The Puzzle

Ari believes the house is empty and might not be empty.
The Puzzle

(1) ?? Ari believes the house is empty and might not be empty.
The Puzzle

(1) ?? Ari believes the house is empty and might not be empty.

- Relevant reading: Ari bel [empty ∧ ◊¬empty]
Uncertain Belief: It’s possible to coherently believe $\varphi$ without being certain that $\varphi$. 
The Puzzle

- **Uncertain Belief**: It’s possible to coherently believe \( \varphi \) without being certain that \( \varphi \).

- **Uncertainty-Possibility Link**: If an agent A is coherent, then if A isn’t certain that \( \varphi \), A believes \( \lozenge \neg \varphi \).
The Puzzle

- **Uncertain Belief**: It’s possible to coherently believe $\varphi$ without being certain that $\varphi$.

- **Uncertainty-Possibility Link**: If an agent $A$ is coherent, then if $A$ isn’t certain that $\varphi$, $A$ believes $\Diamond \neg \varphi$.

- **No Contradictions**: It’s incoherent to believe $(\varphi \land \Diamond \neg \varphi)$.
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Uncertain Belief: It’s possible to coherently believe $\varphi$ without being certain that $\varphi$. 
Uncertain Belief: It’s possible to coherently believe $\phi$ without being certain that $\phi$.

(2) ✓ I believe the movie starts at 7, but I’m not certain.
(3) # I’m certain that the movie starts at 7, but I’m not certain.
(4) ✓ Ari believes that the house is empty, but she’s not certain of it.
Uncertainty-Possibility Link: If an agent A is coherent, then if A isn’t certain that $\varphi$, A believes $\Diamond \neg \varphi$. 
**Uncertainty-Possibility Link:** If an agent $A$ is coherent, then if $A$ isn’t certain that $\varphi$, $A$ believes $\Diamond \neg \varphi$.

(5) ?? I’m not certain the house is empty. But there’s no possibility that it isn’t.
• **Uncertainty-Possibility Link:** If an agent $A$ is coherent, then if $A$ isn’t certain that $\varphi$, $A$ believes $\Diamond \neg \varphi$.

(5) ?? I’m not certain the house is empty. But there’s no possibility that it isn’t.

(6) ?? The detective isn’t certain whether the butler did it. But she thinks there’s no chance the butler didn’t do it.
No Contradictions: It’s incoherent to believe $(\varphi \land \Diamond \neg \varphi)$. 
No Contradictions: It’s incoherent to believe ($\varphi \land \lozenge \neg \varphi$).

(7) ?? Ari believes the house is empty and might not be.
**No Contradictions**: It’s incoherent to believe \((\varphi \land \Diamond \neg \varphi)\).

(7) ?? Ari believes the house is empty and might not be.
No Contradictions: It’s incoherent to believe \((\varphi \land \Diamond \neg \varphi)\).

(7) ?? Ari believes the house is empty and might not be.
(8) ?? Joe thinks it’s raining and might not be.
(9) ?? The detective believes the butler is guilty and might be innocent.
A more general phenomenon:
A more general phenomenon:

(10) ?? It’s raining and it might not be.
A more general phenomenon:

(10) ?? It’s raining and it might not be.

(11) ?? Suppose/imagine that it’s raining and might not be.

(Yalcin 2007; Anand and Hacquard 2013; Dorr and Hawthorne 2013)
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Definition (Contextualism)

$$[\Diamond \varphi]_{c,w}^c = 1 \text{ iff } B_{c,w} \cap [\varphi]^c \neq \emptyset.$$ 

- $B_{c,w} =$ the $c$-determined modal base
- e.g., $[\text{The house might not be empty}]^c_{c,w} = 1$ iff $B_{c,w} \cap [\text{The house isn’t empty}]^c \neq \emptyset$

Contextualism

- What’s the epistemic modal base?
What’s the epistemic modal base?

