Two Technical Corrections to My Coherence Measure

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Correction #1: The Corrected Definition of F

My original definition of $F(h, e)$ was incorrect. I was trying to be facncy about some of the cases involving "paradoxes of entailment". What I should have done is simply given the following, classical version of F.[1]

$$F_{\mathcal{M}}(h,e) =_{df} \begin{cases} \dfrac{\Pr_{\mathcal{M}}(e \mid h) - \Pr_{\mathcal{M}}(e \mid \sim h)}{\Pr_{\mathcal{M}}(e \mid h) + \Pr_{\mathcal{M}}(e \mid \sim h)} & \text{if } e \not\models h \text{ and } e \not\models \sim h. \\ 1 & \text{if } e \models h, \text{ and } e \not\models \bot. \\ -1 & \text{if } e \models \sim h. \end{cases}$$

Note: This is *not* an *ad hoc* change at all. It's simply the natural thing say here – if one thinks of F as a generalization of *classical* logical entailment. The extra complexity I had in my original (incorrect) definition of F was there because I was foolishly trying to encode some non-classical, or "relavant" logical structure in F. I now think this is a mistake, and that I should go with the above, classical account of F. Arguments about relevance logic need to be handled in a different way (and a different context!). And, besides, as Luca Moretti has shown (see below), the original definition of F cannot be the right basis for $\mathscr{C}$! OK, now on to $\mathscr{C}$.

Correction #2: The Corrected Definition of $\mathscr{C}$

Let $\mathbf{S}$ be the set of statements, the coherence of which $\mathscr{C}$ aims to measure. Let $\mathscr{P}$ be the set of all nonempty disjoint subsets of $\mathbf{S}$. And, let $\mathscr{P}^2$ be the set of all ordered pairs of disjoint elements of $\mathscr{P}$.[2] And, let $\mathscr{S}$ be the result of taking conjunctions of each of the (flattened) sets in the pairs in $\mathscr{P}^2$. Now, apply F to each element of $\mathscr{S}$. This yields a set of F-values: $\mathscr{F}$. Finally, $\mathscr{C}(\mathbf{S}) = \text{Mean}(\mathscr{F})$. Here are some examples.

- The two-element set $\mathbf{S}_2 = \{p, q\}$. In this case, we have:
  - $\mathscr{P}_2 = \{\{p\}, \{q\}, \{p, q\}\}$
  - $\mathscr{P}_2^2 = \{\langle\{p\}, \{q\}\rangle, \langle\{q\}, \{p\}\rangle\}$
  - $\mathscr{S}_2 = \{\langle p, q\rangle, \langle q, p\rangle\}$
  - $\mathscr{F}_2 = \{F(p, q), F(q, p)\}$
  - $\mathscr{C}(\mathbf{S}_2) = \text{Mean}(\mathscr{F}_2) = \dfrac{F(p,q) + F(q,p)}{2}$

[1] Here, I have also made explicit the fact that, for me, all claims about F or $\mathscr{C}$ are *relativized* to a *regular*, Kolmogorov probability model $\mathcal{M}$. I need to be explicit about this, since I think it's important for their status as "logical". I will explain *that* in by book!

[2] This step adds some additional structure, not in my original definition of $\mathscr{C}$. Here, I am indebted to Igor Douven who pointed out that this additional structure is necessary to capture the independence/dependence relations for sets with more than three elements.

- The three-element set $\mathbf{S}_3 = \{p, q, r\}$. In this case, we have:

$$\mathscr{P}_3 = \{\{p\}, \{q\}, \{r\}, \{p, q\}, \{p, r\}, \{q, r\}, \{p, q, r\}\}$$
$$\mathscr{P}_3^2 = \{\langle\{p\}, \{q\}\rangle, \langle\{q\}, \{p\}\rangle, \langle\{p\}, \{r\}\rangle, \langle\{r\}, \{p\}\rangle, \langle\{p\}, \{q, r\}\rangle, \langle\{q, r\}, \{p\}\rangle, \langle\{q\}, \{r\}\rangle, \langle\{r\}, \{q\}\rangle, \langle\{q\}, \{p, r\}\rangle, \langle\{p, r\}, \{q\}\rangle, \langle\{r\}, \{p, q\}\rangle, \langle\{p, q\}, \{r\}\rangle\}$$
$$\mathscr{S}_3 = \{\langle p, q\rangle, \langle q, p\rangle, \langle p, r\rangle, \langle r, p\rangle, \langle p, q \mathbin{\&} r\rangle, \langle q \mathbin{\&} r, p\rangle, \langle q, r\rangle, \langle r, q\rangle, \langle q, p \mathbin{\&} r\rangle, \langle p \mathbin{\&} r, q\rangle, \langle r, p \mathbin{\&} q\rangle, \langle p \mathbin{\&} q, r\rangle\}$$
$$\mathscr{F}_3 = \{F(p, q), F(q, p), F(p, r), F(r, p), F(p, q \mathbin{\&} r), F(q \mathbin{\&} r, p), F(q, r), F(r, q), F(q, p \mathbin{\&} r), F(p \mathbin{\&} r, q), F(r, p \mathbin{\&} q), F(p \mathbin{\&} q, r)\}$$
$$\mathscr{C}(\mathbf{S}_3) = \mathrm{Mean}(\mathscr{F}_3)$$

