In[1]:= `<< PrSAT``

■ Carnap's Early Systems: $m^\dagger$ and m^*

Here is the $m^\dagger$ distribution, for the 2 prediate, 2 object case:

In[2]:= `Symbolize[m†2,2];`

In[3]:=
$$m^\dagger_{2,2} = \text{PrSAT}\Big[\Big\{\Pr[Ea\wedge Ga\wedge Eb\wedge Gb]==\frac{1}{16},\ \Pr[Ea\wedge Ga\wedge Eb\wedge\neg Gb]==\frac{1}{16},$$
$$\Pr[Ea\wedge Ga\wedge\neg Eb\wedge Gb]==\frac{1}{16},\ \Pr[Ea\wedge Ga\wedge\neg Eb\wedge\neg Gb]==\frac{1}{16},$$
$$\Pr[Ea\wedge\neg Ga\wedge Eb\wedge Gb]==\frac{1}{16},\ \Pr[Ea\wedge\neg Ga\wedge Eb\wedge\neg Gb]==\frac{1}{16},$$
$$\Pr[Ea\wedge\neg Ga\wedge\neg Eb\wedge Gb]==\frac{1}{16},\ \Pr[Ea\wedge\neg Ga\wedge\neg Eb\wedge\neg Gb]==\frac{1}{16},$$
$$\Pr[\neg Ea\wedge Ga\wedge Eb\wedge Gb]==\frac{1}{16},\ \Pr[\neg Ea\wedge Ga\wedge Eb\wedge\neg Gb]==\frac{1}{16},$$
$$\Pr[\neg Ea\wedge Ga\wedge\neg Eb\wedge Gb]==\frac{1}{16},\ \Pr[\neg Ea\wedge Ga\wedge\neg Eb\wedge\neg Gb]==\frac{1}{16},$$
$$\Pr[\neg Ea\wedge\neg Ga\wedge Eb\wedge Gb]==\frac{1}{16},\ \Pr[\neg Ea\wedge\neg Ga\wedge Eb\wedge\neg Gb]==\frac{1}{16},$$
$$\Pr[\neg Ea\wedge\neg Ga\wedge\neg Eb\wedge Gb]==\frac{1}{16},\ \Pr[\neg Ea\wedge\neg Ga\wedge\neg Eb\wedge\neg Gb]==\frac{1}{16}\Big\}\Big]$$

Out[3]=
$$\Big\{\{Ea\to\{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\},\ Eb\to\{a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}\},$$
$$Ga\to\{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\},\ Gb\to\{a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\},$$
$$\Omega\to\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}\},$$
$$\Big\{a_1\to\frac{1}{16}, a_2\to\frac{1}{16}, a_3\to\frac{1}{16}, a_4\to\frac{1}{16}, a_5\to\frac{1}{16}, a_6\to\frac{1}{16}, a_7\to\frac{1}{16}, a_8\to\frac{1}{16}, a_9\to\frac{1}{16},$$
$$a_{10}\to\frac{1}{16}, a_{11}\to\frac{1}{16}, a_{12}\to\frac{1}{16}, a_{13}\to\frac{1}{16}, a_{14}\to\frac{1}{16}, a_{15}\to\frac{1}{16}, a_{16}\to\frac{1}{16}\Big\}\Big\}$$

In[4]:= **TruthTable**[$m^{\dagger}{}_{2,2}$]

Out[4]//DisplayForm=

Ea	Eb	Ga	Gb	var	Pr
T	T	T	T	a_{16}	$\frac{1}{16}$
T	T	T	F	a_{12}	$\frac{1}{16}$
T	T	F	T	a_{13}	$\frac{1}{16}$
T	T	F	F	a_{6}	$\frac{1}{16}$
T	F	T	T	a_{14}	$\frac{1}{16}$
T	F	T	F	a_{7}	$\frac{1}{16}$
T	F	F	T	a_{8}	$\frac{1}{16}$
T	F	F	F	a_{2}	$\frac{1}{16}$
F	T	T	T	a_{15}	$\frac{1}{16}$
F	T	T	F	a_{9}	$\frac{1}{16}$
F	T	F	T	a_{10}	$\frac{1}{16}$
F	T	F	F	a_{3}	$\frac{1}{16}$
F	F	T	T	a_{11}	$\frac{1}{16}$
F	F	T	F	a_{4}	$\frac{1}{16}$
F	F	F	T	a_{5}	$\frac{1}{16}$
F	F	F	F	a_{1}	$\frac{1}{16}$

