Credence in the Image of Chance *

Michael Caie

Let us say that a doxastic state of an agent is **vindicated** just in case it is objectively as it ought to be. In the case of qualitative belief states, it is plausible that vindication consists in believing only what is true. It is, however, much less obvious what constitutes vindication for credal states. Following Hájek [n.d.], we may put the question as follows. How exactly should we fill in the blank below?

Truth : Belief :: _________ : Credence

According to some (for example, Joyce [1998] and Joyce [2009]) vindication for credal states consists in having credence 1 in truths and credence 0 in falsehoods. According to others (for example, van Fraassen [1983] and Shimony [1988]) vindication for credal states consists in matching the limiting relative frequencies. While still others (for example, Hájek [n.d.]) maintain that vindication for credal states consists in matching the objective chances.

In this paper, I won't try to settle the difficult question of how vindication for credal states should be characterized (though I will have some things to say on the matter). Instead, I want to focus on the following question:

> **Target Question:** What normative constraints, if any, are imposed on an agent's credal state given the assumption that vindication for credal states consists in matching the chances?

This question is worth serious consideration, since it is not implausible that vindication for credal states does consist in matching the objective chances.

It is natural to think that an agent's credences about various events are rationally constrained by her credences in the chances of those events. For example, if one thinks that a coin has 50% chance of coming up heads, then it seems that one ought to have credence 0.5 in the claim that the coin will come up heads. Lewis [1980] argues that a principle such as the following captures the way in which credence about chance imposes rational constraints on our credences in other propositions:[1]:

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*Thanks to Richard Pettigrew for helpful correspondence on this material.

[1]Lewis [1980] provides two formulations of the Principal Principle which he argues are equivalent. Neither of these formulations, however, is the same as our formulation of the Principal Principle. Still, given plausible assumptions, we can show that this formulation is

> **Principal Principle:** Let $ch(\cdot)$ be a possible ur-chance function, i.e., a function describing a possible initial chance distribution. And let C_{ch} be the proposition that claims that the ur-chances are given by $ch(\cdot)$. Then a rational agent's initial credences, i.e., her credences prior receiving any evidence, defined over an algebra containing C_{ch}, ought to be such that:
>
> $Cr(A|C_{ch} \wedge E) = ch(A)$ (given that $Cr(C_{ch} \wedge E) \neq 0$ and $ch(A|E) = ch(A)$).

The Principal Principle certainly *seems* plausible. Still, the only justification that Lewis offers for this putative norm is that it appears to deliver the right results in a number of cases. One might hope, however, that the Principal Principle, if correct, could be given some deeper justification. Pettigrew [2012] attempts to provide just such a justification. There, Pettigrew shows that the Principal Principle follows if we assume that:

> **Initial Vindication:** An agent's initial credences are vindicated just in case they match the ur-chances.

Now it is tempting, I think, to see this argument as providing an answer to our target question. But to conclude this would be a mistake. While the Principal Principle is entailed by Initial Vindication, this normative constraint is not entailed by the assumption that credal vindication consists in matching the chances.

In what follows, however, I'll show that Pettigrew's argument for the Principal Principle may be repurposed in the service of answering our question. In particular, I'll show how a variant of a theorem proved in Pettigrew [2012] may be used to establish a plausible norm connecting chance and credence, given the assumption that credal vindication consists in matching the chances. The norm that is so entailed, however, is interestingly different in kind from the Principal Principle. For, unlike the Principal Principle, the chance-credence norm that is entailed by the claim that credal vindication consists in matching the chances essentially involves *temporal* propositions, i.e., proposition whose truth-values are relativized not just to worlds but also to times. If chance stands to credence as truth to belief, then what normatively constrains our credences are not propositions of the form: **The chances at t are given by $ch(\cdot)$**, whose truth-values are invariant across times, but instead propositions

equivalent to Lewis'. Consider Lewis' second formulation, which says that an agent's initial credences should be such that: $Cr(A|T_w \wedge H_{tw}) = ch_{tw}(A)$, where T_w is the conjunction of history-to-chance counterfactuals true at w, H_{tw} is the history in w up to time t, and $ch_{tw}(\cdot)$ is the chance-function at world w and time t. As Pettigrew [2012] fn.4 shows, this rational constraint is entailed by our Principal Principle. Furthermore, assuming that there is a proposition $H_{t_0 w}$ describing the initial conditions of w which, together with T_w, determines an ur-chance function, then Lewis' principle entails our Principal Principle.

of the form: **The chances are given by** $ch(\cdot)$, which may be true at some times and false at others.

The paper proceeds as follows.

In §1, I begin by outlining Pettigrew's argument for the Principal Principle.

In §2, I show that this argument does not establish that the Principal Principle follows from the assumption that vindication for credences consists in matching the chances.

In §3, I argue that Initial Vindication has features that make it implausible as an account of credal vindication, and that it is more plausible to assume that vindication for credal states consist in matching the chances. Pettigrew's argument, then, doesn't provide a convincing justification of the Principal Principle.

Finally, in §4, I argue that, given the assumption that vindication for credences consists in matching the chances, we can provide a justification for a chance-credence norm which constrains our credences in propositions conditional on temporally self-locating chance propositions.

1 The Principal Principle and Initial Vindication

In this section, I will outline the argument provided in Pettigrew [2012] for the Principal Principle. Before doing so, however, let me highlight a few assumptions that I'll be making.

First, unless otherwise noted, talk of 'possible worlds' throughout should be understood as referring to maximally specific ways the a world might be that cannot ruled out a priori. In certain cases, in order to stress the connection between these possibilities and our epistemic capacities, I will refer to these as 'epistemically possible worlds'.

Second, I will assume here and in the following two sections that credal states and chance functions are defined over algebras of **eternal propositions**. Such propositions determine functions from worlds to truth values. Later we will relax this assumption and allow that credal states and chance functions may be defined over algebras of **temporal propositions**, which determine functions from world-time pairs to truth-values. To avoid unnecessary prolixity, however, in this and the following two sections the term 'proposition' will be reserved for eternal propositions.

Finally, I will assume that each possible ur-chance function $ch(\cdot)$ is a probability function such that $ch(C_{ch}) = 1$.[2] That is, I will take it that ur-

[2] A probability function f is a mapping from an algebra of propositions to real numbers that satisfies the following constraints:

> **Normalization:** For any logical truth $\top$, $f(\top) = 1$
>
> **Non-Negativity:** For any proposition ϕ, $0 \leq f(\phi)$
>
> **Finite Additivity:** If ϕ and ψ are incompatible propositions, then $f(\phi \vee \psi) = f(\phi) + f(\psi)$

See Hájek [n.d.] and Pettigrew [2012] for a defense of the claim that chance functions are chance functions are non self-undermining probability functions. As is well known, the Principal Principle leads to probabilistic incoherence if there are self-undermining chances. Two alternatives to the Principal Principle that allow for self-undermining chances are the New Principle (see Thau [1994], Hall [1994] and Lewis [1994]) and the Generalized Principal Principle (see Ismael [2008]). Pettigrew [2012] offers parallel justifications for both of these principles, given the assumption that chance functions may be self-undermining. Issues concerning self-undermining chances, however, are orthogonal to the main points that I want to make in this paper. For, the points that follow apply, mutatis mutandis, to the putative justifications of the New Principle and the Generalized Principal Principle. I'll, however, leave these details to the interested reader.

Now on to Pettigrew's argument. Let $\mathcal{A}$ be a finite algebra of propositions. Let $\mathcal{B} = \{b : \mathcal{A} \to \mathbb{R}\}$, i.e., the set of functions mapping the members of $\mathcal{A}$ to real numbers. This set will serve as a representation of possible credal states. We will say that a credal state $Cr(\cdot)$ is **potentially vindicated** just in case it is vindicated relative to some epistemically possible world.[3] We let $\mathcal{V} \subseteq \mathcal{B}$ serve as a representation of the potentially vindicated credal states. We let:

$$\mathcal{V}^+ = \{\sum_{v \in \mathcal{V}} \lambda_v v : 0 \leq \lambda_v \leq 1 \text{ and } \sum_{v \in \mathcal{V}} \lambda_v = 1\}.$$

This is the so-called convex hull of $\mathcal{V}$. And, finally, we let $I : \mathcal{B} \times \mathcal{V} \to \mathbb{R}$, be a function measuring the distance of a credal state $b(\cdot)$ from a potentially vindicated credal state $v(\cdot)$. We'll call this the **inaccuracy measure**.

