meanings of the sentences involved, not their truth-values. (ii) The relations are objective, not subjective, in this sense: whether one of these relations does or does not hold in a concrete case is not dependent upon whether or what any person may happen to imagine, think, believe, or know about these sentences. As an example, let i be the sentence 'all swans are white', and j be 'all nonwhite things are nonswans', and suppose we have come to an agreement as to the meaning of all terms occurring. Suppose that a person X believes at the present time that j is a logical consequence of i, while at an earlier time he believed that this was not the case. That the relation is objective is meant in this sense: the change in X's belief about the relation has no effect upon the status of the relation itself; if his present belief is right (as I think it is), then his former belief was wrong; and, if his former belief was right, his present belief is wrong. It does not even make sense to assume that each of the two beliefs was right at its time, i.e., that the relation of logical consequence holds now between the two sentences but did not hold at the former time; this relation is timeless, i.e., it has no time value as argument. I hope that nobody will misinterpret my statement of the objectivity of logical relations as a metaphysical statement of the "subsistence" of these relations in a Platonic heaven (as earlier statements of mine have been misinterpreted). The statement is intended merely to point out the following character which logical concepts share with physical concepts—from which they are fundamentally different in other respects: a sentence which ascribes one of these concepts in a concrete case (e.g., 'j is a consequence of i', like 'this stone is heavier than that') is complete without any reference to the properties or the behavior of any person. (This is not in contradiction to the obvious fact that the *recognition* of a logical or a physical

or any other kind of relation involves a person.) In distinction to logical and physical concepts, certain other concepts are subjective in this sense: their application requires a reference to a person or a kind of person; e.g., 'known', 'familiar', 'pleasant', 'confirmed' (in the pragmatical sense as distinguished from the semantical sense, in which we take the term in our discussions here, see § 8). For example, 'this pattern is familiar' is not a complete sentence; it must be supplemented by something like 'to me', 'to Mr. X', 'to the persons of such and such a class'.

This objectivist conception of logic (in this section always understood in the sense of deductive logic), the view that the concepts of logic and hence the principles and theorems of logic which employ these concepts are objective, is certainly not new. On the contrary, it characterizes the work of practically all logicians. When they lay down their principles and rules or, on this basis, solve a logical problem, they do so in objectivist formulations, from Aristotle on through the Aristotelian tradition, up to modern logic. They say, for instance, 'from premises of the form so and so, a conclusion of the form so and so follows', or '. . . is deducible', or 'the deduction (inference) of . . . from . . . is valid', or the like. Here, for the work within their systems, they would hardly ever use subjectivist formulations, that is, those referring to persons, for instance, 'such and such an inference is valid for me now', or '. . . valid for persons of an introverted type'. And, in order to find out whether a certain conclusion follows from given premises, they do not in fact make psychological experiments about the thinking habits of people but rather analyze the given sentences and show their conceptual relations. However, if we examine not their actual procedure in solving logical problems but their general remarks concerning the task and nature of logic, chiefly in the introductory sections of their books, we often find something entirely different. Here, logic is often characterized as the art of thinking, and the principles of logic are called principles or laws of thought. These and similar formulations refer to thinking and hence are of a subjectivist nature. These references to thinking are in most cases entirely out of tune with what the same author does in the body of his work. Thus we have here a special case of the discrepancy discussed in the beginning of this section. A discrepancy of this kind, where the problems themselves are of an objective nature but the descriptions by which the author intends to give a general characterization of the problems are framed in subjectivist, psychological terms (like 'thinking'), is often called *psychologism*. Thus formulations of the kind mentioned above, frequently occurring in books on logic, are instances of *psychologism in deductive logic*. In some cases we find a situation still worse than that just described. It happens sometimes that the author does not only mislead the readers by his psychologistic general remarks but misleads himself; in this case, we find traces of subjectivism in the logical system itself, in the discussion of the logical problems, mixed with objective logical components; the result is inevitably rather confusing. [The situation is entirely different in cases where not only the general characterization but also the discussion of the problems themselves is consistently subjectivistic. A procedure of this kind, even if its author applies to it the title 'Logic', cannot be criticized as psychologism, because there is no mixture of heterogeneous components; there is merely a terminological difference in the use of the term 'logic'. It seems to me that John Dewey's *Logic, the theory of inquiry* (New York, 1938) is an instance of this kind. This book deals with that kind of behavior which is appropriate to problematic situations and leads to their "solutions"; it does not deal with logic in our sense (except in a few sections which seem somewhat out of place and have little connection with the remainder of the book). The fact that many logicians, that is, men who work in the field of logic in our sense, have erroneously characterized this field as the art of thinking has caused Dewey, who actually works on the art of thinking, that is, the theory and technology of procedures for overcoming problematic situations, to choose the title 'Logic'.]