Here are some salient Eacts about $m^{\dagger}$:

```
In[5]:= x_ ⊃ y_ := ¬ x ⋁ y;
        x_ ≡ y_ := (x ⊃ y) ⋀ (y ⊃ x);

In[7]:= EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb)], m†2,2] // N

Out[7]= 0.5625

In[8]:= EvaluateProbability[Pr[Eb ⋀ Gb], m†2,2] // N

Out[8]= 0.25

In[9]:= EvaluateProbability[Pr[Gb | Ea ⋀ Ga], m†2,2] // N

Out[9]= 0.5

In[10]:= EvaluateProbability[Pr[Gb | Ea ≡ Ga], m†2,2] // N

Out[10]= 0.5

In[11]:= EvaluateProbability[Pr[Gb], m†2,2] // N

Out[11]= 0.5

In[12]:= EvaluateProbability[Pr[Eb ≡ Gb], m†2,2] // N

Out[12]= 0.5

In[13]:= EvaluateProbability[Pr[Eb ≡ Gb | Ea ⋀ Ga], m†2,2] // N

Out[13]= 0.5

In[14]:= EvaluateProbability[Pr[Eb ≡ Gb | Ea ≡ Ga], m†2,2] // N

Out[14]= 0.5
```

```
In[15]:= EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | Ea ⋀ Ga], m†2,2] // N
Out[15]= 0.75

In[16]:= EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | ¬ Ea ⋀ ¬ Ga], m†2,2] // N
Out[16]= 0.75

In[17]:= EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | ¬ Ea ⋀ Ga], m†2,2] // N
Out[17]= 0.75

In[18]:= EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | Ea ⋀ ¬ Ga], m†2,2] // N
Out[18]= 0.

In[19]:= EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | ¬ Ea], m†2,2] // N
Out[19]= 0.75

In[20]:= EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | Ga], m†2,2] // N
Out[20]= 0.75

In[21]:= EvaluateProbability[Pr[Ga | Gb ⋀ (Ea ⋀ ¬ Eb)], m†2,2] // N
Out[21]= 0.5

In[119]:= EvaluateProbability[Pr[Ea | Eb] > Pr[Ea | Eb ⋀ (Ga ⋀ ¬ Gb)] > Pr[Ea], m†2,2]
Out[119]= False

In[22]:= EvaluateProbability[Pr[Ga | Gb], m†2,2] // N
Out[22]= 0.5

In[23]:= EvaluateProbability[Pr[Ga], m†2,2] // N
Out[23]= 0.5
```

In other words, $\mathfrak{m}^{\dagger}$ violates instantial relevance and analogy, but it leads to all Hempelian confirmatory instances for a universal generalization being confirmatory (probabilistically relevant) instances for a universal generalization.

Here is the $\mathfrak{m}^{*}$ distribution:

```
In[24]:= Symbolize[m*2,2];
```

In[25]:= $m^*_{2,2}$ = PrSAT[{Pr[Ea ∧ Ga ∧ Eb ∧ Gb] == $\frac{1}{10}$, Pr[Ea ∧ Ga ∧ Eb ∧ ¬ Gb] == $\frac{1}{20}$,
Pr[Ea ∧ Ga ∧ ¬ Eb ∧ Gb] == $\frac{1}{20}$, Pr[Ea ∧ Ga ∧ ¬ Eb ∧ ¬ Gb] == $\frac{1}{20}$,
Pr[Ea ∧ ¬ Ga ∧ Eb ∧ Gb] == $\frac{1}{20}$, Pr[Ea ∧ ¬ Ga ∧ Eb ∧ ¬ Gb] == $\frac{1}{10}$,
Pr[Ea ∧ ¬ Ga ∧ ¬ Eb ∧ Gb] == $\frac{1}{20}$, Pr[Ea ∧ ¬ Ga ∧ ¬ Eb ∧ ¬ Gb] == $\frac{1}{20}$,
Pr[¬ Ea ∧ Ga ∧ Eb ∧ Gb] == $\frac{1}{20}$, Pr[¬ Ea ∧ Ga ∧ Eb ∧ ¬ Gb] == $\frac{1}{20}$,
Pr[¬ Ea ∧ Ga ∧ ¬ Eb ∧ Gb] == $\frac{1}{10}$, Pr[¬ Ea ∧ Ga ∧ ¬ Eb ∧ ¬ Gb] == $\frac{1}{20}$,
Pr[¬ Ea ∧ ¬ Ga ∧ Eb ∧ Gb] == $\frac{1}{20}$, Pr[¬ Ea ∧ ¬ Ga ∧ Eb ∧ ¬ Gb] == $\frac{1}{20}$,
Pr[¬ Ea ∧ ¬ Ga ∧ ¬ Eb ∧ Gb] == $\frac{1}{20}$, Pr[¬ Ea ∧ ¬ Ga ∧ ¬ Eb ∧ ¬ Gb] == $\frac{1}{10}$}]