- I won't bother with all the intermediate steps in the $\mathbf{S}_4$ case, but the set $\mathscr{F}_4$ is:

$$\{F(p, q), F(q, p), F(p, r), F(r, p), F(p, s), F(s, p), F(p, q \mathbin{\&} r), F(q \mathbin{\&} r, p), F(p, q \mathbin{\&} s), F(q \mathbin{\&} s, p), F(p, r \mathbin{\&} s), F(r \mathbin{\&} s, p), F(p, q \mathbin{\&} r \mathbin{\&} s), F(q \mathbin{\&} r \mathbin{\&} s, p), F(q, r), F(r, q), F(q, s), F(s, q), F(q, p \mathbin{\&} r), F(p \mathbin{\&} r, q), F(q, p \mathbin{\&} s), F(p \mathbin{\&} s, q), F(q, r \mathbin{\&} s), F(r \mathbin{\&} s, q), F(q, p \mathbin{\&} r \mathbin{\&} s), F(p \mathbin{\&} r \mathbin{\&} s, q), F(r, s), F(s, r), F(r, p \mathbin{\&} q), F(p \mathbin{\&} q, r), F(r, p \mathbin{\&} s), F(p \mathbin{\&} s, r), F(r, q \mathbin{\&} s), F(q \mathbin{\&} s, r), F(r, p \mathbin{\&} q \mathbin{\&} s), F(p \mathbin{\&} q \mathbin{\&} s, r), F(s, p \mathbin{\&} q), F(p \mathbin{\&} q, s), F(s, p \mathbin{\&} r), F(p \mathbin{\&} r, s), F(s, q \mathbin{\&} r), F(q \mathbin{\&} r, s), F(s, p \mathbin{\&} q \mathbin{\&} r), F(p \mathbin{\&} q \mathbin{\&} r, s), F(p \mathbin{\&} q, r \mathbin{\&} s), F(r \mathbin{\&} s, p \mathbin{\&} q), F(p \mathbin{\&} r, q \mathbin{\&} s), F(q \mathbin{\&} s, p \mathbin{\&} r), F(p \mathbin{\&} s, q \mathbin{\&} r), F(q \mathbin{\&} r, p \mathbin{\&} s)\}$$

- And, here are the sizes of the sets $\mathscr{F}_n$, for $n = 2$ to $n = 10$:

$\lvert\mathscr{F}_2\rvert$	$\lvert\mathscr{F}_3\rvert$	$\lvert\mathscr{F}_4\rvert$	$\lvert\mathscr{F}_5\rvert$	$\lvert\mathscr{F}_6\rvert$	$\lvert\mathscr{F}_7\rvert$	$\lvert\mathscr{F}_8\rvert$	$\lvert\mathscr{F}_9\rvert$	$\lvert\mathscr{F}_{10}\rvert$
2	12	50	180	602	1932	6050	18660	57002

- So, the combinatorics are a nightmare here (I don't have an analytical solution for $|\mathscr{F}_n|$ yet), but we have (at least) a technically correct construction of $\mathscr{F}$, and hence a technically correct definition of $\mathscr{C}$.

- A *MATHEMATICA*® 5 notebook which generates $\mathscr{F}_n$ for arbitrary n can be downloaded from:

  `http://fitelson.org/coherence2.nb`

- A PDF version of the above *MATHEMATICA*® 5 notebook can be downloaded from:

  `http://fitelson.org/coherence2.nb.pdf`

- In that notebook, the $n = 2$ and $n = 3$ cases of the following theorem are established.

  Theorem. *Let $\mathbf{S}_n$ be a set of size $n \geqslant 2$ of contingent statements. Then, $\mathscr{C}(\mathbf{S}_n) \geqslant 0 \Rightarrow \mathscr{C}(\mathbf{S}_n \cup \{\top\}) \geqslant 0$.*

  That is, adding a tautological statement to a (contingent) coherent set never results in an incoherent set. This was not true on my previous (incorrect) definitions of F and $\mathscr{C}$ (as shown by Luca Moretti).

- Nonetheless, it is still true that on the current definition, $\mathscr{C}(\mathbf{S}_n)$ will often be *less than* $\mathscr{C}(\mathbf{S}_n \cup \{\top\})$, even though they can never differ in their *sign*. It does seem odd that adding tautologies to a set can decrease its coherence. This is an artifact of the *averaging* in the definition of $\mathscr{C}$. If we just take $\mathscr{C}$ to the the *sum* of the values in $\mathscr{F}$, then we avoid this result, and adding tautologies can never decrease the coherence of a (contingent) set. If we take this route, however, we must then keep in mind that $\mathscr{C}(\mathbf{S}_n)$ will be on a $[-|\mathscr{F}_n|, |\mathscr{F}_n|]$ scale, and not on a $[-1, 1]$ scale, which means we have to be careful when we compare the coherence of sets with different sizes. But, perhaps this makes sense on independent grounds (especially, in light of the classes of examples discussed by Luca Moretti and others, which involve comparing sets $\mathbf{S}$ with smaller sets consisting of logical compounds of the elements of $\mathbf{S}$).