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Disquotation and Infinite Conjunctions

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The truth predicate has certain expressive powers akin to those of logical connectives: it allows to express infinitely many sentences at once (the Ps) via generalizations or infinite conjunctions, i.e. expressions of the form

$$\text{"All } P\text{s are true."} \tag{1}$$

"All theorems of arithmetic are true."

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The obvious advantages of such expressive device prompt the search for 'logics' or, more precisely, formal theories of truth, (some syntax at the base must be assumed to talk about sentences).

Such theories should entail all principles and inferences that are necessary to guarantee that the truth predicate can fulfil its logical role, i.e. to allow (1) to express all and only the Ps.

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Several authors, including Horwich [?], Halbach [?], Priest [?], Field [?], Beall [?], Cobreros *et al.* [?], and perhaps Quine [?], maintain that the truth predicate serves this expressive role only in virtue of its *disquotational* or *transparent* nature, i.e. the equivalence between each sentence A and "A' is true', where 'A' is a name for A.

They conclude that what allows the truth predicate Tr in a formal setting to express infinite conjunctions are certain 'transparency' principles that establish this equivalence: e.g. the biconditionals

$$Tr('A') \leftrightarrow A \qquad \text{(T–schema)}$$

or the rules

$$\frac{A}{Tr('A')} \qquad \text{(T–Intro)} \qquad \qquad \frac{Tr('A')}{A} \qquad \text{(T–Elim)}$$

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Due to the paradoxes, a truth predicate satisfying a transparency
principle is not possible in a classical context, on pain of triviality.

Some authors have concluded that classical logic must be abandoned,
and put forward non-classical (paracomplete, paraconsistent,
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Others that stick to classical logic resigned themselves to sacrificing
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We cast doubt on the necessity or sufficiency of transpacency principles to grant truth these expressive powers.

1. We argue that the two most promising accounts of what it means for sentences like (1) to express all the Ps place unreasonable requirements on truth theories.

2. We show that the reasonable bits of both accounts can be met in classical logic, adopting a consistent subprinciple of transparency. We conclude that so far the expression of infinite conjunctions carries no need to abandon classical logic, nor to sacrifice this expressive power if one wishes to remain classical.

3. We show that in certain non-classical systems none of the transparency principles is enough to grant those reasonable bits. Thus, one has to be careful if one still wishes to abandon classical logic and guarantee the logical role of truth.

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Expressing infinite conjunctions

What does it mean that (1) expresses all *P*s and nothing more, or their infinite conjunction?

The 'infinite conjunction' response: (1) expresses all the *P*s as long as it implies all the *P*s and, vice versa, it is implied by all the *P*s.

The 'finite axiomatisation' response (Halbach [?]): (1) expresses all the *P*s as long as (1) and the set of all the *P*s entail the same formulae.

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The infinite conjunction response

Horwich [**?**, p. 3]:

> *Suppose, for example, I have great confidence in Oscar's judgment about food; he has just asserted that eels are good but I didn't quite catch the remark. Which belief might I reasonably acquire? Well, obviously not that eels are good. Rather what is needed is a proposition from which that one would follow, given identification of what Oscar said—a proposition equivalent to*
>
> **If what Oscar said is that eels are good then eels are good, and if he said that milk is white then milk is white, ... and so on;**
>
> *and the raison d'être of the concept of truth is that it supplies us with such a proposition: namely "What Oscar said is true".*

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The infinite conjunction response

In formal terms, that (1) implies and is implied by all the Ps means that the following inferences hold:

$$\frac{\{P(`A`) \rightarrow A : A \in \mathcal{L}_T\}}{\forall x(Px \rightarrow Tr(x))} \qquad (\bigwedge\text{-Intro})$$

$$\frac{\forall x(Px \rightarrow Tr(x)), \ P(`A`)}{A} \qquad (\bigwedge\text{-Elim})$$

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The infinite conjunction response in trouble

It's only reasonable to demand that $\bigwedge$-Elim holds in a formal theory of truth, not $\bigwedge$-Intro.