We find psychologism in deductive logic not only in the literature of traditional logic but also in that of modern logic. A conspicuous example is the title of the book which may be regarded as marking the beginning of modern symbolic logic, Boole's *Laws of thought*. But one of the important achievements in the development of modern logic has been the gradual elimination of psychologism and the gradual clarification of the nature of logic. It seems that the great majority of contemporary writers in modern logic—though not those in logic of the traditional style—are free of psychologism. This is chiefly due to the efforts of the mathematician, Gottlob Frege, and the philosopher, Edmund Husserl, who emphasized the necessity of a clear distinction between empirical psychological problems and nonempirical logical problems and pointed out the confusion caused by psychologism. In this respect, they have also influenced indirectly the attitude of many logicians who have never read their works.

For Frege's emphasis on the objectivity of logic and arithmetic and his rejection of psychologism see his *Grundlagen der Arithmetik* (1884), §§ 26, 27, and *Grundgesetze der Arithmetik*, Vol. I (1893), Preface, pp. xiv ff. Husserl's own position was originally psychologistic (*Philosophie der Arithmetik* [1891]); but later, under the influence of Frege, he became one of the prominent opponents of psychologism (*Logische Untersuchungen*, Vol. I [1900], Preface and chaps. 3–11). Concerning this development of Husserl's views cf. Marvin Farber, *The foundation of phenomenology* (1943).

A primitive psychologistic explanation of the relation of logical consequence would perhaps be somewhat like this. That *j* is a logical consequence of *i* means that, if somebody believes in *i*, he cannot help believing also in *j*. Now, in fact, a psychologistic explanation will hardly ever be given in this crude form, because its inadequacy is too obvious. Taken literally, the explanation given would require us to investigate the statistical results of series of psychological experiments. There are not many logicians who would regard this procedure as appropriate.

A nice illustration, though not meant quite seriously, of primitive psychologism in arithmetic—which is part of deductive logic—is the following passage by P. E. B. Jourdain (*The philosophy of Mr. B*rtr*nd R*ss*ll* [1918], p. 88, quoted by Jeffreys [Probab.], p. 37): "I sometimes feel inclined to apply the historical method to the multiplication table. I should make a statistical inquiry among school children, before their pristine wisdom had been biased by teachers. I should put down their answers as to what 6 times 9 amounts to, I should work out the average of their answers to six places of decimals, and should then decide that, at the present stage of human development, this average is the value of 6 times 9."

Many logicians prefer formulations which may be regarded as a kind of *qualified psychologism*. They admit that logic is not concerned with the actual processes of believing, thinking, inferring, because then it would become a part of psychology. But, still clinging to the belief that there must somehow be a close relation between logic and thinking, they say that logic is concerned with correct or rational thinking. Thus they might explain the relation of logical consequence as meaning: 'if somebody has sufficient reasons to believe in the premise *i*, then the same reasons justify likewise his belief in the conclusion *j*'. It seems to me that psychologism thus diluted has virtually lost its content; the word 'thinking' or 'believing' is still there, but its use seems gratuitous. The explanation of logical consequence just mentioned does not say more than a formulation in nonpsychologistic, objectivist terms, for instance: 'any evidence for *i* is also evidence for *j*'; or: 'if *i* is true, then *j* is necessarily also true' (where 'necessarily' means not more than 'in any possible case, no matter what the facts happen to be'); indeed, we might say that the formulation in terms of justified belief is derivable from this one. Hence that formulation is not wrong. The characterization of logic in terms of correct or rational or justified belief is just as right but not more enlightening than to say that mineralogy tells us how to think correctly about minerals. The reference to thinking may just as well be dropped in both cases. Then we say simply: mineralogy makes statements about minerals, and logic makes statements about logical relations. The activity in any field of knowledge involves, of course, thinking. But this does not mean that thinking belongs to the subject matter of all fields. It belongs to the subject matter of psychology but not to that of logic any more than to that of mineralogy.

