▪ algorithm for generating $\mathcal{F}$[S]

```
Needs["DiscreteMath`Combinatorica`"]
P1[S_] := Flatten[Join[Table[KSubsets[S, i], {i, 1, Length[S]}]], 1];
P2[S_] :=
  Flatten[Permutations /@ Select[KSubsets[P1[S], 2], Length[(#1[[1]]) ⋂ (#1[[2]])] == 0 &], 1];
f[{s1_, s2_}] := F[And @@ s1, And @@ s2];
𝓕[S_] := f /@ P2[S];
```

▪ examples

```
𝓕[{p, q}] // TraditionalForm
```

$\{F(p, q), F(q, p)\}$

```
𝓕[{p, q, r}] // TraditionalForm
```

$\{F(p, q), F(q, p), F(p, r), F(r, p), F(p, q \wedge r), F(q \wedge r, p), F(q, r), F(r, q), F(q, p \wedge r), F(p \wedge r, q), F(r, p \wedge q), F(p \wedge q, r)\}$

```
𝓕[{p, q, r, s}] // TraditionalForm
```

$\{F(p, q), F(q, p), F(p, r), F(r, p), F(p, s), F(s, p), F(p, q \wedge r), F(q \wedge r, p), F(p, q \wedge s), F(q \wedge s, p), F(p, r \wedge s), F(r \wedge s, p), F(p, q \wedge r \wedge s), F(q \wedge r \wedge s, p), F(q, r), F(r, q), F(q, s), F(s, q), F(q, p \wedge r), F(p \wedge r, q), F(q, p \wedge s), F(p \wedge s, q), F(q, r \wedge s), F(r \wedge s, q), F(q, p \wedge r \wedge s), F(p \wedge r \wedge s, q), F(r, s), F(s, r), F(r, p \wedge q), F(p \wedge q, r), F(r, p \wedge s), F(p \wedge s, r), F(r, q \wedge s), F(q \wedge s, r), F(r, p \wedge q \wedge s), F(p \wedge q \wedge s, r), F(s, p \wedge q), F(p \wedge q, s), F(s, p \wedge r), F(p \wedge r, s), F(s, q \wedge r), F(q \wedge r, s), F(s, p \wedge q \wedge r), F(p \wedge q \wedge r, s), F(p \wedge q, r \wedge s), F(r \wedge s, p \wedge q), F(p \wedge r, q \wedge s), F(q \wedge s, p \wedge r), F(p \wedge s, q \wedge r), F(q \wedge r, p \wedge s)\}$

```
Table[Length[𝓕[Range[n]]], {n, 2, 10}]
```

```
{2, 12, 50, 180, 602, 1932, 6050, 18660, 57002}
```

▪ Adding T to a (contingent) coherent set of size 2 can never yield an incoherent set

Here's $\mathcal{F}$[{p,q,T}]:

```
𝓕[{p, q, T}] // TraditionalForm
```

$\{F(p, q), F(q, p), F(p, T), F(T, p), F(p, q \wedge T), F(q \wedge T, p), F(q, T), F(T, q), F(q, p \wedge T), F(p \wedge T, q), F(T, p \wedge q), F(p \wedge q, T)\}$

Any term of the form *F*[_, T] will be zero, by the definition of *F*. So,

```
% //. F[x_, T] → 0 // TraditionalForm
```

$\{F(p, q), F(q, p), 0, F(T, p), F(p, q \wedge T), F(q \wedge T, p), 0, F(T, q), F(q, p \wedge T), F(p \wedge T, q), F(T, p \wedge q), 0\}$

Moreover, any conjunction of the form X & T can be rewritten as X, since it is logically equivalent to X:

```
% //. x_ && T → x // TraditionalForm
```

$\{F(p, q), F(q, p), 0, F(T, p), F(p, q), F(q, p), 0, F(T, q), F(q, p), F(p, q), F(T, p \wedge q), 0\}$

Finally, any term of the form *F*[T,_] will be 1, by the definition of *F*:

```
% //. F[T, x_] → 1 // TraditionalForm
```

