The Problem of Irrelevant Conjunction — Revisited

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- Three (formal) *qualitative* concepts of **confirmation**:
  - **Hypothetico–Deductive confirmation.**
    - E confirms$_h$ H if H entails E.
  - **Confirmation as Firmness.**
    - E confirms$_f$ H iff $\Pr(H \mid E) > t$.
  - **Confirmation as Increase in Firmness.**
    - E confirms$_i$ H iff $\Pr(H \mid E) > \Pr(H)$.
- Two (formal) *comparative* confirmation relations:
  - **Comparative Firmness**.
    - E confirms$_f$ H_1 more strongly than E confirms$_f$ H_2 iff $\Pr(H_1 \mid E) > \Pr(H_2 \mid E)$.
  - **Comparative Increase in Firmness**.
    - E confirms$_i$ H_1 more strongly than E confirms$_i$ H_2 iff $\mathfrak{c}(H_1, E) > \mathfrak{c}(H_2, E)$. [where $\mathfrak{c}$ is some *relevance measure*]
- Two *informal* **evidential support** concepts:
  - E **supports$_1$** H iff E is (positively) evidentially relevant to H.
  - E **supports$_2$** H iff E warrants belief/acceptance of H.

(1) If E confirms H, then E confirms $H \,\&\, X$, for *any* X.

- Clark Glymour [5] raises two worries in connection with (1):

  (1a) [Confirmation$_h$ has property (1).] But we cannot admit, generally, that E will lend plausibility to an arbitrary X. One might ... deny ... the special consequence condition. But ... sometimes ... confirmation does ... follow entailment.

  (1b) As evidence accumulates, we may come to accept [p] ... and when we accept [p] we commit ourselves to accepting all of its logical consequences. So, if [E] could bring us to accept ... H, and whatever confirms H confirms $H \,\&\, X$ ... then ... [E] ... ought, presumably, to bring us to accept X.

- Both of these worries have to do with confirmation/support provided by E "rubbing off" onto an *irrelevant conjunct* X.
- (1b) involves explications of support$_2$, which imply (1).
  - We don't think (1b) is probative. *Nobody* thinks confirms$_h$ is a good explication of support$_2$. We'll focus on support$_1$.
- To that end, let's take a closer look at Glymour's (1a).

- The Special Consequence Condition is:

  (SCC) If E confirms $H \,\&\, X$, then E confirms X.

- If we combine (1) & (SCC), we get an (absurd) consequence: *if E confirms **any** hypothesis, it confirms **every** proposition.*
- So, any theory that entails (1) — *e.g.*, confirms$_h$ — must *not* entail (SCC) — *on pain of triviality*. HD confirmation theory does *not* entail (SCC). But, Glymour wants something more.

☞ Glymour wants an explication of support$_1$ that avoids triviality — but not by a *mere* rejection of (SCC). In (1a), he is demanding a *principled* (and *explanatory*) rejection of (SCC).

- Next, we'll examine two confirmation$_i$-based approaches to "the (1)-problem" — due to Earman and Rosenkrantz.
- After critiquing those approaches, I will discuss some alternative confirmation$_i$-based approaches that I prefer.
- Finally, I'll return to Glymour's (1a), and (time permitting) some recent objections due to Maher and Crupi *et. al.*

- Before getting into confirms$_i$-based approaches to "the problem of irrelevant conjunction" [*i.e.*, "the (1) problem"], we must ask whether there *is* such a problem *for confirms$_i$*.

☞ First, note that the confirms$_i$-analogue of (1) is *false*[1] — *i.e.*:

(2) E confirms$_i$ H $\not\Rightarrow$ E confirms$_i$ $H \,\&\, X$.

- Thus, confirms$_i$ does *not* suffer from a *strictly analogous* problem of irrelevant conjunction. However, we *do* have:

(3) If H *entails* E, then E confirms$_i$ $H \,\&\, X$ (for arbitrary X).

- So, in the *special (deductive) case* where H *entails* E (*i.e.*, where E confirms$_h$ H), confirmation$_i$-theory *does* entail (1).
- Contemporary confirms$_i$-theorists have had various things to say about (3). I will discuss two prominent approaches.
- Then, I'll explain why I don't think these approaches are very satisfying. And, I'll discuss some alternatives.

[1] Of course, the confirms$_f$-analogue of (1) is *also* false, but I won't go there.

- Earman [3] points out that — for *many* $\mathfrak{c}$'s — we have:

(3.1) If H entails E, then $\mathfrak{c}(H \,\&\, X, E) < \mathfrak{c}(H, E)$.