Jim Joyce has shown that, given certain plausible constraints on the inaccuracy measure, the following theorem holds:[4]

> **Dominance Theorem :** For any $b \in \mathcal{B} - \mathcal{V}^+$, there is some $b' \in \mathcal{V}^+$ such that, for every $v \in \mathcal{V}$, $I(b', v) < I(b, v)$.

probability function.

[3]Pettigrew [2012] takes the set of potentially vindicated credal states to be those that are vindicated relative to some "possible world". This, however, leaves it open whether it is metaphysically possible worlds or epistemically possible worlds that are in question. For reasons that will emerge below, I think that for the purposes of justifying various norms we should take the potentially vindicated norms to be those that are vindicated relative to some epistemically possible world. None of the key formal results, however, will turn on which of these two interpretations of $\mathcal{V}$ we employ.

[4]See Joyce [1998]. Pettigrew [2012] provides a useful summary. Joyce [2009] shows, in addition, that the following holds:

> **Converse Dominance Theorem:** For any $b \in \mathcal{V}^+$, there is no $b' \in \mathcal{B} - \mathcal{V}^+$ such that:
>
> (i) For every $v \in \mathcal{V}$, $I(b', v) \leq I(b, v)$.
>
> (ii) For some $v \in \mathcal{V}$, $I(b', v) < I(b, v)$.

This theorem may be used to argue for the following normative constraint on credences:

> **Convexity:** An agent ought to have a credal state that is in the convex hull of the set of potentially vindicated credal states.

How might we use the Dominance Theorem to argue for Convexity? The argument, I think, should go as follows. We begin with the following plausible thought:

> **Vindication Dominance:** An agent is rationally prohibited from having a credal state $Cr(\cdot)$, if the agent is in a position to tell *a priori* that there is some other credal state $Cr'(\cdot)$ that is closer to being vindicated than $Cr(\cdot)$ no matter what the world is like.

Next we note that, since it is a priori knowable which credal states are potentially vindicated, the Dominance Theorem assures us that if one has a credal state $Cr(\cdot)$ that is not in the convex hull of the set of potentially vindicated credal states, then one is in a position to tell *a priori* that there is some other credal state $Cr'(\cdot)$ that is closer to being vindicated than $Cr(\cdot)$ no matter what the world is like.[5] And so it follows from Vindication Dominance that an agent will always be rationally prohibited from having a credal state that is not in the convex hull of the set of potentially vindicated credal states, and so will be rationally required to have a credal state that is in the convex hull of the set of potentially vindicated credal states.[6]

Convexity, then, provides us with the following two-stage strategy for justifying some putative norm of rationality N:

> Stage 1: Argue that the set of potentially vindicated credal states $\mathcal{V}$ is identical to some set S.
>
> Stage 2: Show that if $\mathcal{V} = S$, then every member of $\mathcal{V}^+$ satisfies the conditions imposed by N.

Joyce [1998] and Joyce [2009] offer just such a justification for the following putative norm:

[5]Note that for this justification it is important that the set of potentially vindicated credal states be those that are vindicated relative to some epistemically possible world. If, instead, we took the set of potentially vindicated credal states to be those that are vindicated relative to some metaphysically possible world, we couldn't, at least without significant further argument, assume that it was a priori knowable which worlds are potentially vindicated. Moreover, if it is a priori knowable which worlds are metaphysically possible, then our notion of vindication will be identical to this alternative.

[6]Note that this argument makes no appeal to the Converse Dominance Theorem. Nonetheless, there is an important auxiliary role for this theorem to play. For, given what we've said so far, it is still possible that Vindication Dominance could also rule out as irrational those credal states in the convex hull of the set of vindicated credal states. The Converse Dominance Theorem assures us that this isn't so, and so assures us that Vindication Dominance does not lead to a rational dilemma.

Probabilism: If an agent S has credences defined over some algebra of propositions $\mathcal{A}$, then S's credences over $\mathcal{A}$ ought to be probabilistically coherent.[7]

Joyce's argument depends on the following two claims:

Alethic Vindication: An agent's credal state $Cr(\cdot)$ is vindicated at a world w just in case, for every proposition ϕ over which $Cr(\cdot)$ is defined $w(\phi) = b(\phi)$, where $w(\cdot)$ is the characteristic truth-value function for w. Let $\mathcal{V}_A = \{b :$ For some logically possible distribution of truth-values $w(\cdot)$ over $\mathcal{A}, w(\phi) = b(\phi)$, for every $\phi \in \mathcal{A}\}$. Then we have: $\mathcal{V} = \mathcal{V}_A$.

De Finetti's Theorem: Every $b \in \mathcal{V}_A^+$ is a probability function.[8]

Given Convexity, it follows from these two claims that an agent should always have probabilistically coherent credences.

Pettigrew [2012] employs the same general strategy in service of justifying both the Principal Principle and the claim that a rational agent's initial credences should always be probabilistically coherent.[9] Pettigrew's argument depends on the following two claims:

Initial Vindication: An agent's *initial* credal state $Cr(\cdot)$ is vindicated at a world w just in case, for every proposition ϕ over which $Cr(\cdot)$ is defined, $Cr(\phi) = ch_w(\phi)$, where $ch_w(\cdot)$ is the ur-chance function for w. Let $\mathcal{V}_I = \{b :$ For some possible ur-chance distribution over $\mathcal{A}$, $ch(\cdot), ch(\phi) = b(\phi)$, for every $\phi \in \mathcal{A}\}$. Then we have that the set of potentially vindicated *initial credence functions* $\mathcal{V} = \mathcal{V}_I$.

Pettigrew's Theorem: If each possible chance function $ch(\cdot)$ is a probability function such that $ch(C_{ch}) = 1$, then every $b \in \mathcal{V}_I^+$ is such that:

(i) b is a probability function,

(ii) $b(A|C_{ch}) = ch(A)$ (given that $b(C_{ch})$ is well-defined and $b(C_{ch}) \neq 0$).

[7] We say that $Cr(\cdot)$ is **probabilistically coherent** just in case it can be represented by a probability function.

[8] See de Finetti [1974].

[9] This latter claim is, of course, weaker than Probabilism, since Probabilism demands probabilistic coherence not just for initial credal states but for all credal states. However, if one also endorses the claim that an agent should always update her credences by conditionalization, then Probabilism will follow from the claim that a rational agent's initial credences ought to be probabilistically coherent. For if one's credences are probabilistically coherent and one updates by conditionalization, then one's credences will remain probabilistically coherent.

Since we will want to repurpose a modified version of this theorem later on, it is worthwhile presenting the proof of this result here.[10]

Proof of Pettigrew's Theorem: This result follows from the following two claims:

(1) If each chance function $ch(\cdot)$ is a probability function such that $ch(C_{ch}) = 1$, then each member of $\mathcal{V}_I$ satisfies conditions (i)-(ii).

(2) If b and b' satisfy conditions (i)-(ii), then so does $\lambda b + (1-\lambda)b'$.

Proof of (1): Clearly, if each chance function $ch(\cdot)$ is a probability function, then each member of $\mathcal{V}_I$ satisfies (i). To see that (ii) is also satisfied, note that if each chance function $ch(\cdot)$ is such that $ch(C_{ch}) = 1$, then it follows that for each $C_{ch} \in \mathcal{A}$ and each $b \in \mathcal{V}_I$, if $b(C_{ch}) \neq 0$, then $b(C_{ch}) = 1$ and, for every $\phi \in \mathcal{A}$, $b(\phi) = ch(\phi)$. Thus, if $b(C_{ch})$ is well-defined and $b(C_{ch}) \neq 0$, then $b(A|C_{ch}) = ch(A)$. This suffices to establish that each member of $\mathcal{V}_I$ satisfies condition (ii).

Proof of (2): Let b and b' satisfy (i) and (ii). Given this, we have:

$$\begin{aligned}
\lambda b + (1-\lambda)b'(A|C_{ch}) &= \frac{\lambda b(A \wedge C_{ch}) + (1-\lambda)b'(A \wedge C_{ch})}{\lambda b(C_{ch}) + (1-\lambda)b'(C_{ch})} \\
&= \frac{\lambda b(A|C_{ch})b(C_{ch}) + (1-\lambda)b(A|C_{ch})b(C_{ch})}{\lambda b(C_{ch}) + (1-\lambda)b(C_{ch})} \\
&= \frac{\lambda ch(A)b(C_{ch}) + (1-\lambda)ch(A)b(C_{ch})}{\lambda b(C_{ch}) + (1-\lambda)b(C_{ch})} \\
&= ch(A)
\end{aligned}$$

Given Convexity, then, it follows from Initial Vindication and Pettigrew's Theorem, together with the assumption that each chance function is a non-self-undermining probability function, that an agent should always have an initial credal state that is probabilistically coherent and that, in addition, satisfies the constraints imposed by the Principal Principle.