$\{F(p, q), F(q, p), 0, 1, F(p, q), F(q, p), 0, 1, F(q, p), F(p, q), 1, 0\}$

Now, we can take the Mean, to yield C({p,q,T}):

```
Mean[%] // TraditionalForm
```

$$\frac{1}{12}(3\,F(p, q) + 3\,F(q, p) + 3)$$

Now, let's compare this with C({p,q}):

```
Mean[ℱ[{p, q}]] // TraditionalForm
```

$$\frac{1}{2}(F(p, q) + F(q, p))$$

The following proves that If C({p,q}) ≥ 0, then C({p,q,T}) ≥ 0.

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Needs["Algebra`InequalitySolve`"]
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InequalitySolve[x/2 ≥ 0 && 1/12 (3 x + 3) < 0 && -2 ≤ x ≤ 2, {x}]
```

```
False
```

■ Adding T to a (contingent) coherent set of size 3 can never yield an incoherent set

```
ℱ[{p, q, r, T}] // TraditionalForm
```

$\{F(p, q), F(q, p), F(p, r), F(r, p), F(p, T), F(T, p), F(p, q \wedge r), F(q \wedge r, p), F(p, q \wedge T), F(q \wedge T, p), F(p, r \wedge T), F(r \wedge T, p),$
$F(p, q \wedge r \wedge T), F(q \wedge r \wedge T, p), F(q, r), F(r, q), F(q, T), F(T, q), F(q, p \wedge r), F(p \wedge r, q), F(q, p \wedge T), F(p \wedge T, q), F(q, r \wedge T),$
$F(r \wedge T, q), F(q, p \wedge r \wedge T), F(p \wedge r \wedge T, q), F(r, T), F(T, r), F(r, p \wedge q), F(p \wedge q, r), F(r, p \wedge T), F(p \wedge T, r), F(r, q \wedge T),$
$F(q \wedge T, r), F(r, p \wedge q \wedge T), F(p \wedge q \wedge T, r), F(T, p \wedge q), F(p \wedge q, T), F(T, p \wedge r), F(p \wedge r, T), F(T, q \wedge r), F(q \wedge r, T),$
$F(T, p \wedge q \wedge r), F(p \wedge q \wedge r, T), F(p \wedge q, r \wedge T), F(r \wedge T, p \wedge q), F(p \wedge r, q \wedge T), F(q \wedge T, p \wedge r), F(p \wedge T, q \wedge r), F(q \wedge r, p \wedge T)\}$

```
% //. F[x_, T] → 0 // TraditionalForm
```

$\{F(p, q), F(q, p), F(p, r), F(r, p), 0, F(T, p), F(p, q \wedge r), F(q \wedge r, p), F(p, q \wedge T), F(q \wedge T, p), F(p, r \wedge T),$
$F(r \wedge T, p), F(p, q \wedge r \wedge T), F(q \wedge r \wedge T, p), F(q, r), F(r, q), 0, F(T, q), F(q, p \wedge r), F(p \wedge r, q), F(q, p \wedge T),$
$F(p \wedge T, q), F(q, r \wedge T), F(r \wedge T, q), F(q, p \wedge r \wedge T), F(p \wedge r \wedge T, q), 0, F(T, r), F(r, p \wedge q), F(p \wedge q, r), F(r, p \wedge T),$
$F(p \wedge T, r), F(r, q \wedge T), F(q \wedge T, r), F(r, p \wedge q \wedge T), F(p \wedge q \wedge T, r), F(T, p \wedge q), 0, F(T, p \wedge r), 0, F(T, q \wedge r), 0,$
$F(T, p \wedge q \wedge r), 0, F(p \wedge q, r \wedge T), F(r \wedge T, p \wedge q), F(p \wedge r, q \wedge T), F(q \wedge T, p \wedge r), F(p \wedge T, q \wedge r), F(q \wedge r, p \wedge T)\}$

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% //. x_ && T → x // TraditionalForm
```