(i) Knowledge

(ii) Belief
The Knowledge-Based Approach

- The epistemic modal base = the possibilities compatible with what the relevant agents know (or are in a position to know)
  

- i.e., *Might* $\phi$ is true iff $\phi$ is compatible with what the relevant folks know.
The Knowledge-Based Approach

- The epistemic modal base = the possibilities compatible with what the relevant agents know (or are in a position to know)


- i.e., Might $\varphi$ is true iff $\varphi$ is compatible with what the relevant folks know.

- Con: Has trouble validating No Contradictions.
Believing \((\varphi \land \Diamond \neg \varphi)\) =

Believing \((\varphi \land (\neg \varphi \text{ is compatible with what the relevant agents know}))\)
Believing $(\varphi \land \Box \neg \varphi) =$

Believing $(\varphi \land (\neg \varphi \text{ is compatible with what the relevant agents know}))$

Nothing incoherent about believing $\varphi$, and believing that one’s belief in $\varphi$ doesn’t amount to knowledge.
Possible reply:

Knowledge norm of belief

(Williamson 2000; Sutton 2007; Bird 2007; Huemer 2007; Smithies 2012)
(12) ✓ Thelma believes God exists, and that she doesn’t know God exists.

(13) ✓ Louise believes her ticket will lose, and that she doesn’t know whether her ticket will lose.
The Knowledge-Based Approach

(12) ✓ Thelma believes God exists, and that she doesn’t know God exists.

(13) ✓ Louise believes her ticket will lose, and that she doesn’t know whether her ticket will lose.

(14) ?? Thelma believes God exists and might not exist.

(15) ?? Louise believes her ticket will lose and might win.
The epistemic modal base = the possibilities compatible with what the relevant agents believe
The Belief-Based Approach

- The epistemic modal base = the possibilities compatible with what the relevant agents believe

- **Pro:** Enables us to validate **No Contradictions**.
  - Believing an epistemic contradiction \( \Rightarrow \) having a Moore-paradoxical belief (\( \varphi \land \text{I don’t believe } \varphi \))
The Belief-Based Approach

- **Con:** Forces us to give up either Uncertainty-Possibility Link or Uncertain Belief.
The Belief-Based Approach

- **Con:** Forces us to give up either **Uncertainty-Possibility Link** or **Uncertain Belief**.

- On the belief-based approach, Ari is committed to believing: 
  \((\text{The house is empty and I don’t believe the house is empty})\).
(16) ?? Suppose it’s raining and it might not be raining.

(17) ✓ Suppose it’s raining and I don’t know [/believe] it’s raining.

(18) ?? If it’s raining and it might not be raining, then...

(19) ✓ If it’s raining and I don’t know [/believe] it’s raining, then...
The meaning of $\varphi$ is not $\llbracket \varphi \rrbracket$, the set of worlds where $\varphi$ is true.

The meaning of $\varphi$ is $[\varphi]$, a context change potential.
Update Semantics

Definition (Contexts)
s is a set of possible worlds.

Definition (Update Semantics)
1. $s[\alpha] = s \cap \{w : w(\alpha) = 1\}$
2. $s[\varphi \land \psi] = s[\varphi][\psi]$
3. $s[\neg \varphi] = s - s[\varphi]$
4. $s[\Diamond \varphi] = \{w \in s : s[\varphi] \neq \emptyset\}$. 
Update Semantics

Definition (Support)

s supports $\varphi$ ($s \models \varphi$) iff $s[\varphi] = s$.

Definition (Validity)

$\varphi$ is valid ($\models \varphi$) just in case for every $s$, $s \models \varphi$. 
Update Semantics

Definition (Belief as Support)
\[ s[B_A \varphi] = \{ w \in s | s^w_A \models \varphi \}. \]
- where \( s^w_A \) is the set of worlds compatible with A’s beliefs at \( w \).