Out[25]= {{Ea → {a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}}, Eb → {a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}},
Ga → {a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}}, Gb → {a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}},
Ω → {a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}}},
{$a_1 \to \frac{1}{10}$, $a_2 \to \frac{1}{20}$, $a_3 \to \frac{1}{20}$, $a_4 \to \frac{1}{20}$, $a_5 \to \frac{1}{20}$, $a_6 \to \frac{1}{10}$, $a_7 \to \frac{1}{20}$, $a_8 \to \frac{1}{20}$, $a_9 \to \frac{1}{20}$,
$a_{10} \to \frac{1}{20}$, $a_{11} \to \frac{1}{10}$, $a_{12} \to \frac{1}{20}$, $a_{13} \to \frac{1}{20}$, $a_{14} \to \frac{1}{20}$, $a_{15} \to \frac{1}{20}$, $a_{16} \to \frac{1}{10}$}}

In[26]:= TruthTable[$m^*_{2,2}$]

Out[26]//DisplayForm=

Ea	Eb	Ga	Gb	var	Pr
T	T	T	T	a_{16}	$\frac{1}{10}$
T	T	T	F	a_{12}	$\frac{1}{20}$
T	T	F	T	a_{13}	$\frac{1}{20}$
T	T	F	F	a_6	$\frac{1}{10}$
T	F	T	T	a_{14}	$\frac{1}{20}$
T	F	T	F	a_7	$\frac{1}{20}$
T	F	F	T	a_8	$\frac{1}{20}$
T	F	F	F	a_2	$\frac{1}{20}$
F	T	T	T	a_{15}	$\frac{1}{20}$
F	T	T	F	a_9	$\frac{1}{20}$
F	T	F	T	a_{10}	$\frac{1}{20}$
F	T	F	F	a_3	$\frac{1}{20}$
F	F	T	T	a_{11}	$\frac{1}{10}$
F	F	T	F	a_4	$\frac{1}{20}$
F	F	F	T	a_5	$\frac{1}{20}$
F	F	F	F	a_1	$\frac{1}{10}$

Here are some salient Eacts about m^*:

```
In[27]:= EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb)], m*2,2] // N
Out[27]= 0.6

In[28]:= EvaluateProbability[Pr[Eb ⋀ Gb], m*2,2] // N
Out[28]= 0.25

In[29]:= EvaluateProbability[Pr[Gb | Ea ⋀ Ga], m*2,2] // N
Out[29]= 0.6

In[30]:= EvaluateProbability[Pr[Gb | Ea ≡ Ga], m*2,2] // N
Out[30]= 0.5

In[31]:= EvaluateProbability[Pr[Gb | (Ea ≡ Ga) ⋀ Eb], m*2,2] // N
Out[31]= 0.6

In[32]:= EvaluateProbability[Pr[Gb | Eb], m*2,2] // N
Out[32]= 0.5

In[33]:= EvaluateProbability[Pr[Gb], m*2,2] // N
Out[33]= 0.5

In[34]:= EvaluateProbability[Pr[Eb ≡ Gb], m*2,2] // N
Out[34]= 0.5

In[35]:= EvaluateProbability[Pr[Eb ≡ Gb | Ea ⋀ Ga], m*2,2] // N
Out[35]= 0.6

In[36]:= EvaluateProbability[Pr[Eb ≡ Gb | Ea ≡ Ga], m*2,2] // N
Out[36]= 0.6

In[37]:= EvaluateProbability[Pr[Eb ≡ Gb], m*2,2] // N
Out[37]= 0.5

In[38]:= EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb)], m*2,2] // N
Out[38]= 0.6

In[39]:= EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | Ea ⋀ Ga], m*2,2] // N
Out[39]= 0.8

In[40]:= EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | ¬ Ea ⋀ ¬ Ga], m*2,2] // N
Out[40]= 0.8

In[41]:= EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | ¬ Ea ⋀ Ga], m*2,2] // N
Out[41]= 0.8

In[120]:= EvaluateProbability[Pr[Ea | Eb] > Pr[Ea | Eb ⋀ (Ga ⋀ ¬ Gb)] > Pr[Ea], m*2,2]
Out[120]= False

In[42]:= EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | Ea ⋀ ¬ Ga], m*2,2] // N
Out[42]= 0.

In[43]:= EvaluateProbability[Pr[Ga | Gb ⋀ (Ea ⋀ ¬ Eb)], m*2,2] // N
Out[43]= 0.5
```