$\{F(p, q), F(q, p), F(p, r), F(r, p), 0, F(T, p), F(p, q \wedge r), F(q \wedge r, p), F(p, q), F(q, p), F(p, r), F(r, p), F(p, q \wedge r), F(q \wedge r, p),$
$F(q, r), F(r, q), 0, F(T, q), F(q, p \wedge r), F(p \wedge r, q), F(q, p), F(p, q), F(q, r), F(r, q), F(q, p \wedge r), F(p \wedge r, q), 0,$
$F(T, r), F(r, p \wedge q), F(p \wedge q, r), F(r, p), F(p, r), F(r, q), F(q, r), F(r, p \wedge q), F(p \wedge q, r), F(T, p \wedge q), 0, F(T, p \wedge r),$
$0, F(T, q \wedge r), 0, F(T, p \wedge q \wedge r), 0, F(p \wedge q, r), F(r, p \wedge q), F(p \wedge r, q), F(q, p \wedge r), F(p, q \wedge r), F(q \wedge r, p)\}$

```
% //. F[T, x_] → 1 // TraditionalForm
```

$\{F(p, q), F(q, p), F(p, r), F(r, p), 0, 1, F(p, q\wedge r), F(q\wedge r, p), F(p, q), F(q, p), F(p, r), F(r, p), F(p, q\wedge r), F(q\wedge r, p), F(q, r), F(r, q), 0, 1, F(q, p\wedge r), F(p\wedge r, q), F(q, p), F(p, q), F(q, r), F(r, q), F(q, p\wedge r), F(p\wedge r, q), 0, 1, F(r, p\wedge q), F(p\wedge q, r), F(r, p), F(p, r), F(r, q), F(q, r), F(r, p\wedge q), F(p\wedge q, r), 1, 0, 1, 0, 1, 0, 1, 0, F(p\wedge q, r), F(r, p\wedge q), F(p\wedge r, q), F(q, p\wedge r), F(p, q\wedge r), F(q\wedge r, p)\}$

```
Mean[%] // TraditionalForm
```

$$\frac{1}{50}(3F(p, q) + 3F(p, r) + 3F(p, q\wedge r) + 3F(q, p) + 3F(q, r) + 3F(q, p\wedge r) + 3F(r, p) + 3F(r, q) + 3F(r, p\wedge q) + 3F(p\wedge q, r) + 3F(p\wedge r, q) + 3F(q\wedge r, p) + 7)$$

```
Mean[𝓕[{p, q, r}]] // TraditionalForm
```

$$\frac{1}{12}(F(p, q) + F(p, r) + F(p, q\wedge r) + F(q, p) + F(q, r) + F(q, p\wedge r) + F(r, p) + F(r, q) + F(r, p\wedge q) + F(p\wedge q, r) + F(p\wedge r, q) + F(q\wedge r, p))$$

The following proves that if $C(\{p,q,r\}) \geq 0$, then $C(\{p,q,r,T\}) \geq 0$.

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Needs["Algebra`InequalitySolve`"]
```

$\text{InequalitySolve}\left[\frac{x}{12} \geq 0 \;\&\&\; \frac{1}{50}(3x + 7) < 0 \;\&\&\; -12 \leq x \leq 12, \{x\}\right]$

```
False
```

This can easily be generalized to an inductive proof, for all *n*.

But, we can still have the coherence being DECREASED when we add a tautology. This is seen in the following result (n = 2 case):

In[26]:= $\text{InequalitySolve}\left[\frac{x}{2} > \frac{1}{12}(3x + 3) \;\&\&\; -2 \leq x \leq 2, \{x\}\right]$

Out[26]= $1 < x \leq 2$

This is an artifact of the *averaging* in the definition. If we just take the *sum* of the *F*-values, then this cannot happen. In the n = 2 case:

$$C(\{p, q\}) = F(p, q) + F(q, p)$$

$$C(\{p, q, \text{T}\}) = 3F(p, q) + 3F(q, p) + 3 = 3[C(\{p, q\}) + 1]$$