- What (3.1) says is that, while "irrelevant conjunctions" $H\&X$'s will be confirmed$_i$ by *deductive* evidence for H, such conjunctions will be confirmed *less strongly* than H is.
- Closer scrutiny of Earman's approach reveals:
  (a) The "irrelevance" of X is *irrelevant* to the decrease in $\mathfrak{c}_i$. After all, (3.1) is true for ***all*** X — irrelevant or otherwise.
  (b) (3.1) is *not true for all* $\mathfrak{c}$'s (*e.g.*, it fails for $r(H,E) \stackrel{\text{def}}{=} \frac{\Pr(H|E)}{\Pr(H)}$).
  (c) (3.1) only applies to cases of *deductive* evidence.
    - Arguably, this is not such an important case, since most interesting applications of confirms$_i$ involve *statistical* H's.
    - Moreover, as we'll see below, a *more general problem* of *irrelevant* conjunction plagues confirmation$_i$-theory — in *both* the deductive *and* the non-deductive cases.
- Earman's is not the only confirms$_i$-approach one finds in the literature. Rosenkrantz offers a different approach...

- Rosenkrantz [8] offers a confirms$_i$-approach — based on the following [where $d(H,E) \stackrel{\text{def}}{=} \Pr(H \mid E) - \Pr(H)$]:

(3.2) If H entails E, then $d(H \,\&\, X, E) = \Pr(X \mid H) \cdot d(H,E)$.

- Rosenkrantz does try to address *some* of the problems with Earman's account. In particular, he seems sensitive to (a):

> ...I hope you will agree that the two extreme positions on this issue are equally unpalatable, (i) that a consequence E of H confirms $H \,\&\, X$ not at all, and (ii) that E confirms $H \,\&\, X$ just as strongly as it confirms H alone. ...In general, intuition expects intermediate degrees of confirmation that depend on the degree of compatibility of H with X.

- Adopting $\Pr(X \mid H)$ as his measure of the "degree of compatibility of H with X", and d as his measure of confirmation$_i$ yields the kind of result that Rosenkrantz wants: (3.2). Is this an *improvement* on Earman?
- This depends on whether Rosenkrantz really has adequately addressed worries (a)–(c), above. I don't think he has...

- In a way, Rosenkrantz is *trying* to address (a) here. He seems to be thinking of $\Pr(X \mid H)$ as a kind of measure of "the degree of relevance" of X — *qua conjunct* in $H \,\&\, X$.
- But, this is a *very peculiar* way for a *Bayesian* to explicate "relevance"! Moreover, $\Pr(X \mid H)$ can tell us nothing about "degrees of relevance" involving X, H — *and* E.
- Moreover, when it comes to (b), Rosenkrantz is in even worse shape than Earman. Rosenkrantz's approach works *only* for confirmation$_i$-measures that are *very similar* to d.
- Finally, Rosenkrantz is still only addressing the *deductive* case. So, his account lacks *generality* in the same ways that Earman's approach does. Thus, he has not addressed (c).
- I think $\mathfrak{c}_i$-theorists need to *re-think* the problem of irrelevant conjunction, and its possible resolution(s).
- To that end, let's see how $\mathfrak{c}_i$-theory handles *irrelevant* conjunctions, in the general, *inductive* case...

- First, we need to say what it *means* for X to be an *irrelevant conjunct* to a hypothesis H, with respect to evidence E.
- Here's a natural explication, for a confirmation$_i$-theorist [6]:
  - X is an *irrelevant conjunct to H, with respect to evidence E*, just in case $\Pr(E \mid H \,\&\, X) = \Pr(E \mid H)$ [*i.e.*, *if* $X \perp\!\!\!\perp E \mid H$].
- This is a more natural explication of "irrelevant conjunct" than Rosenkrantz's (implicit) explication, since:
  - It makes use of *probabilistic independence*, which is a standard way for $\mathfrak{c}_i$-theorists to explicate *irrelevance*.
  - It's a relation involving X, H, *and* E (as it intuitively should be).
  - It's a natural (likelihood-based) *generalization* of the special, *deductive* case that has been traditionally discussed.
- With this explication of "irrelevant conjunct" in hand, we can now *state* a *more general problem of irrelevant conjunction — for confirmation$_i$-theory* — as follows:

(4) If E confirms$_i$ H, and X is an irrelevant conjunct to H, with respect to evidence E, then E *also* confirms$_i$ $H \,\&\, X$.

- So, we *have* a (general) "problem of irrelevant conjunction" for for confirmation$_i$-theory. What can be said about it?

(4′) If E confirms$_i$ H, and X is an irrelevant conjunct to H, with respect to evidence E, then $\mathfrak{c}(H \,\&\, X, E) < \mathfrak{c}(H, E)$.

- What (4′) tells us is that — while irrelevant conjunctions will be confirmed$_i$ to *some* degree by (H-confirming evidence) E — adding irrelevant conjuncts will lead to a *decrease* in $\mathfrak{c}_i$.
  - The *precise amount* by which $\mathfrak{c}_i$ is decreased by the addition of irrelevant conjuncts will depend on which relevance measure $\mathfrak{c}$ is used. But, "Rosenkrantz-like" equations [*i.e.*, (3.2)-like equations] can be deduced for each measure.
- (4′) is a generalization of Earman's (3.1). And, like Earman's (3.1), (4′) holds for *most* $\mathfrak{c}$'s (again, a notable exception being r).
- On the next slide, I'll return to Glymour's (1a) and the (SCC).
- Then, I'll address some objections to our approach that have appeared in the recent *Philosophy of Science* literature.