2 The Principal Principle and Chance Vindication

In this section, I'll consider whether Pettigrew's proof may be used to show that the Principal Principle follows from the assumption that vindication for credences consists in matching the chances. I'll argue that the answer is 'no'. Of course, if one accepts Initial Vindication, this fact won't be particularly

[10]For further details see the appendix to Pettigrew [2012].

worrying. In the following section, however, I'll argue that we should not accept Initial Vindication.

Consider, first, vindication as it applies to belief states. Plausibly, an agent's belief state counts as vindicated just in case the agent believes only truths. Since what is true, however, differs from world to world, it follows that vindication itself differs from world to world. More generally, then, we might say:

> **Belief Vindication:** An agent's belief state is vindicated at a world w just in case, for every proposition ϕ over which the agent's belief state is defined, the agent believes ϕ just in case ϕ is true at w.

Like truth-values, chances differ from world to world. Assuming that vindication for credal states consists in matching the chances, credal vindication, like belief vindication, should, then, be relativized to worlds. However, unlike truth-values, chances vary not only across worlds but also across times.[11] What was once chancy may become fixed. For this reason, vindication for credal states should be relativized not only to worlds but also to times.

The motivation here is the same as that which compels us to relativize belief vindication to worlds. Whether a set of beliefs match the truth-values depends on in which world the beliefs are held. Thus, the answer to the question whether an agent's beliefs are vindicated depends on in which world the agent is located. Similarly, whether a credal state matches the chances depends on in which world and at which time the credences are held. Assuming that chance stands to credence as truth to belief, it then follows that the answer to the question whether an agent's credences are vindicated depends on in which world and at which time the agent is located.

We can formulate the claim that credal vindication consists in matching the chances, then, as follows:

> **Chance Vindication:** An agent's credal state $Cr(\cdot)$ is vindicated at a world w and time t just in case, for every proposition ϕ over which $Cr(\cdot)$ is defined, $Cr(\phi) = ch^t_w(\phi)$ (where $ch^t_w(\cdot)$ is the chance-function for world w at time t.) Let $\mathcal{V}_C = \{b :$ For some possible chance distribution over $\mathcal{A}$ at world w and time t, $ch^t_w(\cdot), ch^t_w(\phi) = b(\phi)$, for every $\phi \in \mathcal{A}\}$. Then we have: $\mathcal{V} = \mathcal{V}_C$.

[11] Two points. First, this is only true if we are talking about eternal propositions. Since temporal propositions may change their truth-values over time, if we were to consider belief states whose objects were propositions of this type, then belief vindication would also have to be relativized to both worlds and times.

Second, I am assuming, along with Lewis, Pettigrew and others, that chances are determined by a world and a time. There are, however, those who think that additional features are required in order to determine a chance distribution. See e.g., Meacham [2005]. I won't, however, consider what chance-credence norms should look like on the assumption that chances are determined relative to a richer set of parameters.

The first point to note is that Initial Vindication and Chance Vindication will, in certain cases, issue incompatible verdicts about which credal states are vindicated. To see this, consider some agent Q who is at the beginning of her epistemic life. Let w_q be the world in which Q is located and t_q the time at which her epistemic life begins. Initial Vindication says that Q's credal state $Cr_q(\cdot)$ is vindicated just in case, for every proposition ϕ over which it is defined, $Cr_q(\phi) = ch_{w_q}(\phi)$, where $ch_{w_q}(\cdot)$ is the ur-chance function at w_q. Chance Vindication, on the other hand, says that $Cr_q(\cdot)$ is vindicated just in case, for every proposition ϕ over which it is defined, $Cr_q(\phi) = ch^{t_q}_{w_q}(\phi)$. There is, however, no guarantee, in general, that $ch_{w_q}(\phi)$ will be identical to $ch^{t_q}_{w_q}(\phi)$. For the chances may change over time. Thus, Initial Vindication and Chance Vindication will, in certain cases, issue incompatible verdicts about which credal states count as vindicated.

Now the fact that Initial Vindication and Chance Vindication may disagree about whether, in certain cases, an agent's credal state counts as vindicated does not itself establish that Pettigrew's proof doesn't suffice to show that the Principal Principle follows from Chance Vindication. For Pettigrew's proof depends only on which credal states are in the convex hull of the set of potentially vindicated credal states given Initial Vindication. And, at least in principle, the fact that chances change over time is compatible with the claim that the convex hull of the set of potentially vindicated credal states given Initial Vindication is identical to the convex hull of the set of potentially vindicated credal states given Chance Vindication. If, then, $\mathcal{V}_I^+ = \mathcal{V}_C^+$, Pettigrew's proof would establish that the Principal Principle follows from both Chance Vindication and Initial Vindication.

We can show, however, that $\mathcal{V}_I^+ \neq \mathcal{V}_C^+$. To see this, note that every element $b(\cdot)$ in the convex hull of the set of ur-chance functions is such that $b(C_{ch}) = 1$ iff $b(\cdot) = ch(\cdot)$. Now consider the ur-chance function for a world w, $ch_w(\cdot)$, and the chance function for that world at time t, $ch^t_w(\cdot)$, such that $ch_w(\cdot) \neq ch^t_w(\cdot)$. Since chances change over time, there will be such a pair of chance functions. Now $ch^t_w(\cdot)$ is identical to $ch_w(\cdot|H^t_w)$, where H^t_w is the proposition detailing the total history of w up to time t. Thus, since $ch_w(C_{ch_w}) = 1$, and since conditional probabilities preserve certainties, it follows that $ch_w(C_{ch_w}|H^t_w) = 1 = ch^t_w(C_{ch_w})$. But since $ch_w(\cdot)$ is the only element in the convex hull of the set of ur-chance functions such that $ch_w(C_{ch_w}) = 1$ and, by hypothesis, $ch_w(\cdot) \neq ch^t_w(\cdot)$, it follows that $ch^t_w(\cdot)$ is not identical to a member of the convex hull of the set of ur-chance functions.

This shows, then, that there are possible chance functions that are not in $\mathcal{V}_I^+$. And so there are credal states that are members of $\mathcal{V}_C^+$ that are not members of $\mathcal{V}_I^+$. It follows, then, that Pettigrew's proof that the Principal Principle follows from Initial Vindication does not suffice to show that the Principal Principle follows from Chance Vindication. For it doesn't follow from the fact that every credal state in $\mathcal{V}_I^+$ satisfies the constraints imposed by the Principal Principle, that the same is true of every credal state in $\mathcal{V}_C^+$.

3 Against Initial Vindication

We've seen that Pettigrew's proof does not provide an immediate answer to our target question. For our question is: What normative constraints, if any, are imposed on an agent's credal state given the assumption that vindication for credences consists in matching the chances. But, as we've seen, this is just the question: What normative constraints, if any, are imposed by Chance Vindication? And Pettigrew's proof does not provide an immediate answer to that question.

One may, however, wonder at this point if our question is worth pursuing. For, if there were good reason to prefer Initial Vindication to Chance Vindication, this would undercut much of the motivation for trying to determine what normative constraints follow from the latter principle. In this section, however, I'll argue that there are, in fact, good reasons to prefer Chance Vindication to Initial Vindication.

To begin with, Chance Vindication is, I think, prima facie more plausible than Initial Vindication. For, just as it is hard to see why, say, in the case of belief, facts about the truth-values of propositions at some other world than those at which an agent's beliefs are held should determine whether the agent's beliefs are vindicated, so too, I think, it is hard to see why, in the case of credence, facts about the chances of propositions at some other time than that at which an agent's credences are held should determine whether the agent's credences are vindicated. There is, at the very least, then, a challenge for the proponent of Initial Vindication to say why it is the ur-chances that are relevant to the vindication of initial credences, even if those credences are held at times when the chances differ from the ur-chances. I'm skeptical, however, that there is any adequate justification to be offered here.

Pettigrew, at least, doesn't address this challenge. In support of Initial Vindication, he cites a thought experiment from Hájek [n.d.]. Consider two agents: A and B. Both agents are considering the proposition that a radioactive isotope will decay in the next week. The chance of this happening, let's suppose, is 0.6. B, we'll assume, is certain that the isotope will decay, while A has credence 0.6 that it will decay. We'll further suppose that the isotope does in fact decay within the next week. Pettigrew claims that in this case it is A's credences that are vindicated and I'm inclined to agree. But note that this verdict doesn't support Initial Vindication. For the chances in this case are those at the time that the agent's credences are held. Thus the intuition that A's credences in this sort of situation are vindicated supports, not Initial Vindication, but instead Chance Vindication.[12]

Of course, one may, I think, be reasonably doubtful of the probative force of

[12] It is perhaps worth noting that Hájek considers this type of case as part of a defense of Chance Vindication. Thus: "A degree of belief for a proposition that agrees with the corresponding objective chance for that proposition, *at the time at which the degree of belief is held*, has the virtue that we have been seeking: the probabilistic analogue of truth." Hájek [n.d.] p. 13 (emphasis mine).

these sorts of intuitive considerations. But there is a further, more principled, reason to be skeptical of Initial Vindication.

