Believing Epistemic Contradictions

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Bridges 2 · 2015

Outline

The Puzzle

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(1) ?? Ari believes the house is empty and might not be empty.

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- Relevant reading: Ari bel [$empty \wedge \Diamond\neg empty$]

The Puzzle

- **Uncertain Belief**: It's possible to coherently believe φ without being certain that φ.

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The Puzzle

- **Uncertain Belief**: It's possible to coherently believe φ without being certain that φ.
- **Uncertainty-Possibility Link**: If an agent A is coherent, then if A isn't certain that φ, A believes $\Diamond\neg\varphi$.
- **No Contradictions**: It's incoherent to believe $(\varphi \wedge \Diamond\neg\varphi)$.

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Uncertain Belief

- **Uncertain Belief**: It's possible to coherently believe φ without being certain that φ.

Uncertain Belief

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(2) ✓ I believe the movie starts at 7, but I'm not certain.

(3) # I'm certain that the movie starts at 7, but I'm not certain.

Uncertain Belief

(4) ✓ Ari believes that the house is empty, but she's not certain of it.

Uncertainty-Possibility Link

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Uncertainty-Possibility Link

- **Uncertainty-Possibility Link**: If an agent A is coherent, then if A isn't certain that φ, A believes $\Diamond\neg\varphi$.

(5) ?? I'm not certain the house is empty. But there's no possibility that it isn't.

(6) ?? The detective isn't certain whether the butler did it. But she thinks there's no chance the butler didn't do it.

No Contradictions

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(7) ?? Ari believes the house is empty and might not be.

(8) ?? Joe thinks it's raining and might not be.

(9) ?? The detective believes the butler is guilty and might be innocent.

No Contradictions

- A more general phenomenon:

No Contradictions

- A more general phenomenon:

(10) ?? It's raining and it might not be.

No Contradictions

- A more general phenomenon:

(10) ?? It's raining and it might not be.

(11) ?? Suppose/imagine that it's raining and might not be.

(Yalcin 2007; Anand and Hacquard 2013; Dorr and Hawthorne 2013)

Outline

Contextualism

Definition (Contextualism)

$[\![\Diamond\varphi]\!]^{c,w} = 1$ iff $B_{c,w} \cap [\![\varphi]\!]^{c} \neq \emptyset$.

- $B_{c,w}$ = the c-determined modal base
- e.g., $[\![$The house might not be empty$]\!]^{c,w} = 1$ iff $B_{c,w} \cap [\![$The house isn't empty$]\!]^{c} \neq \emptyset$

  (Kratzer 1981, 1991, 2012)

Contextualism

- What's the epistemic modal base?

Contextualism

- What's the epistemic modal base?
  - (i) Knowledge
  - (ii) Belief

The Knowledge-Based Approach

- The epistemic modal base = the possibilities compatible with what the relevant agents know (or are in a position to know)

  (Hacking 1967; Kratzer 1981, 2012; DeRose 1991; Stanley 2005; Stephenson 2007; Egan and Weatherson 2011; Dorr and Hawthorne 2013)

- i.e., *Might* φ is true iff φ is compatible with what the relevant folks know.

The Knowledge-Based Approach

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- i.e., *Might* φ is true iff φ is compatible with what the relevant folks know.

- **Con:** Has trouble validating **No Contradictions**.

The Knowledge-Based Approach

- Believing $(\varphi \wedge \Diamond\neg\varphi)$ =

  Believing ($\varphi \wedge$ ($\neg\varphi$ is compatible with what the relevant agents know))

The Knowledge-Based Approach

- Believing $(\varphi \wedge \Diamond\neg\varphi) =$

  Believing $(\varphi \wedge (\neg\varphi$ is compatible with what the relevant agents know$))$

- Nothing incoherent about believing φ, and believing that one's belief in φ doesn't amount to knowledge.

The Knowledge-Based Approach

- Possible reply:

  Knowledge norm of belief

  (Williamson 2000; Sutton 2007; Bird 2007; Huemer 2007; Smithies 2012)

The Knowledge-Based Approach

(12) ✓ Thelma believes God exists, and that she doesn't know God exists.

(13) ✓ Louise believes her ticket will lose, and that she doesn't know whether her ticket will lose.

The Knowledge-Based Approach

(12) ✓ Thelma believes God exists, and that she doesn't know God exists.

(13) ✓ Louise believes her ticket will lose, and that she doesn't know whether her ticket will lose.

(14) ?? Thelma believes God exists and might not exist.

(15) ?? Louise believes her ticket will lose and might win.

