Toward probabilistic mental logic

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Plan

- Revive the project of mental logic
- Probabilistic natural logic for syllogistic reasoning
- Weights based in empirical data
- Reflecting ‘complexity/preferability’ of single reasoning rules
- Proof-of-concept providing guidelines for further work
Logic as the theory of reasoning & its challenges

- Logical Omniscience
- Conjunction Fallacy
- Wason Selection Task
- Suppression Task
- etc.
Bayesian Rationality ⊆ Mental Models ⊆ Mental Logic

Reaction:
Mental Logic

- Rips (1994):
  - Formulas as the underlying mental representations
  - Inference rules are the basic operations
  - PSYCOP based on Natural Deduction
  - You can think about proofs as computations.
ML’s shortcomings

- Abstract rules and formal representations
- Based in natural deduction for FOL
- Ad hoc `psychological completeness'`
- Explains only validities, no story on mistakes
- No learning or individual differences
Natural Logic Program

- van Benthem 1986, Sánchez-Valencia 1991:
- Computationally minimal systems
- Following `the surface structure of NL`
- Traditionally monotonicity and semantic containment
- Recently intensively studied, extended, and applied, e.g., by Stanford NLP group
- So, why not build MLs based on these ideas?

IF No aardvark without a keen sense of smell can find food. THEN No aardvark without a sense of smell can find food.
Benchmark Task: arena of syllogistic reasoning

- All A are B: universal affirmative (A)
- Some A are B: particular affirmative (I)
- No A are B: universal negative (E)
- Some A are not B: particular negative (O)

Figure 4

AN ENGINEER
SYLLOGISM

1: I AM GOOD AT
UNDERSTANDING
NUMBERS.

2: THE STOCK
MARKET IS MADE
OF NUMBERS.

3: THEREFORE, I-
WOW, WHERE DID
ALL MY MONEY
JUST GO?

All C are B
AE4O: No B are A
Some A are not C
### Syllogistic reasoning

<table>
<thead>
<tr>
<th>Syllogism</th>
<th>Conclusion</th>
<th>Syllogism</th>
<th>Conclusion</th>
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<td>58 8 1 1 32</td>
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<td>II1</td>
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<td>1 4 1 25 69</td>
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<td>3 8 2 29 58</td>
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<td>0 1 34 1 64</td>
<td>EO1</td>
<td>1 8 8 23 60</td>
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<td>3 3 14 3 77</td>
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<td>0 13 7 11 69</td>
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<td>0 0 18 3 78</td>
<td>EO3</td>
<td>0 0 9 28 63</td>
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<td>EE4</td>
<td>0 3 31 1 65</td>
<td>EO4</td>
<td>0 5 8 12 75</td>
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</table>

Table 2.1: Percentage of times each syllogistic conclusion was endorsed. The data is from a meta-analysis by Chater and Oaksford (1999). “NVC” stands for “No Valid Conclusion”, all numbers have been rounded to the closest integer. A bold number indicates that the corresponding conclusion is valid.

Chater and Oaksford, 1999
Geurts (2003)’s model

- Logic including syllogistics and pivoting on monotonicity with rules:
  - *All-Some:* `All A are B’ implies `Some A are B’.
  - *No-Some not:* `No A are B’ implies `Some A are not B’.
  - *Conversion1:* `Some A are B’ implies `Some B are A’;
  - *Conversion2:* `No A are B’ implies `No B are A”.

- *Monotonicity:* If A entails B, then the A in any upward entailing position can be substituted by a B, and the B in any downward entailing position can be substituted by an A.

- *Extra rule:* `No A are B’ and `Some C are A’ implies `Some C are not B’.
Example for EA2E

No C are B  (1)
All A are B  (2)

No B are C  (3)  Conversion(1)
No A are C  (4)  Monotonicity(2,3)
Geurts’ (2003) model c’td

- The shorter the proof the easier the syllogism.
- Initial budget of 100 units. Each use of the monotonicity rule costs 20, the extra rule costs 30; a proof containing a "Some Not" proposition costs an additional 10 units. Take the remaining budget as an evaluation of the difficulty.
- It gives a good fit with data.

Similiar strategy works for other cognitive tasks, see Gierasimczuk et al. 2014.
Learning the inference rules from the data

Joint work with Fangzhou Zhai and Ivan Titov
Vanilla version

- Geurts’ logic
- Tree representation: states linked by reasoning events
- No vapid transitions
Probabilities

- Tendency value: an easier rule is adopted with higher probability, while a more difficult one is adopted with lower probability.

- Let $T_r$ any rule and $c_r$ the number of ways that it can be adopted at $S$:

$$p_0(S_r | S, \theta_0) = \frac{T_r}{\sum_{r \in R} c_r \cdot T_r}$$
A probability with which a syllogism is endorsed.