In[44]:=
```
EvaluateProbability[Pr[Ga | Gb], m*2,2] // N
```
Out[44]= 0.6

In[45]:=
```
EvaluateProbability[Pr[Ga], m*2,2] // N
```
Out[45]= 0.5

In other words, m^* satisfies instantial relevance and “analogy by similarity“, but it also leads to all Hempelian confirmatory instances for a universal generalization being confirmatory (probabilistically relevant) instances for a universal generalization.

What about “Grue“? Here, the 2-object, 3-predicate case is what we‘d want to look at (E, O, G, and a, b, c). It‘s pretty complex. We need to be more clever now in specifying $m^\dagger_{2,3}$ and $m^*_{2,3}$. We start with $m^\dagger_{2,3}$.

In[46]:=
```
Symbolize[m†2,3];
```
In[47]:=
```
atoms = {Ea, Eb, Oa, Ob, Ga, Gb};
tvs = Flatten[Outer[List, Sequence @@ Table[{True, False}, {6}]], 5];
set[x_] := Table[If[x⟦i⟧ == True, atoms⟦i⟧, ¬ atoms⟦i⟧], {i, 1, Length[atoms]}];
sds = (And @@ # &) /@ (set /@ tvs);
f[s_] := Pr[s] == 1/Length[sds];
m†2,3 = PrSAT[{f /@ sds}, BypassSearch → True];
```
In[53]:=
```
x_ ≡ y_ := (x ⊃ y) ⋀ (y ⊃ x);
```
In[54]:=
```
EvaluateProbability[Pr[(Ea ⊃ (Oa ≡ Ga)) ⋀ (Eb ⊃ (Ob ≡ Gb))], m†2,3] // N
```
Out[54]= 0.5625

In[55]:=
```
EvaluateProbability[Pr[Eb ⋀ Gb], m†2,3] // N
```
Out[55]= 0.25

In[56]:=
```
EvaluateProbability[Pr[Ob | Ea ⋀ Ga], m†2,3] // N
```
Out[56]= 0.5

In[57]:=
```
EvaluateProbability[Pr[Gb | Ea ≡ Ga], m†2,2] // N
```
Out[57]= 0.5

In[58]:=
```
EvaluateProbability[Pr[Gb], m†2,2] // N
```
Out[58]= 0.5

In[59]:=
```
EvaluateProbability[Pr[Eb ≡ Gb], m†2,2] // N
```
Out[59]= 0.5

In[60]:=
```
EvaluateProbability[Pr[Eb ≡ Gb | Ea ⋀ Ga], m†2,2] // N
```
Out[60]= 0.5

In[61]:=
```
EvaluateProbability[Pr[Eb ≡ Gb | Ea ≡ Ga], m†2,2] // N
```
Out[61]= 0.5

In[62]:=
```
EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | Ea ⋀ Ga], m†2,2] // N
```
Out[62]= 0.75

In[63]:=
```
EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | ¬ Ea ⋀ ¬ Ga], m†2,2] // N
```
Out[63]= 0.75

In[64]:=
```
EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | ¬ Ea ⋀ Ga], m†2,2] // N
```
Out[64]= 0.75

In[65]:=
```
EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | Ea ⋀ ¬ Ga], m†2,2] // N
```
Out[65]= 0.

In[66]:=
```
EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | ¬ Ea], m†2,2] // N
```
Out[66]= 0.75

In[67]:=
```
EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | Ga], m†2,2] // N
```
Out[67]= 0.75

In[68]:=
```
EvaluateProbability[Pr[Ga | Gb ⋀ (Ea ⋀ ¬ Eb)], m†2,2] // N
```
Out[68]= 0.5

In[69]:=
```
EvaluateProbability[Pr[Ga | Gb], m†2,2] // N
```
Out[69]= 0.5

In[70]:=
```
EvaluateProbability[Pr[Ga], m†2,2] // N
```
Out[70]= 0.5

Now, for $m^*_{2,3}$, we must be a little more clever. We have to compute the structure descriptions, and then the probabilities for them, etc.