The Belief-Based Approach

- The epistemic modal base = the possibilities compatible with what the relevant agents believe

The Belief-Based Approach

- The epistemic modal base = the possibilities compatible with what the relevant agents believe
- **Pro:** Enables us to validate **No Contradictions**.
    - Believing an epistemic contradiction $\Rightarrow$ having a Moore-paradoxical belief ($\varphi \wedge$ I don't believe φ)

The Belief-Based Approach

- **Con:** Forces us to give up either **Uncertainty-Possibility Link** or **Uncertain Belief**.

The Belief-Based Approach

- **Con:** Forces us to give up either **Uncertainty-Possibility Link** or **Uncertain Belief**.
- On the belief-based approach, Ari is committed to believing: ⟨The house is empty and I don't believe the house is empty⟩.

Outline

Further Embedding Problems

(16) ?? Suppose it's raining and it might not be raining.

(17) ✓ Suppose it's raining and I don't know [/believe] it's raining.

(18) ?? If it's raining and it might not be raining, then...

(19) ✓ If it's raining and I don't know [/believe] it's raining, then...

Update Semantics

- The meaning of φ is not $[\![\varphi]\!]$, the set of worlds where φ is true.
- The meaning of φ is $[\varphi]$, a *context change potential*.

Update Semantics

Definition (Contexts)

s is a set of possible worlds.

Definition (Update Semantics)

1. $s[\alpha] = s \cap \{w : w(\alpha) = 1\}$
2. $s[\varphi \wedge \psi] = s[\varphi][\psi]$
3. $s[\neg\varphi] = s - s[\varphi]$
4. $s[\Diamond\varphi] = \{w \in s : s[\varphi] \neq \emptyset\}$.

Update Semantics

Definition (Support)

s supports φ ($s \models \varphi$) iff $s[\varphi] = s$.

Definition (Validity)

φ is valid ($\models \varphi$) just in case for every s, $s \models \varphi$.

Update Semantics

Definition (*Belief* as Support)

$s[B_A\varphi] = \{w \in s | \ s_A^w \models \varphi\}.$

- where s_A^w is the set of worlds compatible with A's beliefs at w.

Definition (*Certainty* as Support)

$s[C_A\varphi] = \{w \in s | \ c_A^w \models \varphi\}.$

- where c_A^w is the set of worlds compatible with A's certainties at w.

Update Semantics

Fact (No Contradictions)

$\models \neg B_A(\varphi \wedge \Diamond\neg\varphi)$.

Update Semantics

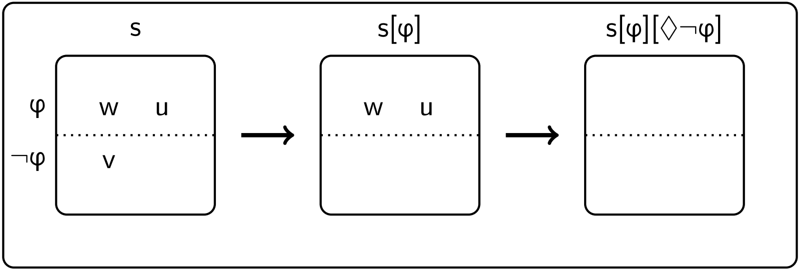

Figure : Updating with $\varphi \wedge \Diamond\neg\varphi$

Update Semantics

- Problem: either **Uncertain Belief** or **Uncertainty-Possibility Link** is invalid.
- $s^w_A = c^w_A \begin{cases} \textbf{Uncertainty-Possibility Link} & \checkmark \\ \textbf{Uncertain Belief} & \times \end{cases}$
- $s^w_A \neq c^w_A \begin{cases} \textbf{Uncertainty-Possibility Link} & \times \\ \textbf{Uncertain Belief} & \checkmark \end{cases}$

Outline

A New Semantics for Belief Reports

- Basic idea:

  an agent believes φ iff they assign a sufficiently high degree of confidence to the result of adding φ to their current information

- Combines a test semantics for epistemic modals with a "Lockean"/threshold view of belief

A New Semantics for Belief Reports

- Let $s_A^w = c_A^w =$ the set of worlds compatible with A's certainties at w (call this 'A's information state at w').
- Let Pr_A^w be A's credence function at w.
- We will hold fixed **Update Semantics** and **Certainty as Support**

Definition (Background: Update Semantics)

1. $s[\alpha] = s \cap \{w : w(\alpha) = 1\}$
2. $s[\varphi \wedge \psi] = s[\varphi][\psi]$
3. $s[\neg\varphi] = s - s[\varphi]$
4. $s[\Diamond\varphi] = \{w \in s : s[\varphi] \neq \emptyset\}$
5. $s[C_A\varphi] = \{w \in s |\ s_A^w \models \varphi\}$.