Because of the frequent discrepancy between introductory general remarks and the actual working theory of an author, we ought to be cautious in judging the latter on the basis of the former. The fact that an author uses occasionally some psychologistic formulations in general remarks about the task of logic, or in preliminary explanations of the meaning of some fundamental terms in logic, is not a sufficient reason for assuming that he has a subjectivistic conception of logic. If those explanations are in terms of correct or rational or justified thinking rather than of actual thinking, then in most cases they are not even subjectivistic. The reference to correctness or justification is presumably meant in the sense of 'in accordance with the rules of logic'; and these rules are regarded as objective by most logicians. The decisive point to examine is the way in which an author solves his logical problems, demonstrates logical theorems. If here his procedure is objectivistic, that is, free from references to the features of actual processes of thinking, then we have to regard his logic as objectivistic. This holds even if we find in his general remarks formulations not only of qualified but of primitive psychologism. If his working procedure is objectivistic, his occasional psychologistic formulations should be regarded as inessential relics from a traditional way of speech rather than as characteristics of his system of logic.

This view concerning the interpretation of psychologistic formulations in deductive logic, where the situation is relatively simple, will help us in understanding the analogous situation in the field of inductive logic, where the situation is at the present time much less clear.

§ 12. Psychologism in Inductive Logic

The situation with respect to psychologism in inductive logic, i.e., in the theory of probability$_1$, is analogous to that in deductive logic. We analyze here the formulations of some authors in two groups. *A*. Those who characterize *probability as a logical relation* similar to logical consequence (e.g., Keynes, Jeffreys). Here we find the systems themselves thoroughly objectivistic, but some general remarks show qualified psychologism, e.g., explanations of probability as degree of reasonable or justified belief; the concept meant is clearly probability$_1$. *B*. Authors of *the classical theory of probability* (e.g., Bernoulli, Laplace). Here, we find, in addition, formulations of primitive psychologism, e.g., explanations of probability as degree of belief or expectation. Nevertheless, it seems to me that their theories themselves were objectivistic; and, further, that they meant in most cases probability$_1$, not probability$_2$.

A. *Probability as a Logical Relation*

Deductive logic may be regarded as the theory of the relation of logical consequence, and inductive logic as the theory of another concept which is likewise objective and logical, viz., probability$_1$ or degree of confirmation. That probability$_1$ is an objective concept means this: if a certain probability$_1$ value holds for a certain hypothesis with respect to a certain evidence, then this value is entirely independent of what any person may happen to think about these sentences, just as the relation of logical consequence is independent in this respect. Consequently, a definition of an explicatum for probability$_1$ must not refer to any person and his beliefs but only to the two sentences and their logical properties within a given language system.

Now we shall show that the situation with respect to psychologism in inductive logic is in all essential respects analogous to that in deductive logic as discussed in the preceding section.

We have previously (§ 9) classified the theories of probability in three groups. In one of these groups the frequency conception of probability is adopted; here, the explicandum is obviously probability$_2$. The other two conceptions are the classical one (Bernoulli, Laplace) and the conception of probability as a logical concept related to deducibility (Keynes, Jeffreys).

Our problem is to discover what is the explicandum for the various authors of these two remaining groups. Let us begin with the last-mentioned group. Here, it will be easy to see that the explicandum is the objective, logical concept of probability$_1$. But even here we shall find psychologistic formulations. This fact will help us later in the analysis of classical authors to look through the deceiving shell of psychologistic formulations to the objectivistic core of their conception.