To see this, first note that there is a striking disanalogy between, on the one hand, the picture of vindication for credal states that emerges from Alethic Vindication and Chance Vindication, and, on the other hand, the picture of vindication for credal states that emerges from Initial Vindication. Given Alethic Vindication and Chance Vindication, whether an agent's credal state is vindicated is an objective matter in the following sense. Any two agents with the same credal state who are located in the same world and same time will be such that either both of their credal states are vindicated or neither are.[13] Given Initial Vindication, however, whether an agent's credal state is vindicated depends not only on objective facts about where in logical and temporal space the agent is located but also on facts about what the agent's evidence is.

Here's why. According to Initial Vindication, an agent's initial credal state, i.e., her credal state prior to receiving any evidence, is vindicated just in case it matches the ur-chances. But the proponent of Initial Vindication can't reasonably hold that, in general, an agent's credal state is vindicated just in case it matches the ur-chances. That is, the proponent of Initial Vindication should not endorse the following general principle:

> **Ur-Chance Vindication:** An agent's credal state $Cr(\cdot)$ is vindicated at a world w just in case, for every proposition ϕ over which $Cr(\cdot)$ is defined, $Cr(\phi) = ch_w(\phi)$ (where $ch_w(\cdot)$ is the ur-chance-function for world w.)

For Ur-Chance Vindication is incompatible with the following principle:

> **Rational Conditionalization:** If one's credences at time t_0, $Cr_{t_0}(\cdot)$, are rational, and between t_0 and t_1 one's total evidence is E, then it is rational for one's credences at t_1 to be such that $Cr_{t_1}(\cdot) = Cr_{t_0}(\cdot|E)$.

Rational Conditionalization, however, is extremely plausible. And so, given the incompatibility between Ur-Chance Vindication and Rational Conditionalization, one should reject the former principle.

> **Claim:** Ur-Chance Vindication and Rational Conditionalization are incompatible.

> **Proof:** Let $A(\cdot,\cdot)$ be our **accuracy measure**. We can think of this as being equal to $-I(\cdot,\cdot)$. Assume, as Ur-Chance Vindication does, that credal vindication is only relativized to worlds. We'll let V_w,

[13] Of course, the proponent of Alethic Vindication will also endorse the stronger claim that any two agents with the same credal state who are located in the same world will be such that either both of their credal states are vindicated or neither are.

then, be the vindicated credal state at w. We require only one very plausible assumption about $A(\cdot, \cdot)$. In particular, we will assume that for every w, $A(V_w, V_w) > A(Cr(\cdot), V_w)$, for every $Cr(\cdot) \neq V_w$.[14]

Def. We say that the **expected accuracy** of a credal state $Cr'(\cdot)$, given a credal state $Cr(\cdot)$, is:

$$E_{Cr}(Cr'(\cdot)) = \sum_w Cr(w)A(Cr'(\cdot), V_w).$$

To show that Ur-Chance Vindication is incompatible with Rational Conditionalization, we'll appeal to the following principle:

Accuracy Expectation: A credal state $Cr(\cdot)$ is irrational if there is some other credal state $Cr'(\cdot)$ such that:
(i) $E_{Cr}(Cr(\cdot)) < E_{Cr}(Cr'(\cdot))$
(ii) For every $Cr''(\cdot) \neq Cr'(\cdot)$, $E_{Cr'}(Cr''(\cdot)) < E_{Cr'}(Cr'(\cdot))$

This principle is, I think, extremely plausible. For a credal state that expects some other credal state to be epistemically better off than itself is self-undermining in a way that would, at least prima facie, seem to preclude it from being rationality adopted. Now we might excuse this sort of self-undermining character, if it were the case that the credal states that were expected to be epistemically better off by the lights of $Cr(\cdot)$, also had this sort of self-undermining character. But condition (ii) rules out this form of exculpation.

To show that Ur-Chance Vindication is incompatible with Rational Conditionalization it suffices, then, to show that, given Ur-Chance Vindication, there are cases in which an agent starts out with a rational credal state and then conditionalizes on some information and winds up with a credal state satisfying conditions (i) and (ii).

Here is such a case.

Suppose we have some agent Q at the beginning of her epistemic life at time t_0 in world w_q. And suppose that her credal state at this time, $Cr_{t_0}(\cdot)$, is such that $Cr_{t_0}(\phi) = ch_{w_q}(\phi)$, for every proposition ϕ over which it is defined. Given Ur-Chance Vindication, Q's credences are vindicated, and so, rational. Now let's suppose that, between t_0 and t, Q gets total evidence E such that $ch_{w_q}(\cdot|E) \neq ch_{w_q}(\cdot)$ And suppose that Q sets her credences at t so that $Cr_t(\cdot) = Cr_{t_0}(\cdot|E)$. Note that this entails that $Cr_{t_0}(\cdot) \neq Cr_t(\cdot)$.

[14] This follows from Joyce's assumptions about the accuracy measure. This is equivalent to the assumption made in Pettigrew [2012], p.254 that the inaccuracy measure is **nontrivial**.

First, we'll show that $E_{Cr_t}(Cr_t(\cdot)) < E_{Cr_t}(Cr_{t_0}(\cdot))$. Since $Cr_{t_0}(\cdot) = ch_{w_q}(\cdot)$ and $ch_{w_q}(C_{ch_{w_q}}) = 1$, it follows that $Cr_{t_0}(C_{ch_{w_q}}) = 1$. And so, since conditionalization preserves certainties, we also have that $Cr_t(C_{ch_{w_q}}) = 1$. This means that, for every w such that $Cr_t(w) \neq 0$, $ch_{w_q}(\cdot)$ is the ur-chance function at w. Thus, for every w such that $Cr_t(w) \neq 0$, $Cr_{t_0}(\cdot)$ is vindicated at w. And so, given Ur-Chance Vindication, we have that for every w such that $Cr_t(w) \neq 0$, $A(Cr_{t_0}(\cdot), V_w) > A(Cr(\cdot), V_w)$ for every $Cr(\cdot) \neq Cr_{t_0}(\cdot)$. And so, since $Cr_{t_0}(\cdot) \neq Cr_t(\cdot)$, this suffices to establish that $E_{Cr_t}(Cr_t(\cdot)) < E_{Cr_t}(Cr_{t_0}(\cdot))$.

Next we'll show that $E_{Cr_{t_0}}(Cr(\cdot)) < E_{Cr_{t_0}}(Cr_{t_0}(\cdot))$, for every $Cr(\cdot) \neq Cr_{t_0}(\cdot)$. To see that this holds, note that the same reasoning as above establishes that, given Ur-Chance Vindication, we have that for every w such that $Cr_{t_0}(w) \neq 0$, $A(Cr_{t_0}(\cdot), V_w) > A(Cr(\cdot), V_w)$ for every $Cr(\cdot) \neq Cr_{t_0}(\cdot)$. This suffices to establish the desired result.

The proponent of Initial Vindication, then, cannot say that, in general, vindication consists in matching the ur-chances. Thus, if one endorses Initial Vindication, one must hold that whether a credal state is vindicated depends both on facts about what the world is like and on additional facts about the agent's epistemic situation, viz., whether or not she has received any evidence.

Why might this be a problem? Well, I take it that we have a grip on an *objective* sense of what an agent ought to believe or what credences an agent ought to have. That is, we have a grip on a sense of *ought* that abstracts away from facts about an agent's evidence and simply considers what would be the ideal belief or credal state given what the world is like where and when the agent is located. Given this sense of *ought*, agents located at the same world, or perhaps the same world and time, ought to have the same doxastic states. We also, I take it, have a grip on a *subjective* sense of what an agent ought to believe or what credences an agent ought to have. That is, we have a grip on a sense of *ought* that abstracts away from what the world is in fact like where and when the agent is located and simply considers what doxastic or credal states are best given the agent's evidence. Given this sense of *ought*, agents cannot differ with respect to what credences or beliefs they ought to have without also differing with respect to their evidential situation. But neither of these corresponds to the sense in which, according to Initial Vindication, an agent who has received no evidence ought to set her credences in line with the ur-chances.