A New Semantics for Belief Reports

- the old version:

Definition (Lockean *belief*)

$[\![B_A\varphi]\!]^w = 1$ iff $Pr_A^w([\![\varphi]\!]) > t$.

A New Semantics for Belief Reports

Definition (Locke Updated)

$s[B_A\varphi] = \{w \in s |\ Pr_A^w(s_A^w[\varphi]) > t\}$.

- **Step 1:** update A's info state at w with φ, giving us: $s_A^w[\varphi]$.
- **Step 2:** Plug this set of worlds ($s_A^w[\varphi]$) into A's credence function Pr_A^w.

Validating **Uncertain Belief**

- **Locke Updated** agrees with **Lockean Belief** when it comes to descriptive (non-modal) beliefs:

Fact (Descriptive Beliefs Are Lockean)

For any descriptive sentence φ*:* $s[B_A\varphi] = \{w \in s|\ Pr^w_A([\![\varphi]\!]) > t\}$.

Validating **Uncertain Belief**

- Together with **Certainty as Support**, this entails **Uncertain Belief**.

Validating **Uncertain Belief**

- Ari's info state = {w, u, v}
  - {w, u} $\subseteq$ {w*: the house is empty at w*}
  - v $\in$ {w*: someone's inside the house at w*}
- Ari's credence in {w, u} = .8
- t = .75
- Ari believes the house is empty.

Validating **Uncertain Belief**

- Ari's info state = $\{w, u, v\}$
  - $\{w, u\} \subseteq \{w^*$: the house is empty at $w^*\}$
  - $v \in \{w^*$: someone's inside the house at $w^*\}$
- Ari's credence in $\{w, u\} = .8$
- $t = .75$
- Ari believes the house is empty.
  - = true, since Ari's credence in $\{w, u\} > t$

Validating **Uncertainty-Possibility Link**

Fact (*Might* Beliefs Are Transparent)

For any descriptive sentence φ*:* $s[B_A \Diamond \varphi] = \{w \in s | \ s_A^w[\varphi] \neq \emptyset\}$.

Validating **Uncertainty-Possibility Link**

- **Fact 2** + **Certainty as Support** ⇒ **Uncertainty-Possibility Link**
- If Ari isn't certain the house is empty, her info state contains at least one not-empty world (v).
- So, by **Fact 2**, Ari believes the house might not be empty.

Validating **No Contradictions**

Fact (No Contradictions)

$\models \neg B_A(\varphi \wedge \Diamond\neg\varphi)$.

Validating **No Contradictions**

(20) ?? Ari believes the house is empty and might not be.

- **Step 1:** Update Ari's info state with *the house is empty*
  - $\{w, u, v\} \rightarrow \{w, u\}$
- **Step 2:** Update Ari's info state with *the house might not be empty*
  - $\{w, u\} \rightarrow \emptyset$
- **Step 3:** Check whether Ari's credence in this set $>$ t

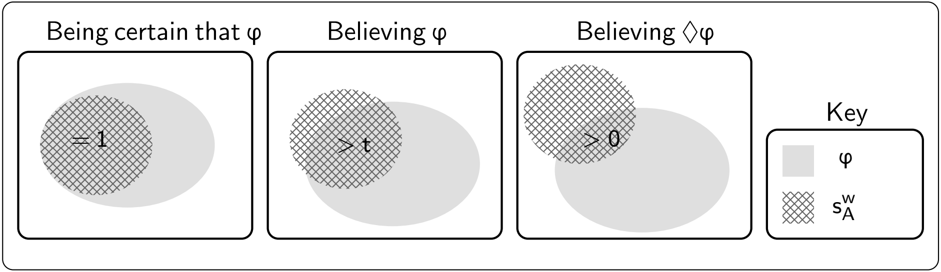

Figure : **Locke Updated**

Outline

Closure

Closure

Definition (Multi-Premise Closure)

If (i) A is rational in believing premises $\varphi_1 \ldots \varphi_n$,
(ii) $\varphi_1 \ldots \varphi_n \models \psi$,
(iii) A competently infers ψ from these premises,
then A's resulting belief in ψ is rational.

Counterexample to Closure

- φ_1 = *the house is empty*.
- φ_2 = *the house might not be empty*.

- Ari rationally believes φ_1, and she rationally believes φ_2.
- But she can't rationally believe $(\varphi_1 \wedge \varphi_2)$.