In[176]:=
```
perm[s_] := s /. {Ea → Eb, Eb → Ea, Ga → Gb, Gb → Ga, Oa → Ob, Ob → Oa};
states = set /@ tvs;

results = {};
For[i = 1, i ≤ Length[states], i++, a = states[[i]]; temp = {};
  AppendTo[temp, i]; For[j = 1, j ≤ Length[states], j++, b = states[[j]];
   If[Sort[a] == Sort[perm[b]], AppendTo[temp, j]]]; AppendTo[results, temp]];
structIndices = Union /@ Union[Sort /@ results];

in[x_, S_] := Or @@ Table[x === S[[i]], {i, 1, Length[S]}];
pr[sn_] := 1 / (36 Length[Flatten[Select[structIndices, in[sn, #1] &]]]);
f[n_] := Pr[And @@ states[[n]]] == pr[n];
m*2,3 = PrSAT[{f /@ Range[64]}, BypassSearch → True];
```

In[185]:=
```
TruthTable[m*2,3]
```

Out[185]//DisplayForm=

Ea	Eb	Ga	Gb	Oa	Ob	var	Pr
T	T	T	T	T	T	a_{64}	$\frac{1}{36}$
T	T	T	T	T	F	a_{58}	$\frac{1}{72}$
T	T	T	T	F	T	a_{59}	$\frac{1}{72}$
T	T	T	T	F	F	a_{43}	$\frac{1}{36}$
T	T	T	F	T	T	a_{60}	$\frac{1}{72}$
T	T	T	F	T	F	a_{44}	$\frac{1}{72}$
T	T	T	F	F	T	a_{45}	$\frac{1}{72}$
T	T	T	F	F	F	a_{23}	$\frac{1}{72}$
T	T	F	T	T	T	a_{61}	$\frac{1}{72}$
T	T	F	T	T	F	a_{46}	$\frac{1}{72}$
T	T	F	T	F	T	a_{47}	$\frac{1}{72}$

T	T	F	T	F	F	a_{24}	$\frac{1}{72}$
T	T	F	F	T	T	a_{48}	$\frac{1}{36}$
T	T	F	F	T	F	a_{25}	$\frac{1}{72}$
T	T	F	F	F	T	a_{26}	$\frac{1}{72}$
T	T	F	F	F	F	a_{8}	$\frac{1}{36}$
T	F	T	T	T	T	a_{62}	$\frac{1}{72}$
T	F	T	T	T	F	a_{49}	$\frac{1}{72}$
T	F	T	T	F	T	a_{50}	$\frac{1}{72}$
T	F	T	T	F	F	a_{27}	$\frac{1}{72}$
T	F	T	F	T	T	a_{51}	$\frac{1}{72}$
T	F	T	F	T	F	a_{28}	$\frac{1}{72}$
T	F	T	F	F	T	a_{29}	$\frac{1}{72}$
T	F	T	F	F	F	a_{9}	$\frac{1}{72}$
T	F	F	T	T	T	a_{52}	$\frac{1}{72}$
T	F	F	T	T	F	a_{30}	$\frac{1}{72}$
T	F	F	T	F	T	a_{31}	$\frac{1}{72}$
T	F	F	T	F	F	a_{10}	$\frac{1}{72}$
T	F	F	F	T	T	a_{32}	$\frac{1}{72}$
T	F	F	F	T	F	a_{11}	$\frac{1}{72}$
T	F	F	F	F	T	a_{12}	$\frac{1}{72}$
T	F	F	F	F	F	a_{2}	$\frac{1}{72}$
F	T	T	T	T	T	a_{63}	$\frac{1}{72}$
F	T	T	T	T	F	a_{53}	$\frac{1}{72}$
F	T	T	T	F	T	a_{54}	$\frac{1}{72}$
F	T	T	T	F	F	a_{33}	$\frac{1}{72}$
F	T	T	F	T	T	a_{55}	$\frac{1}{72}$
F	T	T	F	T	F	a_{34}	$\frac{1}{72}$
F	T	T	F	F	T	a_{35}	$\frac{1}{72}$
F	T	T	F	F	F	a_{13}	$\frac{1}{72}$
F	T	F	T	T	T	a_{56}	$\frac{1}{72}$
F	T	F	T	T	F	a_{36}	$\frac{1}{72}$
F	T	F	T	F	T	a_{37}	$\frac{1}{72}$
F	T	F	T	F	F	a_{14}	$\frac{1}{72}$
F	T	F	F	T	T	a_{38}	$\frac{1}{72}$
F	T	F	F	T	F	a_{15}	$\frac{1}{72}$
F	T	F	F	F	T	a_{16}	$\frac{1}{72}$
F	T	F	F	F	F	a_{3}	$\frac{1}{72}$
F	F	T	T	T	T	a_{57}	$\frac{1}{36}$

