BELIEVING EPISTEMIC CONTRADICTIONS

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1 The Puzzle

(1) ?? Ari believes the house is empty and might not be.

Uncertain Belief	It's possible to coherently believe ϕ without being certain that ϕ.
Uncertainty-Possibility Link	If an agent A is coherent, then if A isn't certain that ϕ, A believes $\Diamond\neg\phi$.
No Contradictions	It's incoherent to believe $(\phi \wedge \Diamond\neg\phi)$.

2 Our Proposal

Definition 1 (Contexts)**.** s is a set of possible worlds. Pr^w_A is A's credence function at w. s^w_A is the set of worlds compatible with A's certainties at w.

Definition 2 (Background: Update Semantics)**.**

1. $s[\alpha] = s \cap \{w : w(\alpha) = 1\}$
2. $s[\phi \wedge \psi] = s[\phi][\psi]$
3. $s[\neg\phi] = s - s[\phi]$
4. $s[\Diamond\phi] = \{w \in s|\ s[\phi] \neq \emptyset\}$. veltman (1996)
5. $s[C_A\phi] = \{w \in s|\ s^w_A \models \phi\}$. ≈ heim (1992)

Definition 3 (Locke Updated)**.** $s[B_A\phi] = \{w \in s|\ Pr^w_A(s^w_A[\phi]) > t\}$.

Definition 4 (Support)**.** s supports ϕ ($s \models \phi$) iff $s[\phi] = s$.

Definition 5 (Validity)**.** ϕ is valid ($\models \phi$) just in case for every s, $s \models \phi$.

Fact 1 (Descriptive Beliefs Are Lockean)**.** For any descriptive (non-modal) sentence ϕ: $s[B_A\phi] = \{w \in s|\ Pr^w_A([\![\phi]\!]) > t\}$.

Proof. By **Locke Updated**, $B_A\phi$ holds at a world w iff A's credence in $s^w_A[\phi]$ exceeds t. To find $s^w_A[\phi]$, we take the set of worlds in A's doxastic state at w (s^w_A) and update this set with ϕ. By **Update Semantics**, when ϕ is descriptive, this is simply the result of intersecting s^w_A with the ϕ worlds ($s^w_A \cap [\![\phi]\!]$). Since every agent assigns credence 1 to the set of worlds in her doxastic state, her credence in $[\![\phi]\!]$ will equal her credence in $s^w_A[\phi]$. □

- Validates **Uncertain Belief**

Fact 2 (*Might* Beliefs Are Transparent)**.** For any descriptive sentence ϕ: $s[B_A\Diamond\phi] = \{w \in s|\ s^w_A[\phi] \neq \emptyset\}$.

Proof. By **Locke Updated**, A believes $\Diamond\phi$ at w just in case she gives sufficiently high credence to $s^w_A[\Diamond\phi]$. By **Update Semantics**, $s^w_A[\Diamond\phi]$ is either s^w_A or $\emptyset$, depending on whether there is a ϕ world in s^w_A. If there is, then

$s^w_A[\Diamond\phi] = s^w_A$, to which A assigns credence 1. Otherwise, $s^w_A[\Diamond\phi] = \emptyset$, to which A assigns credence 0. And so A believes $\Diamond\phi$ just in case her doxastic state includes a ϕ world. □

- Validates **Uncertainty-Possibility Link**

Fact 3 (No Contradictions). $\models \neg B_A(\phi \wedge \Diamond\neg\phi)$.[1]

Proof. By **Locke Updated**, A believes $(\phi \wedge \Diamond\neg\phi)$ at w iff A assigns a sufficiently high credence to $s^w_A[\phi \wedge \Diamond\neg\phi]$. By **Update Semantics**, $s^w_A[\phi \wedge \Diamond\neg\phi] = s^w_A[\phi][\Diamond\neg\phi]$. Now $s^w_A[\phi][\Diamond\neg\phi] = \emptyset$ unless $s^w_A[\phi]$ contains at least one $\neg\phi$ world. But $s^w_A[\phi]$ contains only ϕ worlds. So $s^w_A[\phi \wedge \Diamond\neg\phi] = \emptyset$. Consequently, $Pr^w_A(s^w_A[\phi \wedge \Diamond\neg\phi]) = 0$. □

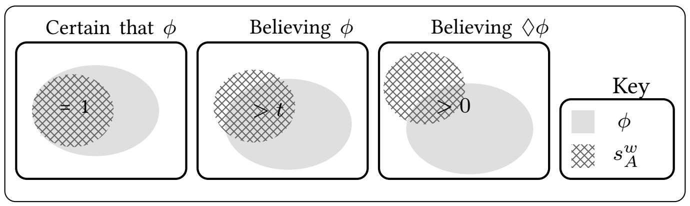

Figure 1: **Locke Updated**

3 Closure

Multi-Premise Closure If (i) A is rational in believing premises $\phi_1...\phi_n$, (ii) $\phi_1...\phi_n \models \psi$, (iii) A competently infers ψ from these premises, then A's resulting belief in ψ is rational.

- ϕ_1 = *the house is empty*; ϕ_2 = *the house might not be empty.*
- Ari rationally believes ϕ_1, and she rationally believes ϕ_2.
- But she can't rationally believe $(\phi_1 \wedge \phi_2)$.

[1]Supposing A is coherent: $s^w_A \neq \emptyset$.

Bayesian Closure If (i) A is rational, and (ii) $\phi_1...\phi_n \models \psi$, then A's uncertainty in ψ isn't greater than her uncertainty in ϕ_1 + her uncertainty in ϕ_2, ..., + her uncertainty in ϕ_n.

Restricted MPC If (i) A is rational in believing descriptive premises $\phi_1...\phi_n$, (ii) $\phi_1...\phi_n \models \psi$, (iii) A competently infers a descriptive conclusion ψ from these premises, then A's resulting belief in ψ is rational.

Definition 6 (Locke Stabilized)**.** $s[B_A\phi] = \{w \in s | \forall\psi : \{\phi, \psi\} \not\models \bot \ \& \ Pr^w_A([\![\psi]\!]) > 0,\ Pr^w_A(s^w_A[\phi] \mid [\![\psi]\!]) > t\}$.

- Validates **Restricted MPC**, but not **MPC**.