Counterexample to Bayesian Closure

Definition (Bayesian Closure)

If (i) A is rational, and
(ii) $\varphi_1 \ldots \varphi_n \models \psi$,
then A's uncertainty in ψ isn't greater than her uncertainty in φ_1 + her uncertainty in φ_2, ..., + her uncertainty in φ_n.

(Adams 1966; Edgington 1997; Sturgeon 2008)

Counterexample to Bayesian Closure

Definition (Bayesian Closure)

If (i) A is rational, and
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(Adams 1966; Edgington 1997; Sturgeon 2008)

- Ari's degree of uncertainty in φ_1 (*the house is empty*) $= .2$.
- Ari's degree of uncertainty in φ_2 (*the house might not be empty*) $= 0$.
- Ari's degree of uncertainty in $\varphi_1 \wedge \varphi_2 = 1$.

Restricting Closure?

Definition (Restricted MPC)

If (i) A is rational in believing descriptive premises $\varphi_1 \ldots \varphi_n$,
(ii) $\varphi_1 \ldots \varphi_n \models \psi$,
(iii) A competently infers a descriptive conclusion ψ from these premises,
then A's resulting belief in ψ is rational.

Restricting Closure?

- Of course, our semantics doesn't validate even Restricted MPC, since it incorporates a Lockean view of belief.

Restricting Closure?

- Of course, our semantics doesn't validate even Restricted MPC, since it incorporates a Lockean view of belief.
- However, there are various ways of trying to modify a Lockean view of belief to preserve closure.
- e.g., A "stability" theory of belief, according to which A believes φ iff A's credence in φ is sufficiently high when conditionalized on any proposition ψ that is compatible with φ and assigned some credence by A (Leitgeb 2014).

Restricting Closure?

- We could impose a similar stability condition on our semantics for *believes*:

Definition (Locke Stabilized)

$s[B_A\varphi] = \{w \in s | \; \forall\psi : \{\varphi, \psi\} \not\models \perp \; \& \; Pr_A^w(\llbracket\psi\rrbracket) > 0, \; Pr_A^w(s_A^w[\varphi] \mid \llbracket\psi\rrbracket) > t\}.$

- This validates Restricted MPC, but not unrestricted MPC.

Conclusion

- Thanks!

References

Ernest Adams. Probability and the logic of conditionals. In Hintikka and Suppes, editors, *Aspects of Inductive Logic*, pages 165–316. North-Holland, Amsterdam, 1966.

Pranav Anand and Valentine Hacquard. Epistemics and attitudes. *Semantics and Pragmatics*, 6:1–59, 2013.

Alexander Bird. Justified judging. *Philosophy and Phenomenological Research*, 74:81–110, 2007.

Keith DeRose. Epistemic possibility. *Philosophical Review*, 100:581–605, 1991.

Cian Dorr and John Hawthorne. Embedding epistemic modals. *Mind*, 488(122):867–913, 2013.

Dorothy Edgington. Vagueness by degrees. In Keefe and Smith, editors, *Vagueness: A Reader*. MIT Press, Cambridge, MA, 1997.

Andy Egan and Brian Weatherson. Epistemic modals and epistemic modality. In Egan and Weatherson, editors, *Epistemic Modality*. Oxford University Press, Oxford, 2011.

Ian Hacking. Possibility. *Philosophical Review*, 76:143–168, 1967.

Michael Huemer. Moore's paradox and the norm of belief. In Nuccetelli and Seay, editors, *Themes from G.E. Moore: New Essays in Epistemology and Ethics*, volume 74, pages 142–157. Clarendon Press, Oxford, 2007.

Angelika Kratzer. The notional category of modality. In Eikmeyer and Rieser, editors, *Words, Worlds, and Contexts: New Approaches in Word Semantics*. W. de Gruyter, Berlin, 1981.

Angelika Kratzer. Modality. In von Stechow and Wunderlich, editors, *Semantics: An International Handbook of Contemporary Research*. W. de Gruyter, Berlin, 1991.

Angelika Kratzer. *Modals and Conditionals*. Oxford University Press, Oxford, 2012.

Hannes Leitgeb. The stability theory of belief. *Philosophical Review*, 123(3):131–171, 2014.

Declan Smithies. The normative role of knowledge. *Noûs*, 46 (2):265–288, 2012.

Jason Stanley. Fallibilism and concessive knowledge attributions. *Analysis*, 65(2):126–131, 2005.

Tamina Stephenson. Judge dependence, epistemic modals, and predicates of personal taste. *Linguistics and Philosophy*, 30(4):487–525, 2007.

Scott Sturgeon. Reason and the grain of belief. *Noûs*, 42(1): 359–396, 2008.

Jonathan Sutton. *Without Justification*. MIT Press, Cambridge, MA, 2007.