Definition (Certainty as Support)
\[ s[C_A \varphi] = \{ w \in s | c^w_A \models \varphi \}. \]
- where \( c^w_A \) is the set of worlds compatible with A’s certainties at \( w \).
Fact (No Contradictions)

\[ \models \neg B_A (\varphi \land \Diamond \neg \varphi). \]
Update Semantics

Figure: Updating with $\varphi \land \Diamond \neg \varphi$
Update Semantics

Problem: either Uncertain Belief or Uncertainty-Possibility Link is invalid.

\[ s^w_A = c^w_A \]

\[ s^w_A \neq c^w_A \]

- Uncertainty-Possibility Link ✓
- Uncertain Belief ✗
- Uncertainty-Possibility Link ✗
- Uncertain Belief ✓
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Basic idea:

*an agent believes $\phi$ iff they assign a sufficiently high degree of confidence to the result of adding $\phi$ to their current information*

Combines a test semantics for epistemic modals with a “Lockean”/threshold view of belief
A New Semantics for Belief Reports

- Let $s^w_A = c^w_A$ = the set of worlds compatible with A’s certainties at $w$ (call this ‘A’s information state at $w$’).
- Let $\Pr^w_A$ be A’s credence function at $w$.
- We will hold fixed Update Semantics and Certainty as Support

**Definition (Background: Update Semantics)**

1. $s[\alpha] = s \cap \{w : \omega(\alpha) = 1\}$
2. $s[\varphi \land \psi] = s[\varphi][\psi]$
3. $s[\neg \varphi] = s - s[\varphi]$
4. $s[\Diamond \varphi] = \{w \in s : s[\varphi] \neq \emptyset\}$
5. $s[C_A \varphi] = \{w \in s | s^w_A \models \varphi\}$.
A New Semantics for Belief Reports

- the old version:

**Definition (Lockean belief)**

\[ [B_A \varphi]^w = 1 \text{ iff } Pr_A^w ([\varphi]) > t. \]
Definition (Locke Updated)

\[ s[B_A \varphi] = \{w \in s \mid Pr^w_A(s^w_A[\varphi]) > t\}. \]

- **Step 1:** update A’s info state at w with \( \varphi \), giving us: \( s^w_A[\varphi] \).
- **Step 2:** Plug this set of worlds (\( s^w_A[\varphi] \)) into A’s credence function \( Pr^w_A \).
Locke Updated agrees with Lockean Belief when it comes to descriptive (non-modal) beliefs:

**Fact (Descriptive Beliefs Are Lockean)**

For any descriptive sentence \( \varphi \): \( s[B_A \varphi] = \{ w \in s \mid Pr_w^A(\llbracket \varphi \rrbracket) > t \} \).
Validating **Uncertain Belief**

- Together with **Certainty as Support**, this entails **Uncertain Belief**.
Validating **Uncertain Belief**

- Ari’s info state = \{w, u, v\}
  - \{w, u\} ⊆ \{w*: the house is empty at w*\}
  - v ∈ \{w*: someone’s inside the house at w*\}

- Ari’s credence in \{w, u\} = .8
- t = .75

- Ari believes the house is empty.
Ari’s info state = \{w, u, v\}

- \{w, u\} \subseteq \{w^*: \text{the house is empty at } w^*\}
- v \in \{w^*: \text{someone's inside the house at } w^*\}

Ari’s credence in \{w, u\} = .8

- t = .75

Ari believes the house is empty.

= true, since Ari’s credence in \{w, u\} > t
Fact (*Might* Beliefs Are Transparent)

For any descriptive sentence $\varphi$: $s[B_A \diamond \varphi] = \{w \in s | s^w_A[\varphi] \neq \emptyset\}$. 
Fact 2 + Certainty as Support ⇒ Uncertainty-Possibility Link

If Ari isn’t certain the house is empty, her info state contains at least one not-empty world (v).

So, by Fact 2, Ari believes the house might not be empty.
Fact (No Contradictions)

\[ \models \neg B_A (\varphi \land \Diamond \neg \varphi). \]
(20) ?? Ari believes the house is empty and might not be.