5 possible conclusions: A, I, E, O, NVC.

Each leaf uniquely determines a path from the root.

We can compute the probability that a given conclusion is drawn.

\[
p'_0(y|R, \theta_0) = \sum_{S \text{ is a leaf consistent with } y} \prod_{0 \leq i < n} p_0(S_{i+1}|S_i, \theta_0)
\]
Training

- Subset of the data from Chater and Oaksford (1999)
- We use the Expectation-Maximization algorithm
- Compute:

\[
\arg\max_{\theta_0} p_0(\{(X_i, y_i)\}_{i \leq n} | \theta_0)
\]
Evaluation

- The Khemlani and Johnson-Laird (2012) method
- Detection theory

<table>
<thead>
<tr>
<th>Predictions \ Exp. Data</th>
<th>&lt; 30%</th>
<th>≥ 30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 30%</td>
<td>Correct Rejection</td>
<td>Miss</td>
</tr>
<tr>
<td>≥ 30%</td>
<td>False Alarm</td>
<td>Hit</td>
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</table>
Performance of Vanilla Version

- 95.8% correct predictions on syllogisms with at least one conclusion.
- 81.6% correct predictions on all syllogisms.
- But no mechanism to explain the errors.
- The models always return NVC for invalid syllogisms.
Adding illicit conversions

- **Conversion**: For every Q, `Q A are B’ implies `Q B are A’.
- Half the number of misses.
- 91.9% correct predictions on all syllogisms.
- For II, IO, EE, OI, OE, OO always returns NVC.
Let’s guess

- Probability of guessing NVC is negatively related to the informativeness of the premises.
- **Atmosphere hypothesis** when there is a negation in the premises, individuals are likely to draw a negative conclusion; when there is `some' in the premises it will be likely in the conclusion; when neither is the case, the conclusion is often affirmative.
Performance

• 95% correct predictions on all syllogisms

• The training gives the informativeness order as assumed by Chater & Oaksford

  A(1.11) > E(0.33) > I(0.199) > O(-0.78)

• And data yields the complexity order:

  Conversion < Monotonicity < All-Some < No-SomeNot
Comparing with other theories

<table>
<thead>
<tr>
<th>Theory</th>
<th>Hit</th>
<th>Miss</th>
<th>False Alarm</th>
<th>Correct Rejection</th>
<th>Correct Predictions</th>
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<tbody>
<tr>
<td>Atmosphere</td>
<td>44</td>
<td>41</td>
<td>20</td>
<td>215</td>
<td>259 / 80.9%</td>
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<tr>
<td>Matching</td>
<td>41</td>
<td>44</td>
<td>55</td>
<td>180</td>
<td>221 / 69.1%</td>
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<tr>
<td>Conversion</td>
<td>52</td>
<td>33</td>
<td>12</td>
<td>223</td>
<td>275 / 85.9%</td>
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<tr>
<td>PHM*</td>
<td>40</td>
<td>45</td>
<td>63</td>
<td>172</td>
<td>212 / 66.3%</td>
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<td>PSYCOP</td>
<td>45</td>
<td>40</td>
<td>26</td>
<td>209</td>
<td>254 / 79.4%</td>
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<td>54</td>
<td>31</td>
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<td>206</td>
<td>260 / 81.2%</td>
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<td>Mental Models*</td>
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<td>55</td>
<td>180</td>
<td>265 / 82.8%</td>
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<td>Generative Model Ver. 1</td>
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<td>26</td>
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<td>9</td>
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<tr>
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<td>85</td>
<td>0</td>
<td>0</td>
<td>235</td>
<td>320 / 100%</td>
</tr>
</tbody>
</table>

Khemlani and Johnson-Laird (2012)
Summary

- Abstract ND rules of ML can be replaced by NL.
- Ad hoc `psychological completeness’ can be derived from data, some rules are unlikely to fire.
- It can give a more systematic take on reasoning errors.
- A way to classify inferences steps wrt cognitive difficulty.
- Yields computationally friendlier systems.
- Modular approach.
How much logic do we need?

(Pratt-Hartmann 2010; Thorne, 2010; Larry Moss, 2010)
Further work

❖ Extend to wider fragments of language.
❖ But also other types of reasoning
  (see, e.g. Gierasimczuk et. al. 2013, Braüner 2013).
❖ Run experiments/train model on better data.
❖ Understand learning and individual differences
  (joint work with N. Gierasimczuk & A.L. Vargas Sandoval).
❖ Think about processing model and its complexity.
❖ ...

Thank you!
Amsterdam Colloquium 2015

Workshop ‘Reasoning in Natural Language: Symbolic and Sub-symbolic Approaches’