Keynes makes it quite clear that he regards probability as an objective, logical concept: "In the sense important to logic, probability is not subjective. It is not, that is to say, subject to human caprice. A proposition is not probable because we think so. When once the facts are given which determine our knowledge, what is probable or improbable in these circumstances has been fixed objectively, and is independent of our opinion. The Theory of Probability is logical, therefore" ([Probab.], p. 4). Keynes admits that probability may also be called subjective in another sense; it seems to me that here the term 'relative', in the sense of 'relating to a second proposition as evidence', would be more appropriate. He says (p. 4, in a passage immediately preceding the above quotation): "A proposition is capable at the same time of varying degrees of this relationship [of probability], depending upon the knowledge to which it is related, so that it is without significance to call a proposition probable unless we specify the knowledge to which we are relating it. To this extent, therefore, probability may be called subjective. But in the sense . . .". Then the preceding quotation follows, which makes it clear that Keynes's concept is in no respect meant as subjective in the sense opposite to objective.

Now it is interesting to see that Keynes, immediately following the passage quoted above in which he explicitly emphasizes the objective, logical nature of his concept, uses formulations of the kind which we have previously called qualified psychologism. He says: "The Theory of Probability is logical, therefore, because it is concerned with the degree of belief which it is *rational* to entertain in given conditions, and not merely with the actual beliefs of particular individuals, which may or may not be rational" (p. 4, italics in the original). His explicit contrasting of rational versus actual degree of belief and the use of 'because' show clearly that the reference to beliefs is not intended to modify in any way the characterization of the concept as a logical one or to bring in a subjective component. This will make us hesitant to interpret similar formulations of other authors as genuine symptoms of a subjectivistic conception. The situation here is analogous to that in deductive logic. Suppose that the hypothesis h has the probability$_1$ q with respect to the evidence e. Then, indeed, it follows that if somebody knows e and nothing else, he is justified in believing in h to the degree q and likewise justified in acting accordingly, e.g., in betting on h with q against $1 - q$. But this reference to belief should be avoided in a characterization of probability$_1$, because it blurs the important boundary line between logical and psychological concepts. Of course, in incidental informal explanations of probability$_1$, references to believing and betting will often facilitate the understanding—as in analogous cases in deductive logic and mathematics—but care should be taken that these references to something extra-logical do not obscure the nature of probability$_1$ as a purely logical concept.

That the objective logical concept meant by Keynes is the same as what we call probability$_1$, i.e., the logical concept of confirmation, becomes quite clear both by numerous preliminary explanations and by his reasonings in the construction of his system. He says, for instance: ". . . a logical connection between one set of propositions which we call our evidence

and which we suppose ourselves to know, and another set which we call our conclusions, and to which we attach more or less weight according to the grounds supplied by the first" (p. 5 f.). Keynes takes the concept in general as nonquantitative, similar to our comparative concept of confirmation; only in special cases does his theory allow the attribution of numerical values like our quantitative concept of degree of confirmation.

It is true, some statements of Keynes concerning his concept of probability are not in agreement with our conception of probability$_1$. He says, for example: "A definition of probability is not possible. . . . We cannot analyze the probability-relation in terms of simpler ideas" (p. 8); later he speaks of "a faculty of direct recognition of many relations of probability" (p. 53) by a kind of "logical intuition" (p. 52). But I do not think that this is evidence against our interpretation of his concept in the sense of our probability$_1$. It is one question whether two persons mean the same by certain terms and quite another question whether or not they agree in their opinions concerning the thing meant.

With other representatives of this group the situation is on the whole similar. We see easily from their systematic constructions and often also from explicit explanations that their explicandum is an objective, logical concept and, more specifically, that it is probability$_1$. Often, but not always, we find also psychologistic formulations, mostly of the qualified form. For the reasons earlier discussed, we do not regard these formulations as symptoms of a genuinely subjectivist conception but merely as vestiges of an old tradition that has been overcome in substance but still lingers on in some forms of speech.

The general remarks just made may be illustrated by some brief references to some authors of this group.