- **Step 1**: Update Ari’s info state with *the house is empty*
  - \{w, u, v\} $\rightarrow$ \{w, u\}

- **Step 2**: Update Ari’s info state with *the house might not be empty*
  - \{w, u\} $\rightarrow$ $\emptyset$

- **Step 3**: Check whether Ari’s credence in this set $> t$
Believing Epistemic Contradictions

Figure: Locke Updated
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Closure

Definition (Multi-Premise Closure)

If (i) A is rational in believing premises $\phi_1 \ldots \phi_n$, (ii) $\phi_1 \ldots \phi_n \models \psi$, (iii) A competently infers $\psi$ from these premises, then A's resulting belief in $\psi$ is rational.
Definition (Multi-Premise Closure)

If (i) A is rational in believing premises $\varphi_1 \ldots \varphi_n$,  
(ii) $\varphi_1 \ldots \varphi_n \models \psi$,  
(iii) A competently infers $\psi$ from these premises,  
then A’s resulting belief in $\psi$ is rational.
Counterexample to Closure

- $\varphi_1 = \text{the house is empty.}$
- $\varphi_2 = \text{the house might not be empty.}$

Ari rationally believes $\varphi_1$, and she rationally believes $\varphi_2$.

But she can’t rationally believe $(\varphi_1 \land \varphi_2)$. 
Definition (Bayesian Closure)

If (i) A is rational, and
(ii) \( \varphi_1 \ldots \varphi_n \models \psi \),
then A’s uncertainty in \( \psi \) isn’t greater than her uncertainty in \( \varphi_1 \) + her uncertainty in \( \varphi_2 \), ..., + her uncertainty in \( \varphi_n \).

(Adams 1966; Edgington 1997; Sturgeon 2008)
Counterexample to Bayesian Closure

Definition (Bayesian Closure)

If (i) A is rational, and
(ii) $\varphi_1 \ldots \varphi_n \models \psi$,
then A’s uncertainty in $\psi$ isn’t greater than her uncertainty in $\varphi_1 +$ her uncertainty in $\varphi_2$, ..., + her uncertainty in $\varphi_n$.

(Adams 1966; Edgington 1997; Sturgeon 2008)

- Ari’s degree of uncertainty in $\varphi_1$ (the house is empty) = .2.
- Ari’s degree of uncertainty in $\varphi_2$ (the house might not be empty) = 0.
- Ari’s degree of uncertainty in $\varphi_1 \land \varphi_2 = 1$. 
Restricting Closure?

Definition (Restricted MPC)

If (i) A is rational in believing descriptive premises $\varphi_1 \ldots \varphi_n$, (ii) $\varphi_1 \ldots \varphi_n \models \psi$, (iii) A competently infers a descriptive conclusion $\psi$ from these premises, then A’s resulting belief in $\psi$ is rational.
Of course, our semantics doesn’t validate even Restricted MPC, since it incorporates a Lockean view of belief.
Of course, our semantics doesn’t validate even Restricted MPC, since it incorporates a Lockean view of belief.

However, there are various ways of trying to modify a Lockean view of belief to preserve closure.

e.g., A “stability” theory of belief, according to which A believes φ iff A’s credence in φ is sufficiently high when conditionalized on any proposition ψ that is compatible with φ and assigned some credence by A (Leitgeb 2014).
We could impose a similar stability condition on our semantics for *believes*:

**Definition (Locke Stabilized)**

\[ s[B_A \phi] = \{ w \in s \mid \forall \psi : \{ \phi, \psi \} \nmid \perp \text{ and } Pr^w_A(\llbracket \psi \rrbracket) > 0, \Pr^w_A(s^w_A[\phi] \mid \llbracket \psi \rrbracket) > t \}. \]

This validates Restricted MPC, but not unrestricted MPC.
Thanks!
References


Let $\triangle_n \varphi$ represent the claim \( \varphi \) is at least \( n\% \) likely. Let \( t \) be the Lockean threshold.

They say: \( C_A \varphi \equiv B_A \varphi \equiv B_A \Box \varphi \).