We can't understand this as a subjective ought. For, according to Initial Vindication, what credences an agent with no evidence ought to have will vary from world to world. But we also can't understand this as an objective ought.[15] For, as we've seen, the demands imposed by Initial Vindication cannot be generalized in a way that ignores facts about an agent's evidence.

[15]Note that this means that my initial gloss on vindication given at the beginning of

The proponent of Initial Vindication, then, requires that there be some non-objective and non-subjective sense in which certain agents ought to set their credences in line with the ur-chances. Such a sense of ought must be *partly* subjective, depending in part on the agent's evidential situation. But it cannot be understood, for example, in terms of it being the case that certain credences seem best in light of the agent's evidence. And such a sense of ought must be *partly* objective, depending in part on where in logical space the agent is. But it cannot be understood, for example, in terms of the ur-chances being the feature of reality that it is objectively best for a credal state to match.

It is, however, quite opaque to me whether there really is a coherent hybrid normative notion meeting this description. At the very least, the proponent of Initial Vindication owes us a story about why we should suppose that there are normative facts that are not, as we standardly think of such facts, objective or subjective. Without such a story, I think there's good reason to be skeptical of Initial Vindication.

Note, however, that the same worry clearly doesn't apply to Chance Vindication. For we can understand this principle as employing an objective sense of *ought*. Chance Vindication tells us that objectively an agent ought to have credences that match the chances as they are at the time those credences are held. Such a principle imposes the same demands on all agents located in the same world at the same time regardless of what evidence they possess.

The fact that we can understand Chance Vindication, but not Initial Vindication, by appeal to an antecedently intelligible notion of how an agent's credences ought to be from an objective point of view, gives us, I think, good reason to prefer the former principle to the latter. We have, then, good motivation to explore what normative constraints on credal states follow on the assumption that Chance Vindication, instead of Initial Vindication, is correct. It is to this question that we now turn.

4 Chance Vindication and the Temporal Principle

In this section, I'll show that both Probabilism and an interesting chance-credence norm follow from Chance Vindication. To show this, we will follow Pettigrew's strategy and establish that the demands imposed by these norms are satisfied by every credal state in $\mathcal{V}_C^+$, i.e., the convex hull of the set of potentially vindicated credal states given Chance Vindication. As we'll see, however, certain modifications to our current framework will be required in order to establish, in this manner, that there is an interesting chance-credence norm that follows from Chance Vindication.

this paper is inappropriate as a characterization of the notion of vindication employed by the proponent of Initial Vindication. The question, then, is what is the appropriate characterization of this notion of vindication. As the following indicates, I'm skeptical that there is any intelligible alternative explication of vindication that will serve the proponent of Initial Vindication.

It will be illuminating, however, to begin by considering some plausible chance-credence norms that do not, in fact, follow from Chance Vindication. Seeing why these fail to be entailed by Chance Vindication will lead us naturally to the amendments that are required to our current framework in order to derive an appropriate chance-credence norm.

Let us begin with the Principal Principle. Pettigrew [2012] showed that this norm follows from Initial Vindication by showing that the constraints that it imposes are satisfied by every credal state in $\mathcal{V}_I^+$, i.e., the convex hull of the set of potentially vindicated credal states given Initial Vindication. Earlier we showed that $\mathcal{V}_C^+ \neq \mathcal{V}_I^+$. We'll now show that there are credal states in $\mathcal{V}_C$, and so in $\mathcal{V}_C^+$, that do not satisfy the constraints imposed by the Principal Principle. The Principal Principle, then, cannot be shown to follow from Chance Vindication in the same manner in which it was shown to follow from Initial Vindication.

Claim: There are credal states in $\mathcal{V}_C$ that do not satisfy the constraints imposed by the Principal Principle.

Proof: To see this, consider the ur-chance function for a world w, $ch_w(\cdot)$, and the chance function for that world at time t, $ch_w^t(\cdot)$, such that $ch_w(\cdot) \neq ch_w^t(\cdot)$. As we noted earlier, $ch_w^t(\cdot)$ is identical to $ch_w(\cdot|H_w^t)$. And, since $ch_w(C_{ch_w}) = 1$ and conditional probabilities preserve certainties, it follows that $ch_w(C_{ch_w}|H_w^t) = 1 = ch_w^t(C_{ch_w})$. But this entails that $ch_w^t(\cdot)$ does not satisfy the Principal Principle. For since $ch_w^t(C_{ch_w}) = 1$, we have: $ch_w^t(\cdot|C_{ch_w}) = ch_w^t(\cdot)$. But since $ch_w^t(\cdot) \neq ch_w(\cdot)$, it follows that $ch_w^t(\cdot|C_{ch_w}) \neq ch_w(\cdot)$, and so $ch_w^t(\cdot)$ doesn't satisfy the Principal Principle. Now, for any algebra $\mathcal{A}$, there will be some member of $\mathcal{V}_C$ that matches $ch_w^t(\cdot)$ over $\mathcal{A}$. Thus, we have that, given an algebra containing C_{ch_w}, there will be some member of $\mathcal{V}_C$ that doesn't satisfy the Principal Principle.

We can generalize this proof in the following way. Let ϕ_{ch} be a proposition concerning a chance function $ch(\cdot)$. Perhaps ϕ_{ch} says that the ur-chances are given by $ch(\cdot)$; or perhaps it says that the chances at t are given by $ch(\cdot)$; or perhaps it says that God's favorite chance function is $ch(\cdot)$. Given minimal assumptions, we can show that there is some $Cr(\cdot) \in \mathcal{V}_C$ such that $Cr(\cdot|\phi_{ch}) \neq ch(\cdot)$.

Claim: If ϕ_{ch} is a chance proposition such that (i) there is some world w and some time t_1 such that $ch_w^{t_1}(\phi_{ch}) = 1$, and (ii) there is some time $t_2 > t_1$ such that $ch_w^{t_1}(\cdot) \neq ch_w^{t_2}(\cdot)$, then there is some $Cr(\cdot) \in \mathcal{V}_C$ such that $Cr(\cdot|\phi_{ch}) \neq ch(\cdot)$.

Proof: Consider the chance functions for a world w at times $t_1 < t_2$. Assume that $ch_w^{t_1}(\phi_{ch}) = 1$ and that $ch_w^{t_1}(\cdot) = ch(\cdot)$. Further assume that $ch_w^{t_1}(\cdot) \neq ch_w^{t_2}(\cdot)$. Then, since $ch_w^{t_2}(\cdot) = ch_w^{t_1}(\cdot|H_w^{t_2})$, and since conditional probabilities preserve certainties in propositions, it follows that $ch_w^{t_2}(\phi_{ch}) = 1$. But since $ch_w^{t_1}(\cdot) \neq ch_w^{t_2}(\cdot)$, it follows that $ch_w^{t_2}(\cdot) \neq ch(\cdot)$. Thus, since, for any algebra $\mathcal{A}$, there will be some member of $\mathcal{V}_C$ that matches $ch_w^{t_2}(\cdot)$ over $\mathcal{A}$, it follows that, given an algebra containing ϕ_{ch}, there is some member of $\mathcal{V}_C$ such that $Cr(\cdot|\phi_{ch}) \neq ch(\cdot)$.

Now many chance propositions which might be thought, at least prima facie, to rationally constrain our credences in the manner in which the Principal Principle claims ur-chance propositions rationally constrain our initial credences, will satisfy conditions (i) and (ii). The list includes, in addition to most ur-chance propositions, most propositions about what the chances are at some time t, as well as most existential propositions to the effect that there is some time at which the chances are represented by a function $ch(\cdot)$.[16] Not only can the Principal Principle, then, not be shown to follow from Chance Vindication in the manner in which it was shown to follow from Initial Vindication, neither can other formally analogous principles that appeal to alternative chance propositions.

Given this, a natural response is to look for some other way of deriving normative principles from Chance Vindication. And there is a tempting thought here. For, according to Chance Vindication, vindication must be relativized to a time. It is natural, then, to think that we might be able to justify certain temporally relativized norms by relativizing the set of potentially vindicated credal states to a time and showing that, for each time t, each member of the set of potentially vindicated credal states at t satisfies the constraints imposed by the appropriate temporally relativized normative principle. Indeed, there is an analogue of Pettigrew's Theorem that would seem to support this thought.

Let $\mathcal{V}_t = \{b$: For some possible chance distribution at t, $ch(\cdot), ch(\phi) = b(\phi)$, for every $\phi \in \mathcal{A}\}$. This is, according to Chance Vindication, the set of potentially vindicated credal states, defined over $\mathcal{A}$, given a time t. If $ch(\cdot)$ is a possible chance function given a time t, we'll let C_{ch}^t be the proposition that says that the chances at t are given by $ch(\cdot)$.