F	F	T	T	T	F	a_{39}	$\frac{1}{72}$
F	F	T	T	F	T	a_{40}	$\frac{1}{72}$
F	F	T	T	F	F	a_{17}	$\frac{1}{36}$
F	F	T	F	T	T	a_{41}	$\frac{1}{72}$
F	F	T	F	T	F	a_{18}	$\frac{1}{72}$
F	F	T	F	F	T	a_{19}	$\frac{1}{72}$
F	F	T	F	F	F	a_{4}	$\frac{1}{72}$
F	F	F	T	T	T	a_{42}	$\frac{1}{72}$
F	F	F	T	T	F	a_{20}	$\frac{1}{72}$
F	F	F	T	F	T	a_{21}	$\frac{1}{72}$
F	F	F	T	F	F	a_{5}	$\frac{1}{72}$
F	F	F	F	T	T	a_{22}	$\frac{1}{36}$
F	F	F	F	T	F	a_{6}	$\frac{1}{72}$
F	F	F	F	F	T	a_{7}	$\frac{1}{72}$
F	F	F	F	F	F	a_{1}	$\frac{1}{36}$

In[186]:= **EvaluateProbability[Pr[(Ea ⊃ (Oa ≡ Ga)) ⋀ (Eb ⊃ (Ob ≡ Gb))], $m^*_{2,3}$] // N**

Out[186]= 0.583333

In[78]:= **EvaluateProbability[Pr[(Ea ⊃ (Oa ≡ Ga)) ⋀ (Eb ⊃ (Ob ≡ Gb)) | Ea ⋀ Oa ⋀ Ga], $m^*_{2,3}$] // N**

Out[78]= 0.703704

In[187]:= **EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | Ea ⋀ Oa ⋀ Ga], $m^*_{2,3}$] // N**

Out[187]= 0.777778

In[188]:= **EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb)], $m^*_{2,3}$] // N**

Out[188]= 0.583333

In[189]:= **EvaluateProbability[Pr[(Ea ⊃ (Oa ≡ Ga)) ⋀ (Eb ⊃ (Ob ≡ Gb)) | Ea ⋀ Ga], $m^*_{2,3}$] // N**

Out[189]= 0.388889

In[190]:= **EvaluateProbability[Pr[(Ea ⊃ (Oa ≡ Ga)) ⋀ (Eb ⊃ (Ob ≡ Gb)) | Ea ⋀ Oa], $m^*_{2,3}$] // N**

Out[190]= 0.388889

In[191]:= **EvaluateProbability[Pr[(Ea ⊃ (Oa ≡ Ga)) ⋀ (Eb ⊃ (Ob ≡ Gb)) | Ga ⋀ Oa], $m^*_{2,3}$] // N**

Out[191]= 0.777778

In[192]:= **EvaluateProbability[Pr[Eb ⊃ (Ob ≡ Gb) | Ea ⊃ (Oa ≡ Ga)], $m^*_{2,3}$] // N**

Out[192]= 0.777778

In[193]:= **EvaluateProbability[Pr[Eb ⊃ (Ob ≡ Gb)], $m^*_{2,3}$] // N**

Out[193]= 0.75

In[194]:= **EvaluateProbability[Pr[Eb ⊃ (Ob ≡ Gb) | Ea ⋀ (Oa ⋀ Ga)], $m^*_{2,3}$] // N**