In classical logic the left-to-right direction of the T-schema, this is,

$$Tr(`A`) \to A \qquad\qquad \text{(T-Out)}$$

or, equivalently, T-Elim, suffices to guarantee the inference $\bigwedge$-Elim.

E.g. in classical logic T-Out is consistent with PA.

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Observation

Let TT be any classical theory extending Q that contains T-Out. Then $\bigwedge$-Elim holds in TT.

Proof:

1.	$\forall x(P(x) \rightarrow Tr(x))$	Premise 1
2.	$P(`A`)$	Premise 2
3.	$P(`A`) \rightarrow Tr(`A`)$	1, universal instantiation
4.	$Tr(`A`)$	2, 3, MP
5.	$Tr(`A`) \rightarrow A$	T-Out
6.	A	4, 5, MP

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The finite axiomatisation response

Halbach [?, p. 13]:

> *[…] disquotationalism should not claim that an infinite conjunction and the respective sentence involving the truth predicate are equivalent sentences in a language; they are only equivalent in their consequences with respect to statements without the truth predicate or infinitely placed connectives.*

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The finite axiomatisation response, typed

Formally, $\forall x(Px \rightarrow Tr(x))$ should finitely axiomatise the set $\{P(`A') \rightarrow A : A \in \mathcal{L}\}$, i.e. for every sentence $C \in \mathcal{L}$,

$$\textsc{tt} + \forall x(Px \rightarrow Tr(x)) \vdash C \quad \text{iff} \quad \textsc{b} + \{P(`A') \rightarrow A : A \in \mathcal{L}\} \vdash C$$

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The finite axiomatisation response, untyped

Formally, $\forall x(Px \to Tr(x))$ should finitely axiomatise the set $\{P(`A') \to A : A \in \mathcal{L}_T\}$, i.e. for every sentence $C \in \mathcal{L}_T$,

$$\tau + \forall x(Px \to Tr(x)) \vdash C \quad \text{iff} \quad \tau + \{P(`A') \to A : A \in \mathcal{L}_T\} \vdash C$$

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The finite axiomatisation response in trouble

In classical logic (typed) T-Out or T-Elim suffice to guarantee finite axiomatizations in the typed case.

In the untyped case it's only reasonable to demand the right-to-left direction of the biconditional

$$\top + \forall x (Px \rightarrow Tr(x)) \vdash C \quad \text{iff} \quad \top + \{P(`A') \rightarrow A : A \in \mathcal{L}_T\} \vdash C$$

because, taking C to be $\forall x (Px \rightarrow Tr(x))$, the opposite direction entails $\bigwedge$-Intro.

For the left-to-right direction (untyped) T-Out or T-Elim are sufficient.

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Limitative results

Theorem
No truth predicate in Tarski's Hierarchy satisfies the elimination rules.

Theorem
The truth predicate of Priest's LPTT *does not satisfy the elimination rules.*

Theorem
The truth predicate of Ripley's STTT *does not satisfy the elimination rules if premises are asserted categorically.*

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Introducing the truth predicate

Is the right–to–left direction of the T–schema, this is,

$$A \rightarrow Tr(\text{'}A\text{'}) \qquad \text{(T–In)}$$

or, equivalently, T–Intro, needed for expressing infinite conjunctions?

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Introducing generalizations

For every finite set of sentences $\{A_1, \ldots, A_n\} = \{A \in \mathcal{L}_T : P(`A`)\}$ of the language, T–In or T–Intro allow for the following inference:

$$\frac{\{A \in \mathcal{L}_T : P(`A`)\}}{\forall x(Px \rightarrow Tr(x))} \qquad (\bigwedge_n\text{-Intro})$$

So $\forall x(Px \rightarrow Tr(x))$ is the 'finite conjunction' of the Ps. But finite conjunctions are already possible in the language without the truth predicate.