Consider, then, the following putative norm:

T-Principle: A rational agent's credences at time t defined over an algebra containing C_{ch}^t, ought to be such that:

$Cr(A|C_{ch}^t \wedge E) = ch(A)$ (given that $Cr(C_{ch}^t \wedge E) \neq 0$ and $ch(A|E) = ch(A)$).

[16] Within each class there will be exceptions when the propositions in question say that the ur-chances, or the chances at t, or the chances at some time, are given by some deterministic $ch(\cdot)$, according to which every proposition has chance 1 or 0. In these cases, if at some time such a proposition has chance 1, then the chances will be fixed for all future times, and so condition (ii) cannot obtain.

Claim: Every member of $\mathcal{V}_t^+$ satisfies the constraints imposed by Probabilism and the T-Principle

> **Proof:** To show this, it suffices to show that each member of $\mathcal{V}_t$ satisfies both Probabilism and the T-Principle. For, the proof of the claim—appealed to the justification of Pettigrew's Theorem—that if two credal states b and b' satisfy Probabilism and the Principal Principle, then so will any convex combination of b and b', also suffices to show that the same holds for any two credal states satisfying Probabilism and the T-Principle
>
> That each member of $\mathcal{V}_t$ satisfies Probabilism follows from the fact that each chance function is a probability function.
>
> To see that each member of $\mathcal{V}_t$ satisfies the T-Principle, note that every possible chance function for time t, $ch(\cdot)$, will be such that $ch(C^t_{ch}) = 1$.[17] It follows that for each $C^t_{ch} \in \mathcal{A}$ and each $b \in \mathcal{V}_t$, if $b(C^t_{ch}) \neq 0$, then $b(C^t_{ch}) = 1$ and, for every $\phi \in \mathcal{A}$, $b(\phi) = ch(\phi)$. Thus, if $b(C^t_{ch})$ is well-defined and $b(C^t_{ch}) \neq 0$, then $b(A|C^t_{ch}) = ch(A)$. This suffices to establish that each member of $\mathcal{V}_t$ satisfies the T-Principle.

Now it is tempting, I think, to suppose that this fact can be marshalled to show that, given Chance Vindication, an agent at time t ought to have a credal state that is probabilistically coherent and, in addition, satisfies the T-Principle. To conclude this, however, would be a mistake. For, in order for the preceding to show that Probabilism and the T-Principle follow from Chance Vindication, the following auxiliary principle would have to hold:

> **T-Convexity:** If an agent is located at time t, then she ought to have a credal state that is in the convex hull of the set of potentially vindicated credal states at t.

But, unlike Convexity, T-Convexity is not at all plausible, at least given Chance Vindication.

To see this, recall the argument for Convexity. That argument rested on the claim that, given Joyce's Dominance Theorem, if an agent has a credal state, $Cr(\cdot)$, that is not in the convex hull of the set of potentially vindicated credal states, then it is a priori knowable for the agent that there is some

[17] To see this consider some chance function, $ch^t_w(\cdot)$, for some world w at some time t. We want to show that $ch^t_w(C^t_{ch^t_w}) = 1$. First note that $C^t_{ch^t_w}$ is logically equivalent to the disjunction $(C_{ch'} \wedge H^t_1) \vee (C_{ch''} \wedge H^t_2) \vee ....$, where each disjunct $C_{ch} \wedge H^t$ is such that $ch(\cdot|H^t) = ch^t_w(\cdot)$. Since chance-functions are probability functions, it follows that if $ch^t_w(\cdot)$ assigns 1 to one of these disjuncts, then it will assign 1 to this disjunction, and so to the logically equivalent proposition $C^t_{ch^t_w}$. Note that $ch^t_w(\cdot) = ch_w(\cdot|H^t_w)$. Since $ch_w(C_{ch_w}) = 1$, we have, then, that $ch_w(C_{ch_w} \wedge H^t_w|H^t_w) = 1$. And so we have $ch^t_w(C_{ch_w} \wedge H^t_w) = 1$. Thus, since $ch_w(\cdot|H^t_w) = ch^t_w(\cdot)$, it follows that $ch^t_w(C^t_{ch^t_w}) = 1$.

other credal state, $Cr'(\cdot)$, that is closer to being vindicated no matter what the world is like. Given Chance Vindication, however, we don't have parallel reason to endorse T-Convexity. For, given Chance Vindication, it doesn't follow from the Dominance Theorem that if an agent is located at some time t and has a credal state, $Cr(\cdot)$, that is not in the convex hull of the set of potentially vindicated credal states at t, then it is a priori knowable for the agent that there is some other credal state, $Cr'(\cdot)$, that is closer to being vindicated no matter what the world is like.

Now it is true that Joyce's Dominance Theorem establishes that for any $Cr(\cdot) \notin \mathcal{V}_t^+$, there is some $Cr'(\cdot) \in \mathcal{V}_t^+$, such that, for every $v(\cdot) \in \mathcal{V}_t$, $I(Cr(\cdot)', v(\cdot)) < I(Cr(\cdot), v(\cdot))$. But this fact doesn't ensure that, for an agent located at t whose credal state, $Cr(\cdot)$, is not in $\mathcal{V}_t^+$, it is a priori knowable that there is some other credal state, $Cr'(\cdot)$, that is closer to being vindicated no matter what the world is like. For, if $t \neq t'$, then the set of possible chance functions at t will not be the same as the set of possible chance functions at t'.[18] And so a credal state may be such that there is some other credal state that is closer to each element of $\mathcal{V}_t$ but not closer to each element of $\mathcal{V}_{t'}$. Given Chance Vindication, then, if an agent is located at time t and her credal state is not in $\mathcal{V}_t^+$ but she does not know that she is located at time t, she need not be in a position to tell that there is some other credal state that is guaranteed to be closer to vindication. And so, since one cannot know a priori at which time one is located, it follows that if Chance Vindication is correct, then despite Joyce's Dominance Theorem an agent whose credal state at t is outside the convex hull of the set of potentially vindicated credal states at t need not be in a position to tell a priori that there is some other credal state that is guaranteed to be closer to vindication.

The lesson to be drawn from this is that if we are going to try to derive some normative principle using Joyce's Dominance Theorem, then we must show that the constraints imposed by the principle in question hold for every member of the convex hull of some set S such that it is a priori that the vindicated credal state is amongst S. Given Chance Vindication, however, $\mathcal{V}_C$ is the smallest set S such that it is a priori that the vindicated credal state is amongst S. The problem, though, is that there are, as we've seen, very strict limitations on the sorts of chance propositions that may serve to constrain each member of $\mathcal{V}_C$ in the manner in which ur-chance propositions constrain each member of $\mathcal{V}_I$.

There is, though, a way around this problem. We've been assuming, in the preceding discussion, that the objects of credence and chance are *eternal propositions*, i.e., propositions that determine truth-values relative to a world

[18] The following suffices to show that if $t \neq t'$, then the set of possible chance functions at t is distinct from the set of possible chance functions at t'. Let $ch_w^t(\cdot)$ be a possible chance function at t. $ch_w^t(\cdot)$ is the unique possible chance function at t such that $ch_w^t(C^t_{ch_w^t}) = 1$. Now let $ch_w^{t'}(\cdot) = ch_w^t(\cdot | H_w^{t'}) \neq ch_w^t(\cdot)$. Then, since conditional probabilities preserve certainties, we have $ch_w^{t'}(C^t_{ch_w^t}) = 1$. But since $ch_w^{t'}(\cdot) \neq ch_w^t(\cdot)$ and since $ch_w^t(\cdot)$ is the unique possible chance function at t such that $ch_w^t(C^t_{ch_w^t}) = 1$, it follows that the set of possible chance functions at t is distinct from the set of possible chance functions at t'.

of evaluation. If, however, we allow that the objects of credence and chance are *temporal propositions*, i.e., propositions that determine truth-values relative to a world and time of evaluation, then we can show that there is in fact an interesting class of chance propositions that serves to constrain each member of $\mathcal{V}_C$ in the manner in which ur-chance propositions constrain each member of $\mathcal{V}_I$.