That *Jeffreys* understands 'probability' in the sense of probability$_1$ becomes abundantly clear through his whole theory. The very first sentence of the preface of his chief work ([Probab.], p. v) describes his aim "to provide a method of drawing inferences from observational data". He begins with a comparative concept with three arguments ("on data p, q is more probable than r", p. 15), from which he develops a quantitative concept by suitable conventions (p. 19). The whole conception is thoroughly objectivistic but accompanied by occasional formulations of qualified psychologism, e.g., "The probability, strictly, is the reasonable degree of confidence" (p. 20), "reasonable degree of belief" (p. 31).

F. P. Ramsey's conception of probability seems at first inspection more psychological and subjectivistic than the conception of most of the other authors ([Truth] and [Considerations], both published in [Foundations];

my references are to the latter book). He says that the theory of probability is "the logic of partial belief" (pp. 159, 166); "we must therefore try to develop a purely psychological method of measuring belief" (p. 166); "I propose to take as a basis a general psychological theory" (p. 173). Thus it is not surprising that many authors have judged Ramsey's conception as a particularly clear case of subjectivism. However, it seems to me that a closer examination is apt to evoke serious doubts about this judgment. It is true that the psychological method of measuring the actual degree of belief of a person in a proposition plays a central role in Ramsey's discussion. But he does not define probability as or identify it with actual degree of belief. He says: "It is not enough to measure probability; in order *to apportion correctly our belief* to the probability we must also be able to measure our belief"; "if the phrase 'a belief two-thirds of certainty' is meaningless, a calculus [viz., the theory of probability] whose sole object is *to enjoin such beliefs* will be meaningless also" (both on p. 166; the italics are mine). Thus, he regards the theory of probability not as a part of psychology describing the actually occurring degrees of belief but rather as a part of logic giving standards or norms which tell us which degrees of belief we should entertain if we want to be rational and consistent in our beliefs. This interpretation seems confirmed by his statement that "the laws of probability are laws of consistency, an extension to partial beliefs of formal logic, the logic of consistency" (p. 182); "having degrees of belief obeying the laws of probability implies a further measure of consistency, namely such a consistency between the odds acceptable on different propositions as shall prevent a book being made against you". This shows that the standard imposed upon our beliefs by the theory of probability is regarded as an objective one, viz., avoiding certain unfavorable results in betting. Later (p. 191) he characterizes logic "as the science of rational thought. We found", he continues, "that the most generally accepted parts of logic, namely, formal logic, mathematics, and the calculus of probabilities, are all concerned simply to ensure that our beliefs are not self-contradictory". This conception of the nature of logic as normative for, rather than descriptive of, beliefs is clearly expressed in the following words: "Logic, we may agree, is concerned not with what men actually believe, but what they ought to believe, or what it would be reasonable to believe" (p. 193). This formulation must clearly be judged as qualified rather than primitive psychologism. Therefore our previous consideration that the step from primitive to qualified psychologism shows an underlying objectivist conception applies also to Ramsey. This judgment seems confirmed by Ramsey's own later remark (written in 1929) concerning his earlier paper ([Truth], written in 1926): "The defect of my paper on probability was that it took partial belief as a psychological phenomenon to be defined and measured by a psychologist" (p. 256).

One of the rare cases in which primitive psychologism with respect to probability is meant literally is to be found in *James Jeans's* discussion of the probability waves in quantum mechanics (*Physics and philosophy* [New York, 1943]). We may leave aside here the question as to whether the concept of probability used in quantum theory is to be understood in the sense of probability$_1$ or of probability$_2$; maybe formulations of both kinds are possible. At any rate, both concepts are objective; the application of the one is a matter of logic, that of the other a matter of physics; neither of them is a psychological concept. Jeans, however, believes that probability in quantum theory is something of a mental nature. Hence he comes to the conclusion that Dirac's waves of probability are waves of knowledge; "the final picture consists wholly of waves, and its ingredients are wholly mental constructs". Consequently, he sees in this development of physics "a pronounced step in the direction of mentalism".