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Embedding generalizations in conditionals

Consider the following definition of knowledge:

$$\forall x(K(x) \leftrightarrow C(x) \wedge Tr(x)) \tag{2}$$

Here the truth predicate allows to finitely express all the instances

$$K(`B') \leftrightarrow C(`B') \wedge B \tag{3}$$

If we know that B and that $C(`B')$, we would like to conclude from (2) that $K(`B')$. That requires T-In or T-Intro.

However, there is no need to generalise on the instances of (3) by (2). We may well do so by a generalisation of the form

$$\forall x(Px \rightarrow Tr(x)) \tag{4}$$

where Px is true exactly of all instances of (3).

In classical logic T-Out or T-Elim allow us to infer $K(`B')$ from (4) and $C(`B') \wedge B$.

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The epistemological role of truth

Generalizations like (1) where P is satisfied only by a finite number of sentences can be useful not for logical but for epistemological reasons.

One might not know or remember the explicit articulation of one or (possibly finitely) many sentences while counting on a property P that applies only to them. Then, one can express the content of these sentences nonetheless, aided by the truth predicate: just utter (1).

"Everything the Pope says is true."

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Field's challenge

Field [**?**, p. 210] claims that some epistemological uses require T-In or T-Intro:

> *Suppose I can't remember exactly what was in the Conyers report on the 2004 election, but say*
>
> *(a) If everything that the Conyers report says is true then the 2004 election was stolen.*
>
> *Suppose that what the Conyers report says is $A_1, \ldots, A_n$. Then, relative to this last supposition, (a) better be* equivalent *to*
>
> *(b) If A_1 and …and A_n then the 2004 election was stolen.*
>
> *And this requires $Tr('A')$ to be intersubstitutable with A even when A is the antecedent of a conditional.*

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Meeting Field's challenge

The full equivalence between $Tr('A')$ and A is needed for (a) and (b) to be equivalent to each other.

But demanding full equivalence is not reasonable in the infinite cases, where a weaker requirement should suffice. This weaker requirement should also suffice for the finite cases then.

It takes an introduction principle even to allow (a) to imply (b), but (a) isn't the only possible way of generalizing (b). As before, we suggest employing a sentence of the form

$$\forall x(Px \rightarrow Tr(x)) \tag{5}$$

where Px is true exactly of (b).

In classical logic T-Out or T-Elim are enough to infer (b) from (5) and the supposition that what the Conyers report says is $A_1, \ldots, A_n$.

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It takes an introduction principle even to allow (a) to imply (b), but (a) isn't the only possible way of generalizing (b). As before, we suggest employing a sentence of the form

$$\forall x(Px \rightarrow Tr(x)) \tag{5}$$

where Px is true exactly of (b).

In classical logic T-Out or T-Elim are enough to infer (b) from (5) and the supposition that what the Conyers report says is $A_1, \ldots, A_n$.

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T-In or T-Intro allow us to introduce only finite generalizations, but these are in principle dispensable.

Many uses of T-In or T-Intro can be mimicked in classical T-Out- or T-Elim-theories.

So far there aren't enoguh reasons to abandon classical logic to have a truth predicate capable of expressing infinitely many sentences at once, nor to sacrifice part of this expressive power to remain classical.

In many cases, abandoning classical logic means that the truth predicate isn't capable of expressing infinite conjunctions, even if transparency principles hold in full.

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References I

[1] Beall, J. C.
Transparent disquotationalism.
In *Deflationism and Paradox*, J. C. Beall and B. Armour-Garb, Eds.
Oxford University Press, 2005, pp. 7–22.

[2] Cobreros, P., Egré, P., Ripley, D., and van Rooij, R.
Reaching Transparent Truth.
Mind 122 (2013), 841–866.

[3] Field, H.
Saving truth from paradox.
Oxford University Press, New York, 2008.

[4] Halbach, V.
Disquotationalism and infinite conjunctions.
Mind 108 (1999), 1–22.

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References II

[5] Horwich, P.
Truth, second ed.
Blackwell, Oxford, 1998.

[6] Priest, G.
In Contradiction.
Oxford University Press, New York, 2006.

[7] Quine, W. V. O.
Philosophy of Logic.
Harvard University Press, 1970.