- To illustrate our approach, consider the following example:
  - Suppose we'll be sampling a card at random from a standard deck. Let E be the proposition that the card is black. Let X be the hypothesis that the card is an ace, and let H be the hypothesis that the card is a spade.
- The preconditions for our (4) and (4′) are met here, since:
  - E confirms$_i$ H.
  - $\Pr(E \mid H \,\&\, X) = \Pr(E \mid H)$.
- Therefore, (4) and (4′) entail the following:
  - E confirms$_i$ $H \,\&\, X$.
  - $\mathfrak{c}(H \,\&\, X, E) < \mathfrak{c}(H, E)$, for "most" relevance measures $\mathfrak{c}$.
- Finally, we *also* have the following:
  - E does *not* confirm$_i$ X.

☞ We think all these predictions of our confirms$_i$-explications line-up well with the support$_1$-relations. ∴ We think ours is *no mere* rejection of (SCC). It's *principled* (and explanatory).

- Patrick Maher [7] complains that our approach doesn't *address* the problem of irrelevant conjunction (PIC), because he thinks the PIC is *grounded on the following intuition*:

($\star$) If X is an irrelevant conjunct to H, with respect to evidence E, then E *does not support*$_1$ $H \,\&\, X$.

- ∴ Maher thinks that *the* way to resolve the PIC is *merely* to point out that ($\star$) is *false* (as we do in our example above).
- We agree that ($\star$) is false. We also agree that *some* people *may* be worried about PIC *because* they accept ($\star$).
- But, we *disagree* with Maher on the following two points:
  - We *don't* think acceptance of ($\star$) is *essential* to the problem and/or its motivation [did ($\star$) ground PIC for *Glmyour*?].
  - We think our approach and analysis *further illuminates* what is going on — from a confirmation$_i$ point of view.
- So, we are not moved by Maher's worries about our approach. Next, we'll discuss a more recent objection…

- Crupi *et al.* have recently argued [2] that our approach yields incorrect predictions — in cases of *dis*confirmation.
- To understand their worry, it helps to state the results we had in mind in a slightly more general (and revealing) way.
- Let's assume that confirmation$_i$-measures ($\mathfrak{c}$) take *negative* values in cases of *dis*confirmation$_i$ and *positive* values in cases of *con*firmation$_i$. And, assume we're talking about the measures $\mathfrak{c}$ we had in mind when we put forward our (4′).
- Given these assumptions, we can *actually* show that:

  (†) If X is an irrelevant conjunct to H, with respect to E, then $|\mathfrak{c}(H \,\&\, X, E)| < |\mathfrak{c}(H, E)|$.

  (4†) ∴ If E *dis*confirms H, and X is an irrelevant conjunct to H, with respect to E, then $\mathfrak{c}(H \,\&\, X, E) > \mathfrak{c}(H, E)$.

☞ For the measures $\mathfrak{c}$ we had in mind when we put forward (4′), we get the result that *adding irrelevant conjuncts to E-**dis**confirmed hypotheses **increases** degree of confirmation.*

- Crupi *et. al.* think our (†) and (4†) are counter-intuitive.
- In fact, they defend the following *contrary* claim:

  ∼(4†) If E *dis*confirms H, and X is an irrelevant conjunct to H, with respect to E, then $\mathfrak{c}(H \,\&\, X, E) < \mathfrak{c}(H, E)$.

- And, they use ∼(4†) to bolster their (pre-existing) case for a "piece-wise" confirmation measure z, which treats *confirmation* and *disconfirmation* as *different functions*:

$$z(H,E) = \begin{cases} \frac{\Pr(H|E)-\Pr(H)}{\Pr(\sim H)} & \text{if } \Pr(H \mid E) \geq \Pr(H) \\ \frac{\Pr(H|E)-\Pr(H)}{\Pr(H)} & \text{if } \Pr(H \mid E) < \Pr(H) \end{cases}$$

- I won't be able to discuss the very clever (independent) argument in favor of z that Crupi *et. al.* had previously published [1]. But, our response *here* is to *bite the bullet.*

  ☞ It seems to us that an irrelevant conjunct (one that doesn't alter the likelihood H attributes to the evidence) adds nothing but "extra mass" to the hypothesis. This "extra mass" just makes the incremental confirmation *and disconfirmation* of $H \,\&\, X$ "more sluggish" than for H alone.

[1] V. Crupi and K. Tentori and M. Gonzalez, *On Bayesian Measures of Evidential Support: Theoretical and Empirical Issues*, *Philosophy of Science*, 2007.

[2] V. Crupi and K. Tentori, *Irrelevant Conjunction: Statement and Solution of a New Paradox*, *Philosophy of Science*, 2010.

[3] J. Earman, *Bayes or Bust?*, MIT Press, 1992.

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[5] C. Glymour, *Theory and Evidence*, Princeton U. Press, 1980.

[6] J. Hawthorne and B. Fitelson, *Re-solving irrelevant conjunction with probabilistic independence*, *Philosophy of Science*, 2005. URL: `http://fitelson.org/ic_2.pdf`.

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