Out[194]= 0.777778

```
In[195]:= EvaluateProbability[Pr[Eb ⋀ (Ob ≡ Gb) | Ea ⋀ (Oa ⋀ Ga)], m*2,3] // N
Out[195]= 0.333333

In[196]:= EvaluateProbability[Pr[Eb ⋀ (Ob ≡ Gb) | Ea ⋀ (Oa ≡ Ga)], m*2,3] // N
Out[196]= 0.333333

In[197]:= EvaluateProbability[Pr[Eb ⋀ (Ob ≡ Gb) | Ea ⋀ (Oa ≡ Ga) ⋀ (Oa ⋀ Ob)], m*2,3] // N
Out[197]= 0.4

In[198]:= EvaluateProbability[Pr[Eb ⋀ (Ob ≡ Gb) | (Oa ⋀ Ob)], m*2,3] // N
Out[198]= 0.25

In[199]:= EvaluateProbability[Pr[Eb ⋀ Gb | Ea ⋀ (Oa ≡ Ga) ⋀ (Oa ⋀ Ob)], m*2,3] // N
Out[199]= 0.4

In[200]:= EvaluateProbability[Pr[Eb ⋀ Gb | Oa ⋀ Ob], m*2,3] // N
Out[200]= 0.25

In[201]:= EvaluateProbability[Pr[Eb ⋀ (Ob ≡ Gb)], m*2,3] // N
Out[201]= 0.25

In[202]:= EvaluateProbability[Pr[Eb ⋀ (Ob ⋀ Gb)], m*2,3] // N
Out[202]= 0.125

In[203]:= EvaluateProbability[Pr[Eb ⋀ (Ob ⋀ Gb) | Ea ⋀ (Oa ⋀ Ga)], m*2,3] // N
Out[203]= 0.222222

In[204]:= EvaluateProbability[Pr[Gb | Ea ≡ Ga], m*2,3] // N
Out[204]= 0.5
```

Comparison with 2,2:

```
In[205]:= EvaluateProbability[Pr[Gb | Ea ≡ Ga], m*2,2] // N
Out[205]= 0.5

In[206]:= EvaluateProbability[Pr[Gb | Ea ≡ Ga], m*2,3] // N
Out[206]= 0.5

In[207]:= EvaluateProbability[Pr[Gb], m*2,2] // N
Out[207]= 0.5

In[208]:= EvaluateProbability[Pr[Gb], m*2,3] // N
Out[208]= 0.5

In[209]:= EvaluateProbability[Pr[Eb ≡ Gb], m*2,2] // N
Out[209]= 0.5

In[210]:= EvaluateProbability[Pr[Eb ≡ Gb], m*2,3] // N
Out[210]= 0.5

In[211]:= EvaluateProbability[Pr[Eb ≡ Gb | Ea ⋀ Ga], m†2,2] // N
Out[211]= 0.5
```

```
In[212]:= EvaluateProbability[Pr[Eb ≡ Gb | Ea ⋀ Ga], m†2,3] // N
Out[212]= 0.5
```

Here‘s a counterexample to Fine‘s L7:

```
In[213]:= EvaluateProbability[Pr[Eb ≡ Gb | Ea ⋀ Ga], m*2,2] // N
Out[213]= 0.6

In[214]:= EvaluateProbability[Pr[Eb ≡ Gb | Ea ⋀ Ga], m*2,3] // N
Out[214]= 0.555556

In[215]:= EvaluateProbability[Pr[Eb ≡ Gb | Ea ≡ Ga], m†2,2] // N
Out[215]= 0.5

In[216]:= EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | Ea ⋀ Ga], m†2,2] // N
Out[216]= 0.75

In[217]:= EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | ¬ Ea ⋀ ¬ Ga], m†2,2] // N
Out[217]= 0.75

In[218]:= EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | ¬ Ea ⋀ Ga], m†2,2] // N
Out[218]= 0.75

In[219]:= EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | Ea ⋀ ¬ Ga], m†2,2] // N
Out[219]= 0.

In[220]:= EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | ¬ Ea], m†2,2] // N
Out[220]= 0.75

In[221]:= EvaluateProbability[Pr[(Ea ⊃ Ga) ⋀ (Eb ⊃ Gb) | Ga], m†2,2] // N
Out[221]= 0.75

In[222]:= EvaluateProbability[Pr[Ga | Gb ⋀ (Ea ⋀ ¬ Eb)], m†2,2] // N
Out[222]= 0.5

In[223]:= EvaluateProbability[Pr[Ga | Gb], m†2,2] // N
Out[223]= 0.5

In[224]:= EvaluateProbability[Pr[Ga], m†2,2] // N
Out[224]= 0.5
```