Timothy Williamson. *Knowledge and its Limits*. Oxford University Press, Oxford, 2000.

Seth Yalcin. Epistemic modals. *Mind*, 116(464):983–1026, 2007.

Probably

- Let $\triangle_n \varphi$ represent the claim φ *is at least n% likely*. Let t be the Lockean threshold.
- They say: $C_A\varphi \equiv B_A\varphi \equiv B_A\Box\varphi$.
- We say: $B_A\varphi \equiv B_A\triangle_t\varphi; C_A\varphi \equiv B_A\Box\varphi$.

Probably

- Yalcin 2012:

Definition (Probabilistic Contexts)

Let $i = \langle s_i, Pr_i \rangle$ be a pair of a set of worlds s_i and a probability function Pr_i, where for any non-absurd context, i $Pr_i(s_i) = 1$. Let i_A^w be A's information state at w $(\langle s_A^w, Pr_A^w \rangle)$.

Definition (Trivial and Absurd Contexts)

Let $\mathbf{1}$ and $\mathbf{0}$ denote the trivial and absurd contexts, respectively:

$\mathbf{1} = \langle W, Pr_W \rangle$, where W is the set of all possible worlds.

$\mathbf{0} = \langle \emptyset, Pr \rangle$, for any probability function Pr.

Probably

Definition (Probabilistic Update Semantics)

1. $i[\alpha] = \langle s_i \cap \{w : w(\alpha) = 1\}, Pr_i(\cdot | \{w : w(\alpha) = 1\} \rangle$
2. $i[\varphi \wedge \psi] = i[\varphi][\psi]$
3. $i[\neg\varphi] = \langle s_i - s_{i[\varphi]}, Pr(\cdot | s_i - s_{i[\varphi]}) \rangle$
4. $i[\Diamond\varphi] = \langle \{w \in s_i : i[\varphi] \neq \mathbf{0}\}, Pr_i \rangle$

Probably

Definition (*Probably, n% likely*)

1. $i[\triangle\varphi] = \langle\{w : Pr_i(s_{i[\varphi]}) > .5\}, Pr_i\rangle$
2. $i[\triangle_n\varphi] = \langle\{w : Pr_i(s_{i[\varphi]}) > n\}, Pr_i\rangle$.

Probably

- Extending with *believes*:

Definition (Locke Reupdated)

$i[B_A\varphi] = \langle s_i \cap \mathscr{B}, Pr_i(\cdot|\mathscr{B}) > t\rangle$ where $\mathscr{B} = \{w : Pr_{i_A^w}(s_{i_A^w[\varphi]}) > t\}$.

Fact (Belief-Probability Link)

$B_A\varphi \equiv B_A\triangle_t\varphi$.

Definition (Locke Simplified)

$i[B_A\varphi] = \langle s_i \cap \mathscr{B}, Pr_i(\cdot|\mathscr{B}) > t\rangle$ where $\mathscr{B} = \{w : i_A^w \models \triangle_t\varphi\}$.

Epistemic Modesty

(21) ? Ari believes the house is empty. She also believes it might not be.

Epistemic Modesty

(21) ? Ari believes the house is empty. She also believes it might not be.

- **No Modesty**: It's incoherent for A to believe φ and believe $\Diamond\neg\varphi$.

Problems for **No Modesty**

- **No Modesty**, **Uncertain Belief**, and **Uncertainty-Possibility Link** $\Longrightarrow \bot$.
- (21) is not as bad as (1). **No Modesty** doesn't explain the felicity *difference*.
- Variants of (21) are ok:

(22) ✓ Ari believes the house is empty. But she realizes that it might not be.

- *concessive belief attributions* are ok:

(23) ✓ I believe the movie starts at 7, but it might start later.

Three Grades of Modal Infelicity

(24) a. # A believes $(\varphi \wedge \Diamond\neg\varphi)$.

b. ? A believes φ. A also believes $\Diamond\neg\varphi$.

c. ✓ A believes φ. But A realizes $\Diamond\neg\varphi$.

d. ✓ I believe φ. But $\Diamond\neg\varphi$.

- One hypothesis: modal subordination.

Order Sensitivity

(25) # Ari believes the house might not be empty and (it) is empty.

- To predict that (25) is bad, we could modify (Update Semantics) by endorsing the 'Consecutive Idempotence' Norm from Yalcin 2015.
- This says roughly that $s[\varphi] = \emptyset$ if any constituent ψ of φ is such that $s[\psi][\psi] \neq s[\psi]$.
- $\Diamond\varphi \wedge \neg\varphi$ is such a constituent.