We say: \( B_A \varphi \equiv B_A \triangle_t \varphi \); \( C_A \varphi \equiv B_A \Box \varphi \).
Yalcin 2012:

**Definition (Probabilistic Contexts)**

Let \( i = \langle s_i, Pr_i \rangle \) be a pair of a set of worlds \( s_i \) and a probability function \( Pr_i \), where for any non-absurd context, \( i Pr_i(s_i) = 1 \). Let \( i^w_A \) be A’s information state at \( w \) (\( \langle s^w_A, Pr^w_A \rangle \)).

**Definition (Trivial and Absurd Contexts)**

Let \( 1 \) and \( 0 \) denote the trivial and absurd contexts, respectively:

- \( 1 = \langle W, Pr_W \rangle \), where \( W \) is the set of all possible worlds.
- \( 0 = \langle \emptyset, Pr \rangle \), for any probability function \( Pr \).
Definition (Probabilistic Update Semantics)

1. \( i[\alpha] = \langle s_i \cap \{ w : w(\alpha) = 1 \}, \Pr_i(\cdot | \{ w : w(\alpha) = 1 \}) \rangle \)
2. \( i[\varphi \land \psi] = i[\varphi][\psi] \)
3. \( i[\neg \varphi] = \langle s_i - s_i[\varphi], \Pr(\cdot | s_i - s_i[\varphi]) \rangle \)
4. \( i[\lozenge \varphi] = \langle \{ w \in s_i : i[\varphi] \neq 0 \}, \Pr_i \rangle \)
Definition (Probably, n% likely)

1. \( i[\triangle \varphi] = \langle \{ w : \Pr_i(s_i[\varphi]) > .5 \}, \Pr_i \rangle \)

2. \( i[\triangle_n \varphi] = \langle \{ w : \Pr_i(s_i[\varphi]) > n \}, \Pr_i \rangle \).
Extending with *believes*:

**Definition (Locke Reupdated)**

\[
i[B_A \varphi] = \langle s_i \cap B, Pr_i(\cdot|B) > t \rangle \quad \text{where} \quad B = \{ w : Pr_i^w(s_i^w[\varphi]) > t \}.
\]

**Fact (Belief-Probability Link)**

\[B_A \varphi \equiv B_A \triangle t \varphi.\]

**Definition (Locke Simplified)**

\[
i[B_A \varphi] = \langle s_i \cap B, Pr_i(\cdot|B) > t \rangle \quad \text{where} \quad B = \{ w : i_A^w \models \triangle t \varphi \}.
\]
(21) Ari believes the house is empty. She also believes it might not be.
(21) ? Ari believes the house is empty. She also believes it might not be.

- **No Modesty**: It’s incoherent for A to believe $\varphi$ and believe $\lozenge \neg \varphi$. 
Problems for No Modesty

- No Modesty, Uncertain Belief, and Uncertainty-Possibility Link $\Rightarrow \bot$.
- (21) is not as bad as (1). No Modesty doesn’t explain the felicity difference.
- Variants of (21) are ok:

  (22) ✓ Ari believes the house is empty. But she realizes that it might not be.

- concessive belief attributions are ok:

  (23) ✓ I believe the movie starts at 7, but it might start later.
Three Grades of Modal Infelicity

(24) a. \# A believes \((\varphi \land \lozenge \neg \varphi)\).

b. ? A believes \(\varphi\). A also believes \(\lozenge \neg \varphi\).

c. ✓ A believes \(\varphi\). But A realizes \(\lozenge \neg \varphi\).

d. ✓ I believe \(\varphi\). But \(\lozenge \neg \varphi\).

One hypothesis: modal subordination.
(25) # Ari believes the house might not be empty and (it) is empty.

- To predict that (25) is bad, we could modify (Update Semantics) by endorsing the ‘Consecutive Idempotence’ Norm from Yalcin 2015.
- This says roughly that $s[\varphi] = \emptyset$ if any constituent $\psi$ of $\varphi$ is such that $s[\psi][\psi] \neq s[\psi]$.
- $\diamond \varphi \land \neg \varphi$ is such a constituent.