B. *The Classical Theory of Probability*

Now let us see to what extent psychologism is to be found in the so-called classical conception of probability, as originated by Jacob Bernoulli and Laplace. This conception shows itself in the definition of probability and in the way in which this definition is used; in other words, in the explicatum of these authors and their followers. Here, however, we shall not discuss their explicatum but their explicandum. We find many psychologistic formulations; probability is explained, for instance, as degree of belief, degree of certainty, and the like. Therefore, many later writers have characterized the classical conception as subjectivistic. If those formulations were taken literally, the theorems on probability would be statements of psychological laws; most of them would be obviously false just as are theorems of deductive logic interpreted as psychological laws, because our beliefs are often influenced by irrational factors. Thus it is understandable that many adherents of the classical conception seem not to feel quite satisfied with these formulations and use, either in addition or instead, those of qualified psychologism, for instance, 'rational degree of belief', and the like. As we have seen earlier, formulations of this kind may be regarded as a step toward the elimination of psychologism and are indeed no longer subjectivistic because they presuppose—in most cases tacitly—objective standards. Therefore, the occurrence of these formulations suggests that perhaps the use of primitive psychologistic formulations is likewise not a proof of a genuinely subjectivist conception but merely a customary, though not quite adequate, way of dealing with concepts which are meant as logical, not psychological.

Jacob Bernoulli makes some general explanatory remarks about the nature and application of probability in the beginning of Part Four of his *Ars conjectandi*, a work that marks the beginning of the systematic study of probability. He declares that "probability is the degree of certainty and differs from it as a part from the whole" (p. 211). The highest certainty is attributed by him to those things which we know by revelation, reasoning, or sensory perception; all other things have a less perfect measure of certainty. All this has a psychologistic sound. It becomes, however, quite clear that Bernoulli's theory of probability which he calls the art of conjecture ("ars conjectandi sive stochastice", p. 213) is not meant as a description of actual processes of reasoning but rather as a guide to correct and useful reasoning. He defines this art as "the art of measuring the probabilities of things as exactly as possible, so that we can always select and heed in our judgments and actions that which appears to us as better, more suitable, more certain or advisable" (p. 213).

Similarly *Laplace* understands 'probability' not in a psychological, subjective sense but in an objective sense. This is clearly shown by some passages near the end of his philosophical work ([Essai]; our quotations are from the edition of 1921). Here he says that the theory of probability makes exact what we feel by a kind of instinct; that it leaves nothing arbitrary in the choice of our opinions, since, with its help, the most advantageous choice can be determined; further, that the theory guides our judgments and protects us from illusions (II, 105 f.).

If the explicandum which the classical authors had in mind was not a subjective concept, which objective concept was it? The logical concept of probability$_1$ and the empirical concept of probability$_2$ are both objective. I am inclined to assume that on most occasions, though perhaps with a few exceptions, they meant something like probability$_1$, that is to say, not an empirical but a logical concept, which characterizes the strength given to a certain hypothesis by some amount of evidence.

Laplace ([Essai], I, 7) discusses an example of three urns—A, B, C. We know that one of them contains only black balls, but we do not know which of the three it is; we know further that the two other urns contain only white balls. Laplace raises the question as to what is the probability that a ball which will be drawn from the urn C will turn out to be black. From our present point of view, the essential fact is that Laplace states

different values of the probability: first one on the basis of the knowledge mentioned; then another value which the probability takes on when we learn that the urn A contains only white balls; and, finally, a third value when we learn, in addition, that B likewise contains only white balls. This shows that Laplace is not speaking about probability$_2$ or any other physical property of the urns, because these properties do not change when we learn more about the urns. What he means must be something that is dependent upon the state of our knowledge; hence it seems likely that he means something like the weight of evidence that our knowledge gives to a certain hypothesis, in other words, something like probability$_1$.