Now there is some good motivation for thinking that the objects of credence may vary in truth-value across times. In particular, considerations of cognitive significance may be used to motivate this claim. Thus, believing say, that it is raining in San Francisco, would seem to be different from believing that it is raining in San Francisco at t, for any value t. For, even if one is located at time t, if one is unaware of what time it is, the latter proposition would seem to be informative, even given belief in the former. And one very simple way to account for this is to allow that the proposition that it is raining in San Francisco varies in truth-value across times and so is distinct from the proposition that it is raining in San Francisco at t, for every value of t.[19]

It may be less clear, though, that chance functions will be well-defined for essentially temporal propositions. There is, however, a perfectly natural way of extending a chance function defined over eternal propositions to a chance function defined over temporal propositions. For simplicity let us think about propositions as being sets of world-time pairs. An eternal proposition can be represented by a sets of world-time pairs ϕ such that, for all w, t, t', $\langle w, t\rangle \in \phi$ iff $\langle w, t'\rangle \in \phi$. A temporal proposition, however, will be represented a set of world-time pairs ϕ such that, for some w, t, t', $\langle w, t\rangle \in \phi$ and $\langle w, t'\rangle \notin \phi$.

> **Def.** Let ϕ be a proposition. We'll call the **t-rigidification of** ϕ: $\phi^t = \{\langle w, t'\rangle : \langle w, t\rangle \in \phi\}$. Note that ϕ^t is an eternal proposition, and that if ϕ is also an eternal proposition, then $\phi = \phi^t$.

Let $ch_t(\cdot)$ be a possible chance function for some time t that is defined for eternal propositions. We'll now show how $ch_t(\cdot)$ may be extended so that it is defined also for temporal propositions. We'll assume the following:

> **Chance Rigidity:** For every temporal proposition ϕ, $ch_t(\phi) = ch_t(\phi^t)$.

This principle tells us that, at t, the chance of any temporal proposition is just the chance of its t-rigidification. For example, consider the temporal proposition that it will rain. We can think of this as the set of world-time pairs where at the world parameter it rains at some time later than time parameter. The t-rigidification of this proposition is the eternal proposition consisting of all worlds where at t it rains later than that time. Chance Rigidity, then, claims that the chance at t of the temporal proposition that it will rain is just the

[19] See, e.g., Kaplan [1989] and Lewis [1979] for this line of thought.

chance at t that it rains at some time later than t. This seems to me to be exactly the right thing to say.

Now consider the following principle:

> **Temporal Principle:** Let $ch(\cdot)$ be some possible chance function, i.e., a function describing a possible chance distribution for some world w and time t. And let T_{ch} be the temporal proposition that claims that the chances are given by $ch(\cdot)$. We can think of this as the set of world-time pairs such that $ch(\cdot)$ is the chance function at the world and time in question. Then a rational agent's credences defined over an algebra containing T_{ch}, ought to be such that:
>
> $Cr(A|T_{ch} \wedge E) = ch(A)$ (given that $Cr(T_{ch} \wedge E) \neq 0$ and $ch(A|E) = ch(A)$).

Claim: Given Chance Rigidity, it follows that each member of $\mathcal{V}_C^+$ satisfies the constraints imposed by Probabilism and the Temporal Principle.

Proof: To show this, it suffices to show that each member of $\mathcal{V}_C$ satisfies both Probabilism and the Temporal Principle. For, again, the proof of the claim—appealed to the justification of Pettigrew's Theorem—that if two credal states b and b' satisfy Probabilism and the Principal Principle, then so will any convex combination of b and b', also suffices to show that the same holds for any two credal states satisfying Probabilism and the Temporal Principle.

First, we'll show that each member of $\mathcal{V}_C$ satisfies Probabilism. We're assuming that every chance function defined over an algebra of eternal propositions is probabilistically coherent. Given this, we'll show that it follows from Chance Rigidity that every chance function defined over an algebra containing also temporal propositions is probabilistically coherent.

To show this we'll show that each such function $ch_t(\cdot)$ is such that (i) $ch_t(\top) = 1$, (ii) $ch_t(\phi) \geq 0$, for every proposition ϕ, and (iii) if ϕ and ψ are mutually-exclusive propositions, then $ch_t(\phi \vee \psi) = ch_t(\phi) + ch_t(\psi)$.

(i) $\top$ is just the set of all world-time pairs, and so is an eternal proposition. And so, given that $ch_t(\cdot)$ is probabilistically coherent over eternal propositions we have that $ch_t(\top) = 1$.

(ii) Given Chance Rigidity, we have that $ch_t(\phi) = ch_t(\phi^t)$. And since $ch_t(\cdot)$ is probabilistically coherent over eternal propositions we have that $ch_t(\phi^t) \geq 0$. And so we have that $ch_t(\phi) \geq 0$.

(iii) Assume that ϕ and ψ are two mutually-exclusive propositions. Given Chance Rigidity, we have that $ch_t(\phi \vee \psi) = ch_t([\phi \vee \psi]^t)$. But

$[\phi \vee \psi]^t = \phi^t \vee \psi^t$. Thus we have that $ch_t(\phi \vee \psi) = ch_t(\phi^t \vee \psi^t)$. Next note that, given that ϕ and ψ are mutually exclusive, it follows that so too are ϕ^t and ψ^t. To see this assume that ϕ^t and ψ^t are not mutually exclusive. Thus there is some w such that, for every t' $\langle w, t'\rangle \in \phi^t$ and $\langle w, t'\rangle \in \psi^t$. But this means that there must be some pair $\langle w, t\rangle$, such that $\langle w, t\rangle \in \phi$ and $\langle w, t\rangle \in \psi$. And so if ϕ^t and ψ^t are not mutually exclusive, neither are ϕ and ψ. Given that ϕ^t and ψ^t are mutually exclusive and $ch_t(\cdot)$ is probabilistically coherent for eternal propositions, we have then that $ch_t(\phi^t \vee \psi^t) = ch_t(\phi^t) + ch_t(\psi^t)$. And so we have $ch_t(\phi \vee \psi) = ch_t(\phi^t) + ch_t(\psi^t)$. Finally, given Chance Rigidity we have that $ch_t(\phi) = ch_t(\phi^t)$ and $ch_t(\psi) = ch_t(\psi^t)$. Thus we have that $ch_t(\phi \vee \psi) = ch_t(\phi) + ch_t(\psi)$.

To see that each member of $\mathcal{V}_C$ satisfies the Temporal Principle, note that, given Chance Rigidity, it follows that every possible chance function $ch(\cdot)$ is such that if $ch(\cdot) = ch'(\cdot)$, then $ch(T_{ch'}) = 1$, and otherwise $ch(T_{ch'}) = 0$.

Why is this? Well, let $ch^t_w(\cdot)$ be the chance function for a world w at time t. We saw earlier that, given that ur-chance functions are non-self-undermining, it follows that for any time t, if $ch^t_w(\cdot) = ch'(\cdot)$, then $ch^t_w(C^t_{ch'}) = 1$, and otherwise $ch^t_w(C^t_{ch'}) = 0$.[20] Now $C^t_{ch'}$ is the t-rigidification of $T_{ch'}$. And so, given Chance Rigidity, we have $ch^t_w(T_{ch'}) = ch^t_w(C^t_{ch'})$. Thus, we have that if $ch^t_w(\cdot) = ch'(\cdot)$, then $ch^t_w(T_{ch'}) = 1$, and otherwise $ch^t_w(T_{ch'}) = 0$.

Since every possible chance function $ch(\cdot)$ is such that if $ch(\cdot) = ch'(\cdot)$, then $ch(T_{ch'}) = 1$, and otherwise $ch(T_{ch'}) = 0$, it follows that for each $T_{ch} \in \mathcal{T}$ and each $b \in \mathcal{V}_C$, if $b(T_{ch}) \neq 0$, then $b(T_{ch}) = 1$ and, for every $\phi \in \mathcal{A}$, $b(\phi) = ch(\phi)$. Thus, if $b(T_{ch})$ is well-defined and $b(T_{ch}) \neq 0$, then $b(A|T_{ch}) = ch(A)$. This suffices to establish that, given Chance Rigidity, each member of $\mathcal{V}_C$ satisfies the Temporal Principle.

Given Chance Vindication, then, it follows that an agent should have a credal state that is probabilistically coherent and, in addition, satisfies the constraints imposed by the Temporal Principle.

Earlier we showed that the Principal Principal fails to follow from Chance Vindication by showing that there are distinct possible chance functions that assign the value 1 to the same proposition C_{ch}. Now, as we've seen, the same is not true for propositions of the form T_{ch}. Indeed, we've seen that every possible chance function $ch(\cdot)$ is such that if $ch(\cdot) = ch'(\cdot)$, then $ch(T_{ch'}) = 1$, and otherwise $ch(T_{ch'}) = 0$. A natural question, then, is where the parallel attempt to prove that there are distinct possible chance functions that assign the value 1 to some proposition T_{ch} breaks down.

[20]See fn. 16 above.