The formulations by which the classical authors intend to explain what they mean by 'probability' vary a good deal, even with the same author, and are often not as clear as we might wish. Thus we must base our interpretation also on the way in which they reason about probability in their theories. Often when we try to interpret an ambiguous term used by an author of another period, in another language or in an unfamiliar terminology, we proceed in the following way. Suppose the author in question is known for many valuable results he has found in the same or a related field; suppose further that he uses the term in question at certain places not in a casual way but in the formulation of theorems which are clearly important to him; suppose, finally, that among the meanings of the term which come into consideration there is one for which these theorems would hold, while they would be false for the other meanings. Then there is some reason to regard these facts as supporting the assumption that the meaning of the term which makes the theorems true is the one intended by the author. Certainly, this method must be used with caution; otherwise it would lead to rather arbitrary interpretations and, in the extreme, to the absurd result that all assertions of all authors seem to agree with our opinions. But as an auxiliary procedure, in combination with a consideration of the author's own explanations of the term, it may sometimes be helpful. Let us apply this to our case. The classical theory of probability contains certain theorems of the following kind. If interpreted in the sense of probability$_2$, these theorems are obviously false (even after certain modifications which seem necessary for any interpretation, e.g., the addition of a second argument of the probability function). Therefore the representatives of the frequency conception have rejected these theorems and have even expressed their amazement that any sensible man should assume such absurdities. These theorems are, of course, also false if interpreted in the sense of the psychological concept of degree of belief, as are practically all theorems. On the other hand, these theorems are true or at least not quite implausible if interpreted in the sense of probability$_1$. (Examples are certain specializations of the controversial principle of indifference; this principle itself in the customary form, however, is too general and leads to contradictions.) It seems to me that this fact lends additional support to our assumption that the explicandum which the classical authors had in mind during most of their discussions is probability$_1$ or something similar to it. I formulate this assumption with these cautious restrictions because it seems to me that there is no one meaning of the term 'probability' which is applied with perfect consistency throughout his work by any of the classical authors. There are some places where, I think, the interpretation as probability$_1$ makes no good sense while the interpretation as probability$_2$ does. (Examples are the references to "unknown probabilities"; see below, § 41D.)

Our interpretation of the classical theory in terms of probability$_1$ is in agreement with the view of Jeffreys, who offers forceful arguments in favor of this interpretation as against one in terms of frequency; one strong argument is simply the characteristic title *Ars conjectandi* of Bernoulli's book. Jeffreys comes to the following conclusion: "I maintain that the work of the pioneers [Bernoulli, Bayes, and Laplace] shows quite clearly that they were concerned with the construction of a consistent theory of reasonable degrees of belief, and in the cases of Bayes and Laplace, with the foundations of common sense or inductive inference" ([Probab.], p. 335).

With respect to those later writers who follow the classical tradition the situation is quite similar. In spite of psychologistic formulations, it is usually quite clear that they have an objectivist conception. We may perhaps have some doubt in this respect in the case of *De Morgan* because of his persistent formulations in terms of primitive psychologism. But even here we find that finally the author not only takes the saving step from primitive to qualified psychologism but regards this step merely as a transition from a natural, though not quite adequate, formulation to a more correct one rather than as a change in the conception itself: " 'It is more probable than improbable' means . . . 'I believe that it will happen more than I believe that it will not happen'. Or rather, 'I *ought* to believe, etc.' " ([Logic], pp. 172 f.). [Incidentally, a formulation like 'It is more probable than improbable that it will rain', used by some authors, seems a somewhat jumbled way of saying 'It is more probable that it will rain than that it will not rain'; it is like saying: 'I believe that it will rain more than I disbelieve that it will rain'.]

It seems to me that, on the basis of the discussions of this section, it is plausible to assume that for most, perhaps for practically all, of those authors on probability who do not accept a frequency conception the following holds. (i) Their theories of probability are objectivistic; the frequent formulations of psychologism, qualified or even primitive, are usually only preliminary remarks not affecting their actual working method. (ii) The objective concept which they mean, clearly or vaguely, as their explicandum is something similar to probability$_1$; in the classical period the explicandum is often not yet quite clear; but it seems that in the course of the historical development the concept of probability$_1$ emerges more and more clearly.

It cannot, of course, be denied that there is also a subjective, psychological concept for which the term 'probability' may be used and sometimes is used. This is the concept of the degree of actual, as distinguished from rational, belief: 'the person X at the time t believes in h to the degree r'. This concept is of importance for the theory of human behavior, hence for psychology, sociology, economics, etc. But it cannot serve as a basis for inductive logic or a calculus of probability applicable as a general tool of science.