To show that there are distinct possible chance functions that assign the value 1 to the same proposition C_{ch}, we began by considering the ur-chance function for a world w, $ch_w(\cdot)$, and the chance function for that world at time t, $ch^t_w(\cdot)$, such that $ch_w(\cdot) \neq ch^t_w(\cdot)$. We, then, noted that $ch^t_w(\cdot)$ is identical to $ch_w(\cdot|H^t_w)$, and so it follows that, while $ch_w(\cdot) \neq ch^t_w(\cdot)$, since $ch_w(C_{ch_w}) = 1$ and since conditional probabilities preserve certainties, we also have $ch^t_w(C_{ch_w}) = 1$. The corresponding argument for propositions of the form T_{ch} breaks down at the point where we assume that $ch^t_w(\cdot)$ is identical to $ch_w(\cdot|H^t_w)$. Now it is true that the values that $ch^t_w(\cdot)$ assigns to *eternal* propositions is the same as the values assigned by $ch_w(\cdot|H^t_w)$. However, given Chance Rigidity, the same will not be true for all temporal propositions. To see this, note that, given Chance Rigidity, there will be temporal propositions ϕ such that $ch_w(\phi) = 1$ but $ch^t_w(\phi) \neq 1$. This, however, is precluded on the assumption that $ch^t_w(\cdot)$ is identical to $ch_w(\cdot|H^t_w)$. As an example, consider the temporal proposition T_{ch_w}. Given Chance Rigidity, we'll have that $ch_w(T_{ch_w}) = 1$, but, assuming that $ch_w(\cdot) \neq ch^t_w(\cdot)$, we will also have that $ch^t_w(T_{ch_w}) = 0$.

Let me close by considering how the Temporal Principle relates to the Principal Principle. We can show, given certain auxiliary constraints, that if an agent's initial credences satisfy the Principal Principle, then they will also satisfy the Temporal Principle. The reverse, however, is not true. There is a precise sense, then, in which the Principal Principle imposes a stronger normative constraint than the Temporal Principle.

Claim: Let $Cr(\cdot)$ be an agent's initial credal state. We assume (i) that $Cr(\cdot)$ is probabilistically coherent, and (ii) that there is a finite set $\mathbf{C} = \{C_{ch_i} : 1 \leq i \leq n\}$ of mutually exclusive propositions C_{ch_i} such that $Cr(C_{ch_i}) > 0$, for each $C_{ch_i} \in \mathbf{C}$, and $\sum_{i=1}^{n} Cr(C_{ch_i}) = 1$. We can show that if $Cr(\cdot)$ satisfies the Principal Principle, then it will also satisfy the Temporal Principle.

Proof: Assume that $Cr(\cdot)$ is such that for every proposition C_{ch} over which it is defined, if $Cr(C_{ch}) \neq 0$, then, $Cr(\cdot|C_{ch}) = ch(\cdot)$. We'll show that for every proposition of the form T_{ch} over which $Cr(\cdot)$ is defined, if $Cr(T_{ch}) \neq 0$, then, $Cr(\cdot|T_{ch}) = ch(\cdot)$.[21]

Given assumptions (i) and (ii), we know that there is a finite set of propositions of the form C_{ch} such that $Cr(C_{ch})$ is well-defined and non-zero. This is the set $\mathbf{C}$. We will first show that for each $C_{ch_i} \in \mathbf{C}$ and each corresponding temporal proposition T_{ch_i}, if $Cr(\cdot)$ is well-defined for T_{ch_i}, then the following hold: $Cr(T_{ch_i}|C_{ch_i}) = 1$, and $Cr(C_{ch_i}|T_{ch_i}) = 1$.

[21] Note that we can show the same on the assumption that the set of propositions $\mathbf{C}$ is countable, if we assume, in addition, that $Cr(\cdot)$ satisfies countable additivity.

First, we'll show that $Cr(T_{ch_i}|C_{ch_i}) = 1$. Let t_0 be the initial time when the ur-chance function gives the actual chances. First note that $T^{t_0}_{ch_i} = C_{ch_i}$. Now, it follows by Chance Rigidity that $ch(T_{ch_i}) = ch(T^{t_0}_{ch_i})$. And so, we have $ch(T_{ch_i}) = ch(C_{ch_i})$. And since $ch(C_{ch_i}) = 1$, we have $ch(T_{ch_i}) = 1$. Thus, it follows from the assumption that $Cr(\cdot|C_{ch_i}) = ch_i(\cdot)$, that $Cr(T_{ch_i}|C_{ch_i}) = 1$.

Next, we'll show that $Cr(C_{ch_i}|T_{ch_i}) = 1$. We've shown that for each $C_{ch_x} \in \mathbf{C}$, $Cr(T_{ch_x}|C_{ch_x}) = 1$. Now it follows from this fact that if $ch(\cdot) \neq ch_x(\cdot)$, then if $Cr(\cdot)$ is well defined for T_{ch}, then $Cr(T_{ch}|C_{ch_x}) = 0$. Furthermore, given assumptions (i) and (ii), we know that $Cr(T_{ch_i}) = \sum_{x=1}^{n} Cr(T_{ch_i}|C_{ch_x}) \times Cr(C_{ch_x})$. But it follows from these two facts that $Cr(T_{ch_i}) = Cr(T_{ch_i} \wedge C_{ch_i})$. And so we have that $Cr(C_{ch_i}|T_{ch_i}) = 1$.

We've established that for each $C_{ch_i} \in \mathbf{C}$, if $Cr(T_{ch_i})$ is well-defined, then $Cr(T_{ch_i}|C_{ch_i}) = 1$, and $Cr(C_{ch_i}|T_{ch_i}) = 1$. This suffices to establish that for each proposition T_{ch_i} corresponding to some $C_{ch_i} \in \mathbf{C}$ for which $Cr(\cdot)$ is well-defined $Cr(\cdot|C_{ch_i}) = Cr(\cdot|T_{ch_i})$. Thus, given our assumption that $Cr(\cdot|C_{ch_i}) = ch(\cdot)$, it follows that, for each proposition T_{ch_i} corresponding to some $C_{ch_i} \in \mathbf{C}$ for which $Cr(\cdot)$ is well-defined, we have that $Cr(\cdot|T_{ch_i}) = ch(\cdot)$.

We'll now complete our proof of the claim that $Cr(\cdot|T_{ch}) = ch(\cdot)$, for every proposition T_{ch} such that $Cr(T_{ch})$ is well-defined and non-zero. We do so by showing that if $Cr(T_{ch})$ is well-defined and $ch(\cdot)$ is distinct from each $ch_i(\cdot)$ such that $C_{ch_i} \in \mathbf{C}$, then $Cr(T_{ch}) = 0$.

This follows from two previously established facts. First, for each $C_{ch_i} \in \mathbf{C}$, if $ch(\cdot) \neq ch_i(\cdot)$, then if $Cr(\cdot)$ is well-defined for T_{ch}, then $Cr(T_{ch}|C_{ch_i}) = 0$. And, second, given assumptions (i) and (ii), $Cr(T_{ch}) = \sum_{i=1}^{n} Cr(T_{ch}|C_{ch_i}) \times Cr(C_{ch_i})$. For if $ch(\cdot)$ is distinct from each $ch_i(\cdot)$ such that $Cr(C_{ch_i}) \in \mathbf{C}$, then, for each $C_{ch_i} \in \mathbf{C}$, we have $Cr(T_{ch}|C_{ch_i}) = 0$. And so $\sum_{i=1}^{n} Cr(T_{ch}|C_{ch_i}) \times Cr(C_{ch_i}) = 0$, and so $Cr(T_{ch}) = 0$.

Since $Cr(\cdot)$ is well-defined and non-zero only for propositions of the form T_{ch_i} corresponding to propositions of the form $C_{ch_i} \in \mathbf{C}$, it follows, then, from the fact that $Cr(\cdot|T_{ch_i}) = ch(\cdot)$, for each proposition T_{ch_i} for which $Cr(\cdot)$ is well-defined, that $Cr(\cdot)$ satisfies the Temporal Principle.

Given certain auxiliary constraints, the Principal Principle, then, entails the Temporal Principle. However, the reverse is not true. For we've seen that while every possible chance function satisfies the Temporal Principle, the same is not true of the Principal Principle. But each possible chance function is a probability function satisfying constraints (i) and (ii), as well as the analogue of (ii) for propositions of the form T_{ch}. There is not, then, an analogous way of showing that the Principal Principle follows from the Temporal Principle. The Temporal Principle, then, provides a weaker normative constraint on our credences than the Principal Principle. It is this weaker principle, though, that follows from Chance Vindication, i.e., from the assumption that vindication for credences consists